Analysis and Refinement of a Temporal-Causal Network Model for Absorption of Emotions[1]

2

> *"Without the aid of trained emotions the intellect is powerless against the animal organism."*
>
> — **C.S. Lewis**
> "The Abolition of Man"[2]

Abstract

An earlier proposed temporal-causal network model for mutual absorption of emotions aims to model emotion contagion in networks using characteristics such as traits of openness and expressiveness of the members of the network, and the strengths of the connections between them. The speed factors describing how fast emotional states change, were modeled based on these characteristics according to a fixed dependence relation. In this paper, particular implications of this choice are analyzed. Based on this analysis, a refinement of the model is proposed, offering alternative ways of modeling speed factors. This refinement is also analyzed and evaluated.

2.1 Introduction

The social phenomenon called emotion contagion indicates the process by which emotions of a person are affected by emotions of other persons when they are interacting in a social network. This concept has a foundation in neurological findings on mirror neurons [6], and can be used to understand emotions, for example in situations where decisions can be affected by the emotional state of a person. This can occur in urgent situations, when events with a short duration can create disturbances in decisions, but also in processes with longer durations, like mood and depression, commitment with work, et cetera.

Different computational models have been proposed to model emotion contagion. Among them are temporal-causal network models [8] such as the absorption model, introduced in [4] and the amplification model introduced in [3]. The current paper focuses on the absorption model. In this model emotion contagion is modeled using personal characteristics (or traits) such as openness (how a person is open to be influenced by others) and expressiveness (how a person expresses him or herself in the social network), and the strength of the connection between persons. This paper presents an analysis and refinement of the absorption model, in particular by considering multiple options for the way in which the speed factor is modeled. In the original absorption model, a fixed dependence relation is used for the speed factor, describing how the speed factor relates to the traits and connections in the network. In the proposed refined absorption model, in addition two alternative ways are offered that relate the speed factor in different ways to these network characteristics. The effects and improvements that are obtained from these alternative options are analyzed and evaluated as well. This work also shows a more in depth mathematical analysis to better understand convergence and stability in the model. These analyses show that the presented temporal-causal network model is trustworthy and can be very useful to understand different contexts of emotions in social networks.

The paper has the following structure: Section 2.2 will explain the original absorption model in detail. In Section 2.3 an analysis is made in particular concerning the speed factor in the model. Section 2.4 presents two possible alternative ways to model the speed factor. A scaled approach and an advanced logistic function approach are the options explored in this section. Section 2.5 presents mathematical analysis of the model regarding monotonicity and equilibria. Section 2.6 presents results using the new approach for the model, and Section 2.7 presents the conclusions and future works.

2.2 Emotion absorption: the temporal-causal network model

In this section, the computational model for mutual absorption of emotions is presented [2, 4]. This model has been developed as a temporal-causal network model; see [8]. First, the most important concepts used in the model are explained, both in terms of a conceptual representation and a numerical representation. The

section concludes with examples of applications of this computational model of emotion contagion.

The model distinguishes some characteristics of persons and the connections between them, represented by parameters. These characteristics affect emotion contagion in the network. The model describes how internal emotion states q_A of persons A affect each other. However, internal states do not affect each other in a direct manner. First, they have to be expressed, after which they can be observed by another person, and in turn such an observation can affect the internal state of this other person. So, internal emotion states q_A affect each other by *contagion as a three-step process*, for which each step has its own characteristics (indicated by ϵ_B, α_{BA}, δ_A, respectively):

ϵ_B	from internal emotion state q_B of B to expressed emotion by B
α_{BA}	from expressed emotion by B to observed emotion by A
δ_A	from observed emotion by A to internal emotion state q_A of A

The characteristic for the extent to which a person B expresses him or herself within the network is captured by the concept of *expressiveness*, modeled by parameter ϵ_B. Similarly, the characteristic for the extent to which a person A is open to be influenced is represented by the *openness*, modeled by parameter δ_A. The strength of the relation between two people in the network is described by the *channel strength*, modeled by parameter α_{BA}. They are formalized by the numerical representations ϵ_B, α_{BA}, and δ_A as real numbers between 0 and 1.

Based on the steps above, the overall contagion process is modeled in terms of the *connection weight* ω_{BA} from sender B to receiver A. This represents the resulting influence of the internal emotion state of sender B on the internal emotion state of receiver A and depends on the three parameters above as shown in equation 2.1.

$$\omega_{BA} = \epsilon_B \alpha_{BA} \delta_A \qquad (2.1)$$

In the model this ω_{BA} is used to determine the strength of the impact from the emotion state of B to the emotion state of A at some time point t, as shown in equation 2.2.

$$\textbf{impact}_{BA}(t) = \omega_{BA} q_B(t) \qquad (2.2)$$

In equation 2.2 $q_B(t)$ is the emotion level of B at time t. The overall contagion strength ω_A to q_A represents the total effect from all nodes that are connected to emotion state q_A of person A; it is modeled as in equation 2.3.

$$\omega_A = \sum_{B \neq A} \omega_{BA} \qquad (2.3)$$

The aggregated impact $\mathbf{aggimpact}_A(t)$ at time t of all connected emotion states q_{B_i} on emotion state q_A is modeled by a *scaled sum function* (see [8]) $\mathbf{ssum}_{\omega_A}(\dots)$ with the overall connection strength ω_A as scaling factor, as shown in equation 2.4.

$$
\begin{aligned}
\mathbf{aggimpact}_A(t) &= \mathbf{ssum}_{\omega_A}(\mathbf{impact}_{B_1 A}(t), \dots, \mathbf{impact}_{B_k, A}(t)) \quad (2.4) \\
&= (\mathbf{impact}_{B_1, A}(t) + \cdots + \mathbf{impact}_{B_k A}(t))/\omega_A \\
&= (\omega_{B_1 A} q_{B_1}(t) + \cdots + \omega_{B_k A} q_{B_k}(t))/\omega_A \\
&= (\omega_{B_1 A}/\omega_A) q_{B_1}(t) + \cdots + (\omega_{B_k A}/\omega_A) q_{B_k}(t)
\end{aligned}
$$

From this it follows that $\mathbf{aggimpact}_A(t)$ is calculated as a weighted average of the emotion levels of the connected states q_B as in 2.5.

$$
\mathbf{aggimpact}_A(t) = \sum_{B \neq A} w_{BA} q_B(t) \quad (2.5)
$$

where

$$
w_{BA} = \frac{\omega_{BA}}{\omega_A} = \frac{\epsilon_B \alpha_{BA} \delta_A}{\sum_{C \neq A} \epsilon_C \alpha_{CA} \delta_A} = \frac{\epsilon_B \alpha_{BA}}{\sum_{C \neq A} \epsilon_B \alpha_{CA}} \quad (2.6)
$$

The sum of these weights is 1. The dynamics for the contagion for this temporal-causal network model (see also [8]) is described in equation 2.7.

$$
\Delta q_A(t + \Delta t) = q_A(t) + \eta_A [\mathbf{aggimpact}_A(t) - q_A(t)] \Delta t \quad (2.7)
$$

We denote with η_A the speed factor of A, which is chosen $\eta_A = \omega_A$ here. Sometimes the aggregated impact $\mathbf{aggimpact}_A(t)$ on A is denoted by the shorter notation $q_A^*(t)$.

This temporal-causal network model of emotion contagion has been investigated further and applied in a number of studies. For example, it was applied to predict the emotion levels of team members, in order to maintain emotional balance within a team [5]. When the teams' emotion level was found to become deficient, the model, which was embedded in an ambient agent, provided support to the team by proposing the team leader to give his employees a pep talk [5]. The pep talk is an example of an intervention strategy. Another study experimented with simulations of changes in the social network structure in order to guide the contagion process in a certain direction [7]. Yet another study used the model to predict changes on Physical Activity levels of a group of friends/acquaintances from the same course, applying the model to behaviour contagion [1].

2.3 Analysis of the absorption model

The absorption model is based on two main assumptions, one of which addresses the *level* of the emotions and the other one the *speed* of change of the emotion levels:

1. The emotion level $q_A(t)$ of a person A is affected linearly by the weighted average $\sum_{B \neq A} w_{BA} q_B(t)$ of the emotion levels of the connected persons B.

2. For each person A the speed of change η_A of his or her emotion level linearly depends on the overall connection strength ω_A within the network: $\eta_A = \omega_A$.

Roughly spoken, assumption 1 entails that the members of the network adapt to an average emotion level in the network. As a consequence, the emotion levels will converge to a common emotion level, which is between the minimal and maximal initial emotion levels of the connected members (see Section 2.3.1 for example simulations showing this, and Section 2.5 for a mathematical proof). This is in contrast to, for example, the amplification model introduced in [3] where assumption 1 is not taken as a point as departure, and as a result emotion contagion spirals can be modeled that reach levels higher (or lower) than any of the initial levels.

The assumption 2 states that the more connected members in the network, the higher the speed of change will be, in a proportional manner. In the current paper, assumption 1 is kept, but assumption 2 is critically analyzed in more depth and loosened in order to create room for alternatives. This second assumption (2) is an answer for the more open question:

How does the speed of change of the emotion level depend on the network structure and size?

Specific variants of this question are the following. If a person has more connections to members with a given average emotion level, will he or she adapt faster to this average emotion level? Has the number of relations in real life effect on your speed of change for adapting to them? If a person has more friends, will his or her emotions be affected faster than the emotions of another person with fewer friends? If so, to which extent? Is this relation linear or proportional, or is it inherently nonlinear? Is this increase of the speed going on indefinitely, or is there some bound for it?

In [3] these questions were answered in a most simple, linear, proportional manner, as expressed by assumption 2 above. However, it is doubtful whether this most simple linear option is the most plausible option for realistic networks.

The initial studies of the absorption model itself in [3] already highlighted two constraints: (a) In dynamic property P3 in Section 5 (referring to Theorem 5 in Section 4) in [3] it is stated that for some initial emotion values the emotion values eventually can run out of their boundaries 0 or 1. Also, (b) ω_A is a cumulative value based on the number of connections and their weights; when this number increases, because of the assumption 2, $\eta_A = \omega_A$, also the speed factor η_A increases in a proportional manner, without any limitation. For (b) Bosse et al. [3] used

adaptations to the choice of the step size Δt to control that the model stays within the boundaries. That can work well for a few nodes, but this entails that all the time a smaller value for Δt has to be chosen, when the number of nodes becomes larger. This is possible, but not very practical.

The hypothesis is that problem (a) relates to the strongly increasing value for the speed factor η_A for larger networks entailed by the choice of taking it equal to ω_A. Some experiments were run for analysis keeping the same characteristics of the experiment done by Bosse et al. [3] but with more nodes. The idea is to analyze if the choice for ω_A as a speed factor η_A indeed is the bottleneck for issues (a) and (b) of the absorption model.

2.3.1 Analysis of the original model

The same simulations found at [3] were run again, with more nodes added to the scenario in order to better understand how the model works, and what alternatives are possible. Seven scenarios were created according to the Appendix A of the paper (`http://www.few.vu.nl/~tbosse/emotion/ECMS09.pdf`). The scenarios are:

1. All members have $\omega = 1$ – fully open channels (1a)

2. All members have $\omega > 0$ – big openness for all (1b)

3. All members have $\omega > 0$ – small openness for all (1c)

4. All members have $\omega = 0$ – no changes on emotional levels (2)

5. One member has $\omega = 0$ ($\delta = 0$) (3)

6. Only one member has $\omega \neq 0$ (all other members have $\delta = 0$) (4)

7. One member has $\omega = 0$ ($\delta = 0$ and $\epsilon = 0$) (5)

Below a brief analysis of the effects on scenario 1a is made, showing what happens when the number of nodes is increased. The results of the other scenarios can be found at Appendix A [3]. The first scenario (1a) considers the maximal contagion that can happen. For that, all the parameters (expressiveness, openness and channel strengths) are set to 1. Moreover, Δt is set to 0.1. Figure 2.1 shows the differences between graphics for different numbers of members, 3, 9, and 18 nodes. As the initial values for the emotion levels for the tests have been generated at random, different convergence points, according to the average of the initial values are shown in the graphics. The convergence value for all the nodes that emerges is an average of the initial emotion levels, as shown in [3].

As all parameters are equal to 1, the speed factor η_A for each member A is the in-degree of the nodes minus 1: $\eta_A = n - 1$, with n = number of nodes. As the speed factor η_A determines the next emotion level for all the nodes (equation 2.7),

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[3]The file can also be found at `http://www.few.vu.nl/~efo600/iccci16/ICCCI16_A.pdf`
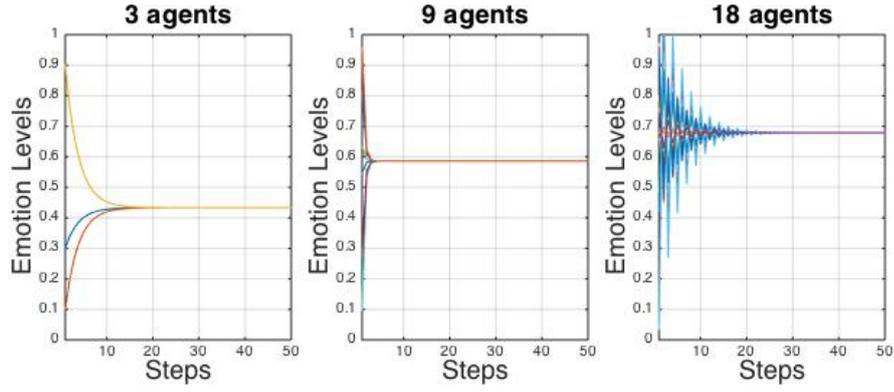
Fig. 2.1: Full channel connections for 3 to 18 members.

the emotion levels converge faster, and at some network size (after 12 members) oscillation in the emotional levels occurs due to the sudden changes caused by the high speed factor. Note that to see this effect Δt was not decreased, what normally would be a measure taken; it was kept at 0.1. Such a decrease would be possible; however, decreasing Δt with the size of the network indefinitely is neither practical nor desirable.

2.3.2 Mathematical analysis of the problem

This section addresses mathematical analysis of the problem concerning the oscillation of the emotion levels. The sudden changes will be explained showing the effect of the increase in the value of the choice of the speed factor $\eta_A = \omega_A$ for larger networks.

The equation for ω_A is the sum of all the connection strengths generated by the nodes in-connected to q_A. The strength by which the emotion from each B is received by A is calculated by $\omega_{BA} = \epsilon_B \alpha_{BA} \delta_A$, as seen before. If all nodes have ϵ and δ higher than zero, and if the network is fully connected (in other words, there is no $\delta_{BA} = 0$ to any pair B, A), and the number of neighbors in the network increases, the value of A also increases proportionally and by the assumption (2) $\eta_A = \omega_A$ the same holds for η_A.

While the speed factor increases, the changes from $q_A(t)$ at the next emotion level $q_A(\Delta t + t)$ become more sudden, and less realistic. As a matter of illustration, imagine a fully connected network (all the channel strengths α_{BA} equal to 1), where every member has openness and expressiveness equals 0.5. So, in that case

$$\omega_{BA} = \epsilon_B \alpha_{BA} \delta_A = 0.5 \times 1 \times 0.5 = 0.25 \qquad (2.8)$$

Therefore, $\omega_{BA} = 0.25$ is the connection strength between the emotion states q_A for each of the members that are connected. So, if n is the number of nodes, ω_A for any of the fully connected network will be $(n-1) \times 0.25$. If the number of nodes is increased around 10 times, the speed factor $\eta_A = \omega_A$ will be increased 10 times as well. For an increase to 1000 nodes, the speed factor will be 100 times bigger.

Figure 2.2 (left graph) shows that this increase follows a linear tendency. As it may be doubted that this indefinite increase of η_A is realistic, in Section 4 alternative options for the speed factor η_A will be considered, with patterns corresponding to the other graphs in Figure 2.2, where there is some bound in the increase of the speed factor.
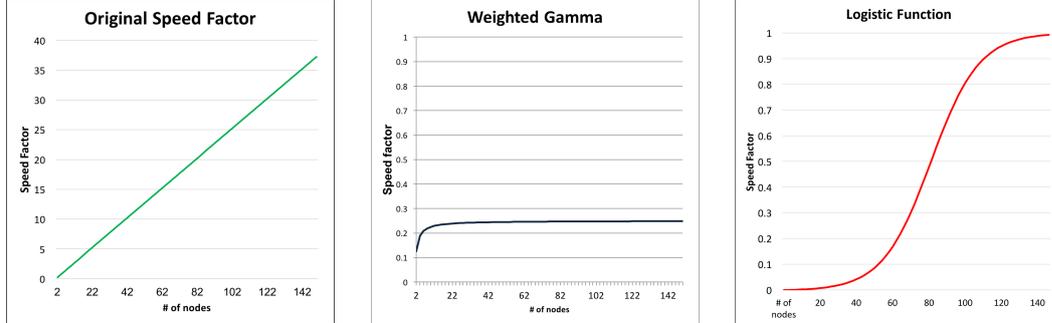

Fig. 2.2: Three types of relations for the speed factor depending on number of nodes:
left: η_A increasing linearly as the number of members increases
middle: η_A increases up to a limit, due to a weighted speed factor
right: η_A increases according to an advanced logistic function

So assuming the speed factor $\eta_A = \omega_A$ (assumption 2 of the original absorption model) causes an unbalanced behaviour of the model. In order to avoid abrupt changes, usually the value of Δt is made smaller and smaller for larger networks. This approach is conceptually and practically inadequate as the speed factor refers to the velocity of the changes in the emotions, whereas Δt refers to the time steps of the model, and has no relation with the speed factor η_A. That incompatibility is a reason to consider alternative answers on the main question concerning speed factors, as discussed next.

2.4 Alternative ways to model the speed factor η_A

In this section two alternative ways of modeling the speed factor η_A are explored. In both cases the speed factor increases with the size of the network, but stays under a certain bound, according to patterns as shown in Figure 2.2 middle and right graphs. The first option is by modeling the speed factor as a *scaled* ω_A, with scaling factor the number n of nodes in the network:

$$\eta_A = \frac{\omega_A}{n} \tag{2.9}$$

This option avoids the effect caused by the increasing in the number of members. In this case, and using the same network as in Figure 2.2, it can be seen that the value for η_A converges to the ω_{BA} which is the same for all nodes in this scenario. Figure 2.2 middle graph shows the new situation when the above scaled model is used for η_A.

Using this option will assure that the speed factor η_A has boundaries defined according to the following mathematical analysis. For the new calculation for η_A, the following holds for all A:

$$0 \leq \eta_A \leq 1 \tag{2.10}$$

This can be verified as follows. It holds

$$\sum_{B \neq A} \epsilon_B \alpha_{BA} \delta_A \leq n - 1 \tag{2.11}$$

as each of the terms of this sum is less or equal 1. Therefore

$$\eta_A = \frac{\omega_A}{n} = \frac{\sum_{B \neq A} \epsilon_B \alpha_{BA} \delta_A}{n} \leq \frac{n-1}{n} < 1 \tag{2.12}$$

In other words the speed factor η_A is now bounded by 1. Note that by a slight modification this bound can be set to any number η by multiplying this by an extra parameter η (the same holds for the second alternative discussed below):

$$\eta_A = \eta \frac{\omega_A}{n} \tag{2.13}$$

A second alternative is to use an advanced logistic function in order to gradually increase the speed with network size but keep the values for the speed factors within some bound. The advanced logistic function has a S shape, or sigmoid curve, and is described by the equation 2.14.

$$\textbf{alogistic}_{\sigma,\tau} = \left[\frac{1}{1 + e^{-\sigma(\eta_A - \tau)}} - \frac{1}{1 + e^{\sigma\tau}} \right] (1 + \exp^{-\sigma\tau}) \tag{2.14}$$

Here σ is the steepness and τ is the threshold value. The values for σ and τ can be chosen according to a person's traits. While some persons respond gradually to the increasing influence of people to whom they are connected, other persons may respond by flare-ups. For the situation of a person that responds mode linearly to the increasing on their cumulative ω_A, a low steepness value such as $\sigma = 0.3$ can be chosen, and, for example, $\tau = 20$. The results for this situation can be seen at Figure 2.2, right graph.

As can be seen, the logistic function also keeps the speed factor values between 0 and 1, and if the parameters of the function are well adjusted, the equation can give more realistic outcomes.

2.5 Mathematical Analysis

This section presents some of the results of a mathematical analysis for the model after the changes at the speed factor calculation.

Definition 1. *A network is called strongly connected if for every two nodes A and B there is a directed path from A to B and vice versa.*

Lemma 1. *Let a temporal-causal network model be given based on scaled sum functions for states q_A:*

$$\boldsymbol{d}q_A/\boldsymbol{d}t = \eta_A \left[\frac{\sum_{B \neq A} \omega_{BA} q_B}{\omega_A - q_A} \right]$$

Then the following hold.

(a) If for some state q_A at time t for all states q_B connected toward q_A it holds $q_B(t) \geq q_A(t)$, then $q_A(t)$ is increasing at t: $\boldsymbol{d}q_A(t)/\boldsymbol{d}t \geq 0$; if for all states B connected toward A it holds $q_B(t) \leq q_A(t)$, then $X_A(t)$ is decreasing at t: $\boldsymbol{d}q_A(t)/\boldsymbol{d}t \leq 0$.

(b) If for all states q_B connected toward q_A it holds $q_B(t) \geq q_A(t)$, and at least one state X_B connected toward q_A exists with $q_C(t) > q_A(t)$ then $q_A(t)$ is strictly increasing at t: $\boldsymbol{d}q_A(t)/\boldsymbol{d}t > 0$. If for all states X_B connected toward q_A it holds $q_B(t) \leq q_A(t)$, and at least one state q_B connected toward q_A exists with $q_C(t) < q_A(t)$ then $q_A(t)$ is strictly decreasing at t: $\boldsymbol{d}q_A(t)/\boldsymbol{d}t < 0$.

Proof of Lemma 1. *From the differential equation for $q_A(t)$*

$$
\begin{aligned}
\boldsymbol{d}q_A/\boldsymbol{q}_t &= \eta_A \left[\frac{\sum_{B \neq A} \omega_{BA} q_B}{\omega_A} - q_A \right] & (2.15) \\
&= \eta_A \left[\frac{\sum_{B \neq A} \omega_{BA} q_B - \omega_A q_A}{\omega_A} \right] \\
&= \eta_A \left[\frac{\sum_{B \neq A} \omega_{BA} q_B - \sum_{B \neq A} \omega_{BA} q_A}{\omega_A} \right] \\
&= \eta_A \left[\frac{\sum_{B \neq A} \omega_{BA}(q_B - q_A)}{\omega_A} \right]
\end{aligned}
$$

it follows that $\boldsymbol{d}q_A(t)/\boldsymbol{d}_t \geq 0$, so $q_A(t)$ is increasing at t. Similar for decreasing.

For (b) it follows that $\boldsymbol{d}q_A(t)/\boldsymbol{d}_t > 0$, so $q_A(t)$ is strictly increasing. Similar for decreasing. ∎

Theorem 1 (convergence to one value). *Let a strongly connected temporal-causal network model be given based on scaled sum functions for the states q_A*

$$\boldsymbol{d}q_A/\boldsymbol{d}_t = \eta_A \left[\frac{\sum_{B \neq A} \omega_{BA} q_B}{\omega_A - q_A} \right] \qquad (2.16)$$

and with equilibrium values $\underline{q}_A$. Then for all A and B the equilibrium values $\underline{q}_A$ and $\underline{X}_B$ are equal: $\underline{q}_A = \underline{q}_B$. Moreover, this equilibrium state is attracting.

Proof of Theorem 1. *Take a state q_A with highest value $\underline{q}_A$. Then for all states q_C it holds $\underline{q}_C \leq \underline{q}_A$. Suppose for some state q_B connected toward q_A it holds $\underline{q}_B < \underline{q}_A$. Take a time point t and assume $q_C(t) = \underline{q}_C$ for all states q_C.*

Now apply Lemma 1(b) to state q_A. It follows that $\boldsymbol{d}q_A(t)/\boldsymbol{d}_t < 0$, so $q_A(t)$ is not in equilibrium for this value $\underline{q}_A$. This contradicts that this $\underline{q}_A$ is an equilibrium value for q_A. Therefore the assumption that for some state q_B connected toward q_A it holds $\underline{q}_B < \underline{X}_A$ cannot be true.

This shows that $\underline{q}_B = \underline{q}_A$ for all states connected towards q_A. Now this argument can be repeated for all states connected toward q_A. By iteration every other state in the network is reached, due to the strong connectivity assumption; it follows that all other states in the temporal causal network model have the same equilibrium value as q_A. From Lemma 1(b) it follows that such an equilibrium state is attracting: if for any state the value is deviating it will move to the equilibrium value. ∎

Proposition 1 (Monotonicity Conditions). *It follows that:*

(a) If $q_A^*(t) \leq q_A(t)$ then $q_A(t)$ is monotonically decreasing; it is strictly decreasing when $q_A^*(t) < q_A(t)$.
(b) If $q_B^*(t) \geq q_B(t)$ then $q_B(t)$ is monotonically increasing; it is strictly increasing when $q_B^*(t) > q_B(t)$.

Proof of Proposition 1. *This follows from the differential equation*

$$\boldsymbol{d}q_C(t)/\boldsymbol{d}t = \eta_C(q_C^*(t) - q_c(t)) \tag{2.17}$$

and the fact that $0 \leq q_C(t) \leq 1$ and $0 \leq q_C^*(t) \leq 1$. ∎

Lemma 2. *Suppose all ω_{CD} are nonzero. Then for an equilibrium the following hold:*

(a) $q_A^* = 0$ if and only if $q_C = 0$ for all $C \neq A$
(b) $q_B^* = 1$ if and only if $q_C = 1$ for all $C \neq B$

Proof of Lemma 2. *It follows that*

(a) From

$$q_A^* = \sum_{C \in G\setminus\{A\}} w_{CA}q_C = 0 \tag{2.18}$$

and the fact that all terms are nonnegative it follows that $w_{CA}q_C = 0$ for all $C \neq A$ and conversely.

(b) From

$$q_B^* = \sum_{C \in G\setminus\{B\}} w_{CB}q_C = 1 \tag{2.19}$$

and the fact that

$$\sum_{C \in G\setminus\{B\}} w_{CB} = 1 \tag{2.20}$$

it follows that $q_C = 1$ for all $C \neq B$ and conversely. ∎

Lemma 3. *For an equilibrium for any member the following hold:*

(a) If $q_A = 0$ then $q_A^* = 0$
(b) If $q_B = 1$ then $q_B^* = 1$

Proof of Lemma 3. *It follows that*

(a) From

$$q_A^* - q_A = 0 \qquad\qquad (2.21)$$

with $q_A = 0$ it follows

$$q_A^* = 0 \qquad\qquad (2.22)$$

(b) From

$$q_B^* - q_B = 0 \qquad\qquad (2.23)$$

with $q_B = 1$ it follows

$$q_B^* = 1 \qquad\qquad (2.24)$$

∎

Proposition 2. *Suppose some A is given and all w_{BA} are nonzero. Then for an equilibrium the following hold:*

(a) If $q_A = 0$ then $q_C = 0$ for all C.
(b) If $q_B = 1$ then $q_C = 1$ for all C.

Proof of Proposition 2. *This immediately follows from Lemma 2 and Lemma 3.* ∎

Proposition 3. *Suppose all w_{DC} are nonzero. Then for an equilibrium it holds*

(a) If $q_A = 0$ for some A then $q_C = 0$ for all $C \in G$.
(b) If $q_B = 1$ for some B then $q_C = 0$ for all $C \in G$.

Proof of Proposition 3. *This immediately follows from Proposition 2.* ∎

2.6 Results

Using the alternative models for the speed factors η_A and comparing them with the outcomes for the original model, it turns out that the oscillation is not present anymore in any of the new approaches. For scenario 1(a), Figure 2.3, it is possible to observe that for 3 members the logistic function delays the convergence point, especially because the logistic function will give a lower value when ω_A is lower.

For scenario 3, Figure 2.4, it is clear how the use of both the scaled and or logistic model for the speed factor corrects the awkward slopes from the original model without needing any change on the time step Δt used. As noticed at scenario 1(a), for fewer nodes, the logistic function still keeps the convergence point later. This can be handled at the logistic function itself through steepness and threshold adjustments.
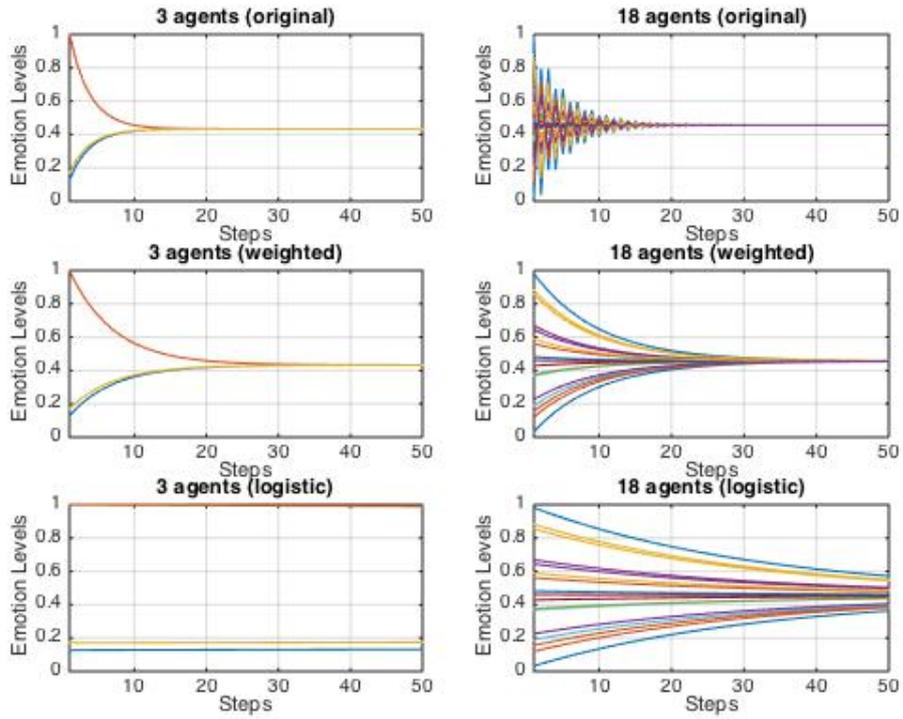
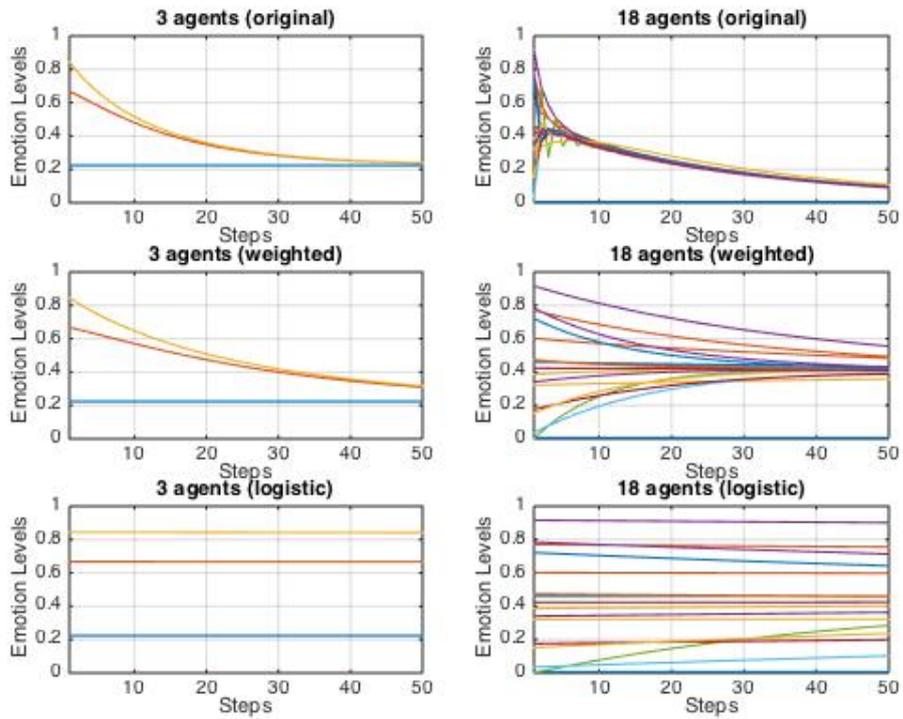**Fig. 2.3:** Comparison for scenario 1(a) between the 3 speed factors

Fig. 2.4: Scenario 3 and the different speed factors used to calculate the emotion levels

More results and analysis for the other scenarios and graphics can be found at Appendix B[4]. As it can be observed, the issue with the oscillation is fixed for all the scenarios explored using weighted or logistic function as a speed factor for the model.

2.7 Conclusion

Mathematical models are used in order to mimic the real world. Regarding the temporal-causal network model for absorption of emotions in a network introduced by Duell et al. [5] it has become clear that the assumption made about the speed factor isn't perfect, and gives room to alternatives. Two of such alternatives were explored here: a scaled model and an advanced logistic model. The expressiveness, openness to changes, and the strength of links still play a role in modeling the speed of the change of the emotion level. By these alternative models the speed can be well regulated between boundaries and do not lead to sudden changes that conflict with our understanding of emotional evolution over time. Limiting the value of speed factor η_A between 0 and 1 creates a stable slope in the emotion changes in networks, what brings the model closer to what it is expected to do.

A mathematical analysis also shows some of the features of the model. Part of the analysis explains characteristics of the model such as convergence and stability.

Future work can be done to investigate how these alternative models for the speed factor affect the results of previous research, and how they can be combined with the model for emotion contagion spirals from [4].

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[4]This appendix can also be accessed at `http://www.few.vu.nl/~efo600/iccci16/ICCCI16_B.pdf`

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