Chapter 6

General discussion and conclusion
Aims and Summary

The overall research question of this thesis was: “What is the ISI-CK of pre-service primary school teachers, how does their ISI-CK develop during an intervention of limited length, and what ISI-CK do they show during teaching ISI in primary school?” The first aim of this thesis was to provide a comprehensive overview of the beginning pre-service teachers’ ISI-CK. The second aim of this thesis was to understand how their ISI-CK can be fostered.

Chapter 2 describes the results of a large scale survey of beginning pre-service teachers’ knowledge of all ISI components. We conducted two interventional studies that both aimed to foster pre-service teachers’ ISI-CK. Chapter 3 reports about an exploratory study into the informal inferential reasoning of pre-service teachers from three classes engaged in a growing samples activity. The findings of Chapters 2 and 3 informed the design of the intervention in Chapter 4. During this short intervention, we investigated the ISI-CK development in a class of 21 pre-service teachers. Part of the intervention, the participants taught an ISI lesson in primary school. Chapter 5 describes the ISI-CK of three of the participating pre-service teachers while teaching this ISI lesson.

In this thesis, we use an extended version of the ISI framework of Makar and Rubin (2009). This framework consists of three components: “data as evidence”, “generalization beyond the data”, and “probabilistic language”. We subdivided the “data as evidence” component into two subcomponents, “using data” and “describing data”, and we subdivided the “probabilistic language” component into four subcomponents, “sampling variability”, “sampling method”, “sample size” and “uncertainty”.

In the following, we provide an overview of the main results from the four studies, organized by the components of ISI.

“Using data”: In all studies, the majority of the pre-service teachers used the data as evidence.

“Describing data” and “generalization beyond the data”: The survey and the growing samples activity showed that most pre-service teachers used suitable descriptive statistics to describe the data, while they tended not to generalize beyond the data. Therefore, in the design of the intervention, we put less emphasis on the “describing data” subcomponent and more emphasis on the
“generalization” component. The activities used during the intervention and in the lesson taught by the pre-service teachers indeed helped the pre-service teachers to be engaged in making generalizations beyond the data.

“Sampling variability”: In the survey and during the growing samples activity, a majority of the pre-service teachers did not understand why making inferences is possible. During the intervention, pre-service teachers developed their understanding of sampling variability. At the same time, the pre-service teachers who taught an ISI lesson had difficulty using their understanding of sampling variability when they had to explain primary school students why making inferences is possible.

“Sampling method”: A majority of the pre-service teachers agreed that random sampling is a valid sampling method, but at the same time distributed sampling (i.e. purposefully selecting individuals to obtain a distributed sample across critical population characteristics) was their preferred sampling strategy.

“Sample size”: Throughout the thesis, pre-service teachers seemed to overestimate how large a representative sample needs to be.

“Uncertainty”: Most pre-service teachers appeared to understand that a larger sample yields more certainty. However, the intervention showed that the participants seemed to lack tools to express the certainty of their inferences. This became problematic during teaching an ISI lesson, since without such tools they could not explain primary school students why making inferences is possible.

Discussion of Main Findings

Making generalizations beyond the data
The survey and the growing samples activity showed that a vast majority of the pre-service teachers, although asked to generalize beyond the sample data to a population, tended to restrict their attention to descriptive analyses. In the growing samples activity, only after being probed by the teacher educator, they engaged in inferential reasoning. The tendency to restrict attention to descriptive analyses has also been found in other studies among (pre-service) teachers and other types of learners (Leavy, 2010; Makar & Rubin, 2009; Pratt, Johnston-Wilder, Ainley, & Mason, 2008). These findings raise the question
why tasks that are meant to elicit generalizations, failed to do so, although they are designed in line with ISI design principles (Makar & Rubin, 2018; Zieffler, Garfield, delMas, & Reading, 2008). A first reason could be that the phase of descriptive analyses demands all time and attention, at the expense of the subsequent phase of interpretation of the results and inferential reasoning (Leavy, 2010). A second reason may be that the population is not visible, while the sample data are usually visible in the form of tables or graphs. The elements of the population thus remain “mere ghosts of data-in-the-future” (G. W. Cobb & Moore, 1997, p. 815).

In contrast to the survey and the growing samples activity and to previous research, the pre-service teachers involved in the intervention did make generalizations beyond the data. These different findings may be attributed to the activities used in the intervention. In the first activity, the participants were asked to search for a news item that made a claim about a population based on a sample. This could have created awareness regarding inference. During a subsequent lesson, called “What is the most frequently used word?”, the pre-service teachers conducted an in-class statistical investigation. The lesson centered on a large pile of children’s novels, and on the question of which word would be observed most frequently in the pile. To answer this question, the participants drew a sample of books from this pile, checked how often five most likely words occurred in a number of lines or pages, and determined the sample’s mode. The visibility of the population and the sample (the pile of books and the books sampled) may have helped to elicit generalizations. Moreover, since only the mode had to be determined, descriptive analyses may not have distracted the pre-service teachers from inferential reasoning.

In contrast to previous studies (Leavy, 2010; Makar & Rubin, 2009), when teaching the same activity in their placement school classrooms, the pre-service teachers were consciously making generalizations beyond data. We attribute these diverging results to the fact that in previous research (pre-service) teachers themselves designed lessons. In general, these lessons did not contain sufficient affordances to be engaged in ISI. In contrast, we provided a lesson to the pre-service teachers that contained sufficient affordances to engage in ISI. This finding suggests that providing pre-service teachers with a lesson plan that contains sufficient affordances may help pre-service teachers to engage in ISI when they introduce primary school students to ISI.
Explaining why making inferences is possible

The pre-service teachers who taught the lesson about the most frequently used word in primary school had difficulties to explain why making inferences is possible. This explanation proved to be critical part of their lessons because the pre-service teachers’ failure to provide a convincing explanation lead students to doubt, or even reject the possibility to infer from a sample.

The pre-service teachers’ difficulties in explaining why making inferences is possible, could be due to their inability to transfer what they had learned about this issue during the intervention, to the ISI lesson they taught in primary school. During the intervention, the teacher educator had illustrated sampling variability and the law of large numbers, and this illustration appeared to have helped the pre-service teachers to understand issue. However, this explanation was not provided in the context of the lesson “What is the most frequently used word?”, nor did not the lesson plan provide them with tools to explain why making inferences is possible. So, for a successful explanation to primary school students of the question why making is possible, it might not suffice that pre-service teachers understand this issue themselves. The lesson plan they use should contain tools that help them explain why making inferences is possible.

Examples of primary classrooms engaged in a discussion about why making inferences is possible have not been reported in the literature (Makar & Rubin, 2009; Paparistodemou & Meletiou-Mavrotheris, 2008). Two possible explanations could account for why in our study this question was discussed, while in previous research this question did not arise. A first reason may be that small groups of students collected samples that resulted in different most frequently used words. These different sample results have made the students question how one could use the results of one sample to draw a conclusion about the entire population. A second reason why this issue arose was that the lesson plan that the pre-service teachers used, suggested to discuss whether another sample would yield a different or similar result. Our findings thus suggest how the crucial question why making inferences is possible can be brought to attention in ISI lessons for primary school students.
Sampling methods and representativeness
The survey showed that many pre-service teachers favored distributed sampling over random sampling. This preference remained even after having received instructions about random sampling in the intervention. Although the preference for distributed sampling suggests that the pre-service teachers acknowledged that a sample should be representative of the population, the problem of this approach is that it does not consider the population distribution of the characteristics of which quota are sampled. In case of a non-uniform population distribution, this would result in a biased sample. For instance, pre-service teachers suggested to sample from each province the same number of persons, even though the population density differs between the provinces. This finding is in line with the findings of Schwartz, Goldman, Vye, and Barron (1998), who reported that fifth and sixth grade students tended to accept random sampling in chance contexts, but preferred distributed sampling in opinion research contexts. One explanation for their preference for distributed sampling might be that the pre-service teachers felt a loss of control when using random sampling. Another reason might be that they lacked an understanding of the workings of random sampling and distributed sampling (Chi, 2013). This result raises the question how ISI education for pre-service teachers can help them understand how random sampling can result in an unbiased sample. As solution may be to include an explicit comparison of the workings of distributed and random sampling in future interventions.

Methodological Reflections

ISI questionnaire
A main methodological contribution of this thesis to the ISI literature is the development of a questionnaire that covers all components of ISI. The design of the instrument was such that it can be used to describe the ISI-CK of a large sample of pre-service teachers. The complexity of this knowledge is captured by a combination of open-ended questions and true/false statements. The open-ended questions allow respondents to give their own answers without steering them into a particular direction, while the true/false statements allow to probe for information that pre-service teachers may not provide in the open-ended
questions. In our study, the large sample helped us to gain a representative picture of beginning Dutch pre-service teachers’ ISI-CK.

We found some evidence for the reliability of the instrument. First, there were very few inconsistencies between the open-ended responses and the true/false statements. Second, the cognitive interviews used in the design phase of the questionnaire showed that the pilot respondents interpreted the statements as intended. After further study of the instrument’s reliability, the instrument can be put to good use to efficiently gain a picture of pre-service teachers’ ISI-CK.

**Practice-based research**

We aimed to design an intervention that could be implemented in mathematics education curricula of Dutch teacher colleges. This was achieved by setting strict limitations on intervention time and by working with classes of regular pre-service teachers who had limited pre-existing knowledge of ISI. The activities used in the intervention can, therefore, be implemented with relative ease into regular mathematics education curricula of Dutch primary education teacher colleges.

Most of the limited intervention time could be spent on fostering the pre-service teachers’ ISI-CK because we provided the pre-service teachers with a lesson plan that they could use when teaching ISI in primary school. Nonetheless, the short length of the intervention limited the opportunities to familiarize the pre-service teachers with ISI. The intervention, for example, allowed the pre-service teachers to have only a few experiences in sampling variability simulations, and for teaching an ISI lesson once. A deeper understanding ISI may be required, also because the inquiry-based teaching approach of the ISI education puts high demands on their ISI-CK (Groth & Meletiou-Mavrotheris, 2018). This kind of teaching requires to respond adequately to students’ suggestions and questions, to steer the discussion and to provide convincing explanations to unforeseen questions. These teaching skills are among the most difficult to conduct well (Rowland & Turner, 2007). While longer interventions are expected to be more effective in terms of learning (Slavin, 2008), our research shows it is possible to engage pre-service teachers in ISI and have them teach an ISI lesson. If the goal of teacher education is to foster their position as life-long learners, also a short
intervention might stimulate pre-service teachers to become more proficient in teaching ISI after the intervention (Mickelson & Heaton, 2004).

Agency
The agency of the pre-service teachers played an important role in the design of our research. In the introduction of this thesis, we described how we tried to foster the pre-service teachers’ agency. For example, we told them about the societal relevance of ISI and stressed their role as research informants. Overall, we were successful in fostering the agency of most pre-service teachers. The pre-service teachers were willing to participate in the activities, and provided us with valuable feedback. They seemed to acknowledge the relevance of ISI. For example, one pre-service teacher remarked during the evaluation session of the intervention that the phrase “How about representativeness?” had become a catchphrase for the class and was used in all kinds of settings outside the intervention. Also, a couple of months later, one of the participants told that the intervention had made her watch the news with different eyes.

The pre-service teachers’ agency was also seen in the way the pre-service teachers prepared the ISI lesson they taught in primary school. Although the lesson plan was structured and prescriptive in the suggested activities, these pre-service teachers tailored the lesson plan to their classes’ needs. For instance, various pre-service teachers added giving a definition of a sample to the lesson plan, because they found it important to teach their students relevant jargon. As a second example, one of the pre-service teachers who taught an ISI lesson in primary school showed that she had reflected how to best teach ISI. For example, during the reflection interview, she criticized how this same lesson was modelled during the teacher college sessions. To her, the modelled lesson did not provide a clear answer to the question why making inferences is possible. In preparing her own lesson, she had been reading the lesson plan over and over again to find such an answer to this question. These examples show that at least some pre-service teachers were not merely executing the prescribed lesson plan, but acted as agentic teachers in preparing and teaching the ISI lesson.

The pre-service teachers accepted the challenge to participate in this “adventurous teaching” project (Cohen, 1988), even though many expressed at least some learner and teacher angst (Heaton & Mickelson, 2002). This
combination of anxiety and willingness is nicely summarized in Demi’s remark after having taught an ISI lesson:

I found it interesting indeed. Anyway, I find it difficult. [...] Still, it feels, felt like a challenge. I think I mastered the subject just well enough to teach this lesson, really just well enough. But, that is also because I find it difficult myself, but that’s why I find it interesting to teach this lesson because it’s like: I can take a nosedive.

The way we dealt with the pre-service teachers’ agency in an intervention that introduced new curriculum matter might be of relevance for those involved in the challenge to engage (pre-service) teachers in innovative teaching approaches or in teaching new curriculum material (see for example Sloot, 2018).

Reliability and validity of the findings

We tried to warrant the reliability and internal validity of the results in the following ways. First, because I combined the roles of researcher and teacher educator, it was important to create distance between both roles. To this end, I adopted a strategy to change roles (Ball, 2000). For example, before entering the classrooms where I would give the sessions reported in the intervention, I tried to mentally switch roles. I told myself that my job as researcher was done for the moment. As a teacher educator, I would teach the lesson designed provided to me by my researcher self as good as possible, but I would also attend to the pre-service teachers’ (emotional) needs. Second, in all studies, we used a second researcher to check the coding and analysis process and to validate whether the conclusions were warranted by the data. Third, both during and afterwards the intervention, the sessions and the pre-service teachers’ ISI-CK development were evaluated with the participating pre-service teachers. Finally, reflection interviews were held with the pre-service teachers whose ISI lessons we observed; their reflections on the lessons were used to triangulate our interpretations.

In terms of the generalizability of our research, it has to be born in mind that our studies were set in the Dutch context, where students enter teacher college immediately after secondary education and where completion leads to the attainment of vocational bachelor degree. Although similar processes may occur in countries where students enter teacher college with similar
backgrounds and with similar statistics curricula in primary and secondary education, our results are dependent on the specific context and the pre-service teachers’ ISI-CK is contingent on the specific tasks used in this research.

**Implications for Theory and Research**

In the following, we highlight three ideas for further investigation. First, a natural progression would be to validate the findings of our research in other settings and to conduct effect studies to study the learning effects of the intervention. Before such studies could be productively conducted, the intervention requires further development. Pre-service teachers should be allowed to develop sufficient understanding of sampling variability to enable them to explain to primary school students why making inferences is possible. In order to achieve this, it could be investigated how pre-service teachers can have multiple experiences in sampling variability, by means of simulation activities (Arnold, Pfannkuch, Wild, Regan, & Budgett, 2011) and hands-on activities (Zapata-Cardona, 2015), within a limited frame.

Second, the task to teach an ISI lesson in primary school is a complex one for pre-service teachers with limited experience in ISI. Therefore, it may be worthwhile to explore whether making the progression of ISI lessons more predictable could support the pre-service teachers in teaching. For example, the pre-service teachers in our study asked students an open question to discuss the sampling method and sample size. As an alternative, the pre-service teachers could offer the students a limited number of options to choose from. This could make the lesson more predictable and less complex to teach.

Third, we hypothesize that the first times that primary school students are engaged in making inferences as part of in-class statistical investigations, the focus should be on the question why making inferences is possible. This would help students to understand the core idea of ISI: Making probabilistic generalizations based on sample data (Makar, Bakker, & Ben-Zvi, 2011). During such an introduction, restricting attention to other aspects of ISI, such as discussing the sampling method and sample size to be used and attention for descriptive analyses, may help pre-service teachers and students
concentrate on inferential reasoning. This would be a fruitful area for future research.

Implications for Educational Practice

ISI design heuristics for primary education teacher colleges
Based on our results, we have formulated eight tentative design heuristics for ISI education at the primary education teacher college. These design heuristics fit into design heuristics for statistics education in general (P. Cobb & McClain, 2004) and align with an inquiry-based approach to teaching ISI. The eight design heuristics are:

1. Create awareness of inferential claims and of the distinction between sample and population, before engaging pre-service teachers in ISI via statistical investigations (Chapters 2, 3 and 4).
2. Use tangible samples and populations when conducting ISI investigations with pre-service teachers (Chapter 4).
3. Use data that require little descriptive analyses, for example, when the parameter of interest is the mode of a nominal variable when conducting ISI investigations with pre-service teachers (Chapter 4).
4. Use a context for the ISI investigation, on which outcome pre-service teachers have no clear expectations, or which outcome challenges pre-service teachers’ current beliefs (Chapter 4; Makar et al., 2011).
5. Discuss why making inferences is possible using growing samples (Bakker, 2004) and repeated sampling activities (Saldanha & Thompson, 2002). If time allows, use such activities multiple times in different contexts, and give pre-service teachers the opportunity to conduct simulations (Chapters 3 and 4).
6. Explain how random sampling can yield unbiased samples and why distributed sampling might lead to biased samples, by building on pre-service teachers’ intuitive notions of representativeness (Chapters 2 and 4).
7. Provide pre-service teachers who introduce primary school students to ISI with a well-structured lesson plan that yields sufficient affordances
to engage students in ISI. The lesson “What is the most frequently used word?” could serve this purpose (Chapter 5).

8. Provide pre-service teachers who introduce primary school students to ISI with a lesson plan that contains a tool that helps them to explain why making inferences is possible (Chapter 5). We offer such a tool in the revised version of the lesson “What is the most frequently used word?”, described in the next section.

“What is the most frequently used word? – revisited”

The lesson “What is the most frequently used word?” was useful to engage pre-service teachers and primary school students in ISI. However, the lesson did not enable pre-service teachers to explain to primary school students why making inferences is possible. Therefore, we suggest some adjustments to the lesson that allow pre-service teachers to explain this issue. The new version of the lesson combines a growing samples (Bakker & Gravemeijer, 2004) and repeated sampling approach (Saldanha & Thompson, 2002), and allows for a quantification of the confidence level. If this confidence level is sufficiently high, primary school students may accept it is possible to make inferences based on a sample.

If students first draw small samples individually and then pool their results in multiple large samples, it could be shown that the proportion that yield the same most frequently used word is greater for large samples than for small samples. If the proportion of samples with the same most frequently used word is high, students could be willing to accept the possibility of making uncertain inferences based on one sample.

Incorporating this tool in the lesson “What is the most frequently used word?”, makes this tool readily available to explain to primary school students why making inferences is possible.

Conclusion

Our studies show promising results of pre-service teachers developing their ISI-CK within a limited time frame. They were able to make generalizations
about a population based on sample data when teaching an ISI lesson. Moreover, most of them showed the agency to learn and to teach ISI.

We are optimistic about the opportunities for the implementation of ISI education in Dutch mathematics teacher education in the future. First, the secondary education mathematics course has recently been revised and now puts more emphasis of learning statistics in the context of statistical investigations and pays attention to ISI, including the discussion of the law of large numbers. The majority of Dutch pre-service teachers takes this mathematics course in secondary school and future Dutch pre-service teachers may thus enter teacher college better prepared than the pre-service teachers who participated in our studies. Second, the mathematics education curriculum for primary education teacher colleges has recently been revised. Although ISI is not included in the obligatory learning objectives for primary school teachers, it now mentions the growing societal relevance of statistics and includes ISI as an example (Boersma, Keijzer, & Wösten, 2018). If in the future ISI would indeed become part of teacher college curricula, this could create the institutional prerequisites for introducing ISI to pre-service teachers. In that case, the insights from this thesis about the learning and teaching of ISI can be put to fruitful use in the implementation of ISI education in primary education teacher colleges, and ultimately help to prepare primary school students to participate in society.
References


Sloot, E. (2018, January 22). Geen uitgever voor lessen die niet om geld draaien. [No publisher for lessons that are not all about money]. Nederlands Dagblad, p. 3.

