4

CP violation in mixing of $B^0$ mesons

The amount of CP violation in mixing of $B^0$ mesons is quantified by the parameter $a_{sll}^d$. The LHCb measurement of $a_{sll}^d$ described in this chapter is published in Ref. [101]. The definition of this phenomenological parameter is given in Sec. 1.2.8, and the methods used to determine the detection asymmetries are described in Sec. 3.

4.1 Method

The $B^0$ decays are reconstructed in two semileptonic decay channels,

- $B^0 \to D^{*-} \mu^+ \nu_\mu X$, with $D^{*-} \to \bar{D}_s^0 \pi^-$ and $\bar{D}_s^0 \to K^+ \pi^-$,
- $B^0 \to D^{-} \mu^+ \nu_\mu X$, with $D^- \to K^+ \pi^- \pi^-$

and their charge-conjugate modes, where $X$ represents any number of additional particles that are not explicitly reconstructed in the decay, in addition to the neutrino (which is always missing and explicitly stated in the decay channel). The decay topologies are illustrated in Fig. 4.1. As mentioned in Sec. 1.2.8 the non-reconstruction of these additional particles results in a broad $B^0$ invariant mass peak. Backgrounds like $B^+ \to D^- \pi^+ \mu^+ \nu_\mu$ therefore contribute to the signal sample, where in this case $X = \pi^+$. The detailed selection of the signal sample is described in Sec. 4.2. The contributions from various backgrounds are determined in Sec. 4.3.

The value of $a_{sll}^d$ and the production asymmetry $A_P$ are determined from the untagged yields of charge-conjugate signal candidates as a function of the $B^0$ decay time as discussed in Sec. 1.2.8. The resulting measured asymmetry is repeated here for convenience,

$$A_{\text{meas}}(t) = \frac{N(f,t) - N(\bar{f},t)}{N(f,t) + N(\bar{f},t)} \approx A_{\text{det}} + \frac{a_{sll}^d}{2} + \left(A_P - \frac{a_{sll}^d}{2}\right) \cos(\Delta m_d t), \quad (4.1.1)$$

where $f$ and $\bar{f}$ are determined by the charge of the final state muon, and the yields $N$ are determined from fits to the invariant mass of the $D^-$ and $D^+$ candidates. The $B^0$ lifetime is corrected for the missing momentum in the reconstruction using so-called $k$-factors. This is discussed in Sec. 4.4. In this analysis, the candidates are weighted such that the
kinematic distribution of the highest-momentum pions match those of the muons, in order to minimize the $\mu^+\pi^-$ detection asymmetry. The determination of the remaining detection asymmetry $A_{\text{det}}$ using the methods described in Chapter 3 is the focus of Sec. 4.5. This is followed by the measured values of $a_{\text{sl}}^d$ and the production asymmetry $A_P$ in Sec. 4.6 and a discussion of the systematic uncertainties in Sec. 4.7.

4.2 Selection

The analysis makes use of the full LHCb data set obtained during run 1 of the LHC. The data set corresponds to integrated luminosities of 1.0 fb$^{-1}$ at a centre-of-mass energy of 7 TeV obtained in 2011, and 2.0 fb$^{-1}$ at 8 TeV obtained in 2012. Since $A_P$ depends on the hadronic environment, it is expected to be slightly different between both data-taking periods, and is thus reported for each year individually.

The recording of the event is triggered by the signal muon at the hardware- and the first software level. In the second software level the topology of the $B^0$ decay is required to be consistent with that of a multibody $B^0$ decay. The demands made in these trigger lines are described in more detail in Sec. 2.4. In the offline selection, standard quality requirements are made on the reconstructed tracks and vertex to form $D^-$ or $D^*$- candidates. These are then combined with a muon of opposite charge to form a good-quality $B^0$ vertex. All final-state tracks should be inconsistent with originating from the primary vertex. A further selection is applied to reduce the background that is visible in the $D^-$ or $\bar{D}^0$ invariant mass distribution. This background is also referred to as combinatorial background, and

Figure 4.1: Decay topologies of the two signal decay modes.
4.2. Selection

dominates in the $D^-$ or $\overline{D}^0$ candidate invariant mass distribution left and right of the signal peak. A tight window is applied to the reconstructed invariant mass difference between the $D^{*-}$ and the $\overline{D}^0$ candidates in the $B^0 \to D^{*-} \mu^+ \nu_\mu X$ mode, to remove combinatorial background contributions. In the $B^0 \to D^- \mu^+ \nu_\mu X$ mode, the combinatorial background is reduced by requiring that the $D^-$ vertex is downstream of the $B^0$ vertex. The most important requirements are summarized in Table 4.1.

4.2.1 Removal of identifiable backgrounds

Decays of $B \to J/\psi X$ form a source of background when one of the muons from the $J/\psi \to \mu^+ \mu^-$ decay is misidentified as a pion, and combined with the other particles in the decay to form a fake $D^{(*)\pm}$ candidate of a signal event. Fortunately, this background is easily identified by applying the muon mass hypothesis to the reconstructed pions, and looking at the invariant mass of either of the pions combined with the muon. This is shown in the left plot of Fig. 4.2 for a limited mass range that includes the $J/\psi$ mass. If the invariant mass of the $\mu^+ \mu^-$ pair is around the $J/\psi$ mass [3070–3150] MeV/c$^2$ and the pion candidate has hits in the muon stations, the candidate is rejected. This veto reduces the total background by about 4% in the $D^-$ mode, and 1% in the $D^{*-}$ mode.

A similar identifiable background originates from $\Lambda_b^0 \to \Lambda^+_c \mu^- X$ decays, followed by

Table 4.1: Most important selection requirements on the data, on top of the trigger requirement. The PID DLL variables are explained in Sec. 2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>$B^0 \to D^{*-} \mu^+ \nu_\mu X$</th>
<th>$B^0 \to D^- \mu^+ \nu_\mu X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offline cuts</td>
<td>$M(D^{(*)\pm} \mu^\mp)$</td>
<td>[3.0, 5.2] GeV/c$^2$</td>
</tr>
<tr>
<td></td>
<td>$M(D^{*-}) - M(\overline{D}^0)$</td>
<td>[144, 147] MeV/c$^2$</td>
</tr>
<tr>
<td></td>
<td>$\tau(\overline{D}^{0,\pm})$</td>
<td>&gt; 0.1 ps</td>
</tr>
<tr>
<td></td>
<td>$z(D^0) - z(B^0)$</td>
<td>&gt; -3.0</td>
</tr>
<tr>
<td></td>
<td>$J/\psi$ veto</td>
<td>See text</td>
</tr>
<tr>
<td></td>
<td>$\Lambda_c^+ \text{ veto}$</td>
<td>See text</td>
</tr>
<tr>
<td>Calibration cuts</td>
<td>$p$ all tracks</td>
<td>&gt; 3 GeV/c</td>
</tr>
<tr>
<td></td>
<td>$p (\mu^\pm$ and higher $p_T \pi^\pm)$</td>
<td>&gt; 6 GeV/c</td>
</tr>
<tr>
<td></td>
<td>$p_T (\text{lower} \ p_T \pi^\pm)$</td>
<td>&gt; 300 MeV/c</td>
</tr>
<tr>
<td></td>
<td>$p_T (K^\pm)$</td>
<td>[300, 7000] MeV/c</td>
</tr>
<tr>
<td></td>
<td>$p_T (\mu^\pm)$</td>
<td>&gt; 1.2 GeV/c</td>
</tr>
<tr>
<td></td>
<td>$p_T (\text{higher} \ p_T \pi^\pm)$</td>
<td>&gt; 1.2 GeV/c</td>
</tr>
<tr>
<td>PID cuts</td>
<td>DLL$_{K^\pm \pi}(K^\pm)$</td>
<td>&gt; 7</td>
</tr>
<tr>
<td></td>
<td>DLL$_{K^\pm \pi}(\text{lower} \ p_T \pi^\pm)$</td>
<td>&lt; 3</td>
</tr>
<tr>
<td></td>
<td>DLL$_{K^\pm \pi}(\text{higher} \ p_T \pi^\pm)$</td>
<td>&lt; 10</td>
</tr>
<tr>
<td></td>
<td>DLL$_{\mu^- \pi}(\mu^\pm)$</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

[Note: Table 4.1 is not fully transcribed here due to the limitations of the text format. The table is meant to be a summary of the most important selection requirements for the $B^0 \to D^{*-} \mu^+ \nu_\mu X$ and $B^0 \to D^- \mu^+ \nu_\mu X$ modes. The requirements listed include both offline cuts and calibration cuts, as well as PID cuts. Each requirement is specified with specific values or ranges that are typical for such analyses.]
$\Lambda_c^+ \to pK^-\pi^+$, and the proton is misidentified as a pion, such that the $\Lambda_c^+$ decay products form a $D^{(*)^-}$ candidate. Again the proton mass hypothesis is applied to the pions, and $pK^-\pi^+$ candidates with an invariant mass around the $\Lambda_c^+$ mass $[2260 - 2310]\text{MeV}/c^2$ are rejected if the pion satisfies a tight proton PID requirement ($\text{DLL}_{p-\pi} > 10$), as shown in the left plot of Fig. 4.2. This veto reduces the background by about 5%. In the $D^{*-}$ sample the contribution of this background is small, and a veto is not applied.

Simulation studies (Sec. 4.3) indicate that combinatorial background in the $D^-$ or $D^0$ invariant mass mostly originates from real $b$-hadron decays and not from the PV, such that they have a short reconstructed $D^-$ or $D^0$ decay time. Figure 4.3 shows the background-subtracted decay-time distribution of $D^-$ and $D^0$ candidates in black, and for candidates in the mass sidebands around the $D^-$ and $D^0$ signal peak in red. A minimum decay time of 0.1 ps is required to reduce the contribution of this background, in particular in the $B^0 \to D^-\mu^+\nu_\mu X$ mode.

Finally, there is a contribution from real $D^-$ or $D^0$ mesons that do not originate from a $b$-hadron decay but from the PV, and are combined with a random muon. These so-called prompt backgrounds peak in the invariant $D^-$ or $D^0$ invariant mass, but point back to the PV instead of a secondary vertex. Simulation studies of inclusive $D$ decays (Sec. 4.3.2) show the size of this contribution to be about 1%. In data, part of this contribution can be identified using the impact parameter (IP) of the $D^-$ of same-sign combinations $D^+\mu^+$, as shown in Fig. 4.4 (left). These combinations have an enhanced relative contribution of prompt candidates. From the number of same-sign combinations this contribution is estimated to be about 0.2% with respect to the total amount of right-sign signal events. Another contribution of prompt candidates originates from real $D^-$ or $D^0$ mesons combined with a muon that originates from a semileptonic decay of the other charmed hadron in the

![Figure 4.2: The invariant mass distributions of (left) the muon and the highest-momentum pion and (right) the $K^+\pi^-\pi^-$ combination where the lowest-momentum pion has the proton hypothesis, for candidates in the sideband of the $B^0 \to D^-\mu^+\nu_\mu X$ data, (black) before and (red) after the corresponding veto. The candidates that fall within the (left) $J/\psi$ and (right) $\Lambda_c^+$ mass window are removed if they satisfy muon-like resp. proton-like PID for the pion in question. Note that not all candidates fall inside the mass range of the left plot.](image)
4.2. Selection

c-c-event. In that case, the muon has the right sign. From a fit to the log(IP) distribution, shown in Fig. 4.4 (right), the total contribution of prompt charm backgrounds in the right-sign data is estimated to be about 0.7%. The prompt charm contribution is reduced to a negligible level of 0.1% when demanding log(IP/mm) > −3.0 for the $D^−$ or $D^0$.

The calibration samples used to determine the detection asymmetries have certain (kinematic) constraints in their respective selection, described in the appropriate sections. In order to match the requirements of these samples with the signal samples, additional

Figure 4.3: Reconstructed decay time of sideband-subtracted signal events in black, and events in the $D^0$ or $D^−$ mass sidebands in red, for (left) the $B^0 \rightarrow D^+\mu^+\nu_{\mu}X$ and (right) the $B^0 \rightarrow D^−\mu^+\nu_{\mu}X$. Due to the requirement that the $D^−$ vertex should be downstream of the $B^0$ vertex in the $B^0 \rightarrow D^−\mu^+\nu_{\mu}X$ mode, the lower decay time is affected by the decay-time resolution.

Figure 4.4: Logarithm of the impact parameter (IP) of the $D^−$ in (left) the same-sign $D^+\mu^+$ data sample, and (right) the right-sign $B^0 \rightarrow D^−\mu^+\nu_{\mu}X$ data. The result of the fit is overlaid, where the green line represents prompt $D^−$ decays, the red line represents secondary $D^−$ decays and the blue line is the total. In the fit to the right-sign data, the prompt shape is fixed to that of the wrong-sign data.
cuts on the momenta and PID of the final-state particles are made. These are also listed in Table 4.1.

4.2.2 Mass fit and signal yields

The signal yields after all selection steps are determined with a fit to the invariant mass distributions of the $D^-$ and $\bar{D}^0$ in data, and are given in Table 4.2. The $D^-$ mode contains about six times more candidates than the $D^{*-}$ mode. Note that these yields still include peaking background contributions from other $b$-hadron decays to real $D^-$ and $\bar{D}^0$ mesons. The $D^-$ and $\bar{D}^0$ invariant mass shapes are modelled with the sum of a Crystal Ball function (Eq. 3.4.2) and a second Gaussian function, with a shared mean. The combinatorial background is described with an exponential function. The fits to the combined 2011 and 2012 data and both magnet polarities are shown in Fig. 4.5. The fits to the individual data samples are shown in Appendix B.

![Graph showing invariant mass distributions](image)

Figure 4.5: The invariant mass distributions of (top) the $D^-$ candidates in the $B^0 \rightarrow D^- \mu^+ \nu_\mu X$ and (bottom) the $\bar{D}^0$ candidates in the $B^0 \rightarrow D^{*-} \mu^+ \nu_\mu X$ mode. The fit shape is overlaid. The contribution of $D^-$ or $\bar{D}^0$ candidates corresponding to the peaking $B^+$ background (see Sec. 4.3.2) is projected as the red curve.
Table 4.2: Signal yields for the $B^0 \to D^{*-} \mu^+ \nu_\mu X$ and $B^0 \to D^- \mu^+ \nu_\mu X$ samples for the two magnet polarities and the two data-taking periods, obtained from fits to the $D^-$ and $\bar{D}^0$ invariant mass distributions, after all selection cuts. The bottom two rows indicate the effective signal yield after applying $\mu \tau$ weights, as described in Sec. 4.5.

<table>
<thead>
<tr>
<th>Channel</th>
<th>2011 Up</th>
<th>2011 Down</th>
<th>2012 Up</th>
<th>2012 Down</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to D^{*-} \mu^+ \nu_\mu X$</td>
<td>53 089</td>
<td>71 462</td>
<td>179 411</td>
<td>168 005</td>
<td>471 967</td>
</tr>
<tr>
<td>$B^0 \to D^- \mu^+ \nu_\mu X$</td>
<td>352 705</td>
<td>477 661</td>
<td>1 187 579</td>
<td>1 103 893</td>
<td>3 121 838</td>
</tr>
<tr>
<td>$B^0 \to D^{*-} \mu^+ \nu_\mu X$ (eff.)</td>
<td>40 020</td>
<td>54 208</td>
<td>140 515</td>
<td>131 647</td>
<td>366 389</td>
</tr>
<tr>
<td>$B^0 \to D^- \mu^+ \nu_\mu X$ (eff.)</td>
<td>226 153</td>
<td>306 205</td>
<td>768 088</td>
<td>714 399</td>
<td>2 014 845</td>
</tr>
</tbody>
</table>

### 4.3 Simulation and background studies

Simulated events with 2011 and 2012 data-taking conditions are generated using the software described in Sec. 2.5. They are used to determine the contribution of backgrounds in the data sample (Sec. 4.3.3), to calculate the $k$-factors for the correction of the $B^0$ lifetime (Sec. 4.4.1) and to verify the time-dependent fit (Sec. 4.4).

#### 4.3.1 Signal simulation

Signal samples of $B^0 \to D^{*-} \mu^+ \nu_\mu X$ and $B^0 \to D^- \mu^+ \nu_\mu X$ decays are generated. The additional particles in the final state can originate from higher charmed resonances, $B^0 \to D^{(*)+} \tau$ decays with $\tau \to \mu X$, and non-resonant higher multiplicity decays which have additional pions in the final state. Therefore, the samples are enriched by including contributions from such decays. The absolute branching ratios used in the creation of these “cocktail” samples come from Ref. [94], and isospin symmetry is assumed to determine the branching ratios of yet unmeasured decay modes. In the $B^0 \to D^- \mu^+ \nu_\mu X$ mode cocktail, exclusive $B^0 \to D^- \mu^+ \nu_\mu$ decays contribute 44%, while decays of $B^0$ through higher charmed resonances contribute 46%, and are dominated by the $B^0 \to D^{*-} (\to D^- \{\pi^0, \gamma\}) \mu^+ \nu_\mu$ decay (33%). Tauonic decay modes (with $\tau \to \mu X$) contribute 5%, and non-resonant decays with higher multiplicity contribute 2%.

In contrast, the $B^0 \to D^{*-} \mu^+ \nu_\mu X$ mode cocktail is dominated by the lowest-multiplicity $B^0 \to D^{*-} (\to \bar{D}^0 \pi^-) \mu^+ \nu_\mu$ decay, with 83%. Higher charmed resonances contribute 6%, tau decays with 4% and higher multiplicity modes with 6%. A comparison between the kinematic distributions of the $B^0$ candidates of data and simulation is shown in Fig. 4.6, where a good agreement is observed.

In total, 60 million $B^0 \to D^{*-} \mu^+ \nu_\mu X$ events, and 30 million $B^0 \to D^- \mu^+ \nu_\mu X$ events are generated. The overall efficiency due to acceptance, trigger and signal selection criteria is found to be 0.25% for the $B^0 \to D^- \mu^+ \nu_\mu X$ mode and 0.20% for the $B^0 \to D^{*-} \mu^+ \nu_\mu X$ mode, with statistical uncertainties of $\mathcal{O}(0.001\%)$. This includes the statistical error degradation due to the application of event weights in the data sample, in order to match
4. Chapter 4. \textit{CP} violation in mixing of $B^0$ mesons

![Graphs of normalized kinematic distributions](image)

Figure 4.6: The normalized kinematic distributions of the reconstructed $B^0$ candidate in the $B^0 \rightarrow D^- \mu^+\nu_\mu X$ mode for data and simulation (MC), for both versions of PYTHIA. A similar agreement between data and simulation is found for the $B^0 \rightarrow D^{*-} \mu^+\nu_\mu X$ mode.

...the pion kinematic distributions to those of the muon, as explained in Sec. 4.5.

4.3.2 Contributions from peaking backgrounds

In order to investigate the various sources of background involving real $D$ mesons, two samples of 15 million of both $D^- \rightarrow K^+\pi^-\pi^-$ and $D^{*-} \rightarrow \bar{D}^0(\rightarrow K^+\pi^-)\pi^-$ decays are generated, which include $D^-$ and $\bar{D}^0$ mesons from $b$-hadron decays. After full selection has been applied, 84\% (83\%) of all candidates in the $D^{*-}$ ($D^-$) mode match to generated signal decays. Most background events originate from real $b$ hadrons decaying to $D^-$ or $\bar{D}^0$, and are dominated by semileptonic $B^+$ decays. These decays are hard to reduce since they peak in the $D^-$ or $\bar{D}^0$ invariant mass. The $B^+$ contribution is discussed in Sec. 4.3.3 and has a separate component in the decay-time fit. Another significant background is combinatoric $D^-$ or $\bar{D}^0$ candidates from real $b$-hadron decays, that involve a fake track. This background appears as combinatorial background in the $D^-$ or $\bar{D}^0$ invariant mass distribution, which is also treated as a dedicated component in the decay-time fit.

The contribution from $B^0 \rightarrow D^{(*)-} \pi^+ X$ with a $\pi^+ \rightarrow \mu^+\nu_\mu$ decay-in-flight, as well as
4.3. Simulation and background studies

$B^0 \to D^{(*)-}D^{(*)+}X$ decays where the $D^{(*)+}$ decays semileptonically, contribute $(0.9 \pm 0.4)\%$ to both signal modes. They have the right sign combination, and are treated as signal decays. Due to additional missing momentum a reduction in time resolution is expected, especially for the latter. This effect is studied in Sec. 4.7.2.

The contribution from (Cabibbo-suppressed) $B_0^0$ decays is about $(1.5 \pm 0.5)\%$ in both modes, and is not modelled. Its effect is considered as a systematic uncertainty, and is described in Sec. 4.7. Finally, the contribution of $D^+ \to D^{(*)-} \mu^+ \nu_{\mu}X$ decays is estimated in Sec. 4.7.2 and considered as a source of systematic error. Other sources of real $D^{(*)-}$ or $D^-$ mesons are found to be negligible.

4.3.3 Determination of the $B^+$ fraction

The dominant contribution of other $b$-hadron decays to the signal sample originates from $B^+$ decays. A cocktail of semileptonic $B^+$ decays is generated to determine the size of this contribution after all selection steps. The branching ratios of contributing decays are determined in the same way as those of the signal cocktails (using Ref. 94 and isospin). The $B^+$ mesons are forced to decay to a $D^- (\to K^+\pi^-\pi^-)$ or a $D^0 \to K^+\pi^-$, including resonant states. The latter includes $B^+ \to D^{(*)-} \mu^+ \nu_{\mu}X$ decays. In total, 22.5 million $B^+ \to D^0 \mu^+ \nu_{\mu}X^+$ events and 10 million $B^+ \to D^- \mu^+ \nu_{\mu}X^+$ events are generated.

Under $(u,d)$ isospin symmetry the production rates of $B^+$ and $B^0$ are equal. With this assumption, the relative fraction of $B^+$ events in the data can be obtained from the selection efficiencies and the total branching fractions of the decays present in the $B^0$ and $B^+$ cocktails. These are summarized in Table 4.3, with the first error is due to the known error on the measured branching fractions, and the second is due to the change in central value when varying the branching ratios of the yet unmeasured decays by a factor two. The $B^+$ fractions in the signal data after selection is defined as

$$f_{B^+} = \frac{N_{B^+}}{N_{B^0} + N_{B^+}},$$

and are found to be

$$f_{B^+}(B^0 \to D^{(*)-} \mu^+ \nu_{\mu}X \text{ mode}) = (8.8 \pm 2.0 \pm 1.0)\%,$$
$$f_{B^+}(B^0 \to D^- \mu^+ \nu_{\mu}X \text{ mode}) = (12.7 \pm 2.1 \pm 0.6)\%,$$

where the first uncertainty is due to the branching fractions, and the second is due to the efficiency variations between Pythia 6 and Pythia 8.

The estimated $B^+$ fractions are verified in data using the corrected mass to distinguish between $B^0$ and $B^+$ decays. The corrected mass is defined as

$$M_{\text{corr}} = \sqrt{M(D^{(*)-}\mu^+)^2 + |p_{T,\text{missing}}|^2 + |p_{T,\text{missing}}|},$$

where $p_{T,\text{missing}}$ represents the transverse momentum of an additional massless particle originating from the $B^0$ vertex, such that the reconstructed $B^0$ flight direction points back to the PV. For the dominant signal modes, which miss only a neutrino, this correction
Table 4.3: The total branching ratios corresponding to the generated cocktails. The first uncertainty comes from the measured uncertainties on the individual branching fractions present in the cocktail. The second uncertainty comes from varying the branching ratios of yet unmeasured decays. The last column displays the selection efficiency. For the $B^+ \rightarrow D^{*-} \mu^+ \nu_{\mu} X$ sample, this efficiency contains the fraction of $\bar{D}^0$ mesons that originated from a $D^{*-}$ decay.

<table>
<thead>
<tr>
<th>Cocktail sample</th>
<th>Branching fraction</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal $B^0 \rightarrow D^{*-} \mu^+ \nu_{\mu} X$</td>
<td>$(4.07 \pm 0.20 \pm 0.08)%$</td>
<td>$\times B(D^0 \rightarrow K^-\pi^+)\times B(D^- \rightarrow K^+\pi^-\pi^-)$</td>
</tr>
<tr>
<td>Signal $B^0 \rightarrow D^- \mu^+ \nu_{\mu} X$</td>
<td>$(4.95 \pm 0.23 \pm 0.08)%$</td>
<td>$\times B(D^- \rightarrow K^+\pi^-\pi^-)$</td>
</tr>
<tr>
<td>Bkg $B^+ \rightarrow D^{*-} \mu^+ \nu_{\mu} X$</td>
<td>$(10.33 \pm 0.41 \pm 0.10)%$</td>
<td>$\times B(D^0 \rightarrow K^-\pi^+)$</td>
</tr>
<tr>
<td>Bkg $B^+ \rightarrow D^- \mu^+ \nu_{\mu} X$</td>
<td>$(0.97 \pm 0.16 \pm 0.08)%$</td>
<td>$\times B(D^- \rightarrow K^+\pi^-\pi^-)$</td>
</tr>
</tbody>
</table>

works better than for the $B^+$ background, which involves at least one additional massive particle. Therefore the corrected mass is expected to be lower for the $B^+$ background than for the signal modes. The distributions of the corrected mass are shown in Fig. 4.7. A fit to data is performed using template shapes for $B^+$ and $B^0$ fixed from simulation, and the shape of the combinatorial background obtained from the corrected mass of wrong-sign $D^\pm \mu^\pm$ combinations. The $B^+$ fractions obtained are $(16.1 \pm 0.1)\%$ for the $B^0 \rightarrow D^- \mu^+ \nu_{\mu} X$ mode, and $(8.0 \pm 0.2)\%$ for the $B^0 \rightarrow D^{*-} \mu^+ \nu_{\mu} X$ mode, where the errors are the statistical errors from the fit only. The errors on the template shapes are not taken into account. In addition, components in the fit that are not modelled (i.e. other $b$-hadron decays) contribute a few percent. When considering the possible systematic effects, the $B^+$ fractions obtained from data are compatible with Eq. 4.3.2.

### 4.4 Decay-time model

The decay time of the $B^0$ candidates is calculated via the measured decay length $L$ between the production vertex (primary vertex) and the decay vertex (secondary vertex),

$$t_{\text{rec}} = \frac{M_{B^0} L}{p_{B^0} c}, \quad (4.4.1)$$

where $M_{B^0}$ is the world-average $B^0$ mass taken from Ref. 2, and $p_{B^0}$ is the momentum of the $B^0$ candidate as obtained from the reconstructed final-state particles. This momentum is biased due to the missing neutrino and possibly other particles in the reconstruction, and is therefore corrected using a so-called $k$-factor method.
4.4. Decay-time model

Figure 4.7: Corrected $B$ mass in (left) the $B^0 \rightarrow D^- \mu^+ \nu \mu X$ sample and (right) the $B^0 \rightarrow D^+ \mu^+ \nu \mu X$ sample. Distributions from data are shown in black points. A fit to the data is done, using shapes for the $B^0$ and $B^+$ components as obtained from simulated samples, including a shape for the combinatorial background as obtained from same-sign combinations in data.

4.4.1 Determination of the $k$-factors

In order to correct the $B^0$ momentum in the decay-time model, a correction factor, called a $k$-factor, is constructed using the simulated events,

$$k = \frac{p_{\text{rec}}}{p_{\text{true}}}, \quad (4.4.2)$$

where $p_{\text{rec}}$ is the reconstructed momentum of the $B^0$, and $p_{\text{true}}$ is the generated momentum. The distributions of the $k$-factors for both decay modes are shown in the top row of Fig. 4.8. No significant dependence of the $k$-factor distribution on the true $B^0$ decay time is found, as is shown in Fig. 4.9 (left).

The dependence of the average $k$-factor on the reconstructed $B^0$ mass is shown in Fig. 4.9 (right). It can be described with a second-order polynomial,

$$\langle k \rangle(M_{\text{rec}}) = p_0 + p_1 \left( \frac{M_{\text{rec}}}{M_{B^0}} - 1 \right) + p_2 \left( \frac{M_{\text{rec}}}{M_{B^0}} - 1 \right)^2, \quad (4.4.3)$$

where $p_{0,1,2}$ are the fit parameters, $M_{\text{rec}}$ is the reconstructed $B^0$ mass and $\langle k \rangle(M_{\text{rec}})$ is the fitted average $k$-factor. The reconstructed decay time of each candidate is multiplied with $\langle k \rangle(M_{\text{rec}})$,

$$t_{\text{corr}} = \langle k \rangle(M_{\text{rec}}) t_{\text{rec}}. \quad (4.4.4)$$

The corrected decay time, $t_{\text{corr}}$, corresponds on average to the real decay time. This is shown in Fig. 4.10 where the good overlap of $t_{\text{corr}}$ with $t_{\text{true}}$ is visible. The small discrepancy at decay times $< 1$ ns is irrelevant due to a decay-time window of $[1,15]$ ps that is applied later on in the fit.
Chapter 4. \textit{CP} violation in mixing of $B^0$ mesons

![Simulation $B^0 \to D^{*+} \mu^+ \nu_\mu X$](image)

![Simulation $B^0 \to D^- \mu^+ \nu_\mu X$](image)

Figure 4.8: The normalized distribution of (top) the $k$-factors, and (bottom) the corrected $k$-factors, as obtained from simulation, for (left) the $B^0 \to D^{*-} \mu^+ \nu_\mu X$ mode and (right) the $B^0 \to D^{-} \mu^+ \nu_\mu X$ mode.

### 4.4.2 Decay-time resolution

Experimentally, there are two sources of uncertainty that smear the decay time in Eq. 4.4.1. The first originates from the error on the decay length $L$, dominated by the error on the secondary vertex position. The second comes from the uncertainty on the $B^0$ momentum, which is dominated by the spread in $k$-factors. As these two sources of uncertainty are uncorrelated the total error is given by

$$
\sigma_t = \sqrt{\left(\frac{M}{p c} \sigma_L\right)^2 + \left(\frac{\sigma_p}{p}\right)^2},
$$

(4.4.5)

where the first contribution results in a constant offset, and the second contribution increases linearly with time. These effects can be seen in simulation. Figure 4.11 shows the mean and width of the decay-time resolution as function of the true decay time. The decay-time resolution is obtained by fitting a single Gaussian function to the difference in true and reconstructed decay times in each time bin. The mean of the decay time resolution as function of true decay time is flat and close to zero. The width of the
4.4. Decay-time model

Figure 4.9: The $k$-factor distribution of the $B^0 \to D^- \mu^+ \nu_\mu X$ mode, as function of (left) the true $B$ decay time, and (right) the reconstructed $B^0$ mass, with the second-order polynomial fit overlaid. The error bars represent the spread of the $k$-factor distribution in each bin. Similar distributions are obtained for the $B^0 \to D^+ \mu^- \nu_\mu X$ mode.

Figure 4.10: Decay-time distributions in signal simulation for $t_{\text{true}}$, $t_{\text{rec}}$ and $t_{\text{corr}}$.

Gaussian increases linearly with the true decay time, and there is a barely visible offset at $t = 0$ due to the uncertainty on the decay length. The size of this offset is determined from simulation. To separately determine the resolution due to the decay length, the reconstructed decay time using the true $B^0$ momentum is compared to the true decay time, shown for both $B^0 \to D^- \mu^+ \nu_\mu X$ and $B^0 \to D^+ \mu^- \nu_\mu X$ modes in Fig. 4.12. No bias is observed, and the average resolution is about 70 fs.

The uncertainty on the momentum dominates the error on the decay time above 0.4 ps,
Chapter 4. \textit{CP} violation in mixing of $B^0$ mesons

Figure 4.11: (left) The mean and (right) the width of the decay-time resolution in simulation, as determined from single Gaussian fits to $t_{\text{rec}} - t_{\text{true}}$, as a function of $t_{\text{true}}$, for the $B^0 \rightarrow D^- \mu^+ \nu_\mu X$ mode. Similar behaviour is observed for the $B^0 \rightarrow D^{*-} \mu^+ \nu_\mu X$ mode.

Figure 4.12: Decay-time resolution in simulation when taking the true $B^0$ momentum for both modes. To avoid any bias from a decay-time acceptance (Eq. 4.4.11), only events with a decay time above 2.0 ps are used. A triple Gaussian function is fitted to the data. The weighted average resolution of the three Gaussian functions is indicated in the plots.
and originates from the spread in $k$-factors,

$$\frac{\Delta p}{p} = k_{\text{corr}} - 1,$$

(4.4.6)

where $k_{\text{corr}}$ is the $k$-factor corrected for the average $k$-factor as function of the reconstructed $B^0$ mass,

$$k_{\text{corr}} = \frac{k}{\langle k \rangle(M_{\text{rec}})}.$$

(4.4.7)

The distribution of the corrected $k$-factor behaves more symmetric than the original $k$-factors, as shown in Fig. 4.8.

### 4.4.3 Signal model

The number of decays to final states $f$ and $\bar{f}$ can be written as (Eq.1.2.42)

$$N(f, t) = N e^{-t/\tau_d} \left( 1 + A_{\text{det}} + \frac{\alpha_d}{2} + \left( A_P - \frac{\alpha_d}{2} \right) \cos(\Delta m_d t) \right),$$

$$N(\bar{f}, t) = N e^{-t/\tau_d} \left( 1 - A_{\text{det}} + \frac{\alpha_d}{2} - \left( A_P - \frac{\alpha_d}{2} \right) \cos(\Delta m_d t) \right),$$

(4.4.8)

where $N$ is a normalization factor, and $\tau_d$ is the lifetime of the $B^0$ meson. The above is implemented in a two-dimensional fit, with $t$ and the charge of the final-state muon ($f$ or $\bar{f}$) as dimensions, and $t$ is the true decay time.

Two convolutions are done in order to take into account the decay-time resolution. The first is due to the error on the decay length, which is taken into account by analytically convolving the decay time in Eq. 4.4.8 with a triple Gaussian resolution function $R(t')$ obtained from the fit in Fig. 4.12, denoted as

$$\mathcal{P}_{L-\text{conv}}(f, t) = N(f, t-t') \otimes R(t').$$

(4.4.9)

The second contribution is due to the spread of the $k$-factors, and is multiplicative in the decay time itself (see Eq. 4.4.5). This convolution is done by dividing the true decay time in Eq. 4.4.8 by the corrected $k$-factor $k_{\text{corr}}$, and summing the decay-time distribution over all bins $i$ of the corrected $k$-factor distribution, weighted by the normalized probability (i.e. the bin height, $F(k_{\text{corr}}^i)$). Finally, for the correct normalization one has to multiply the number of events by the Jacobian, $dt_{\text{true}}/dt_{\text{rec}} = k_{\text{corr}}^i$. To summarize, the convolution with the $k$-factor distribution is done as

$$\mathcal{P}_{k-\text{conv}}(f, t) = \sum_i N(f, t/k_{\text{corr}}^i)k_{\text{corr}}^i F(k_{\text{corr}}^i).$$

(4.4.10)

The decay-time acceptance of LHCb is discussed in Appendix. A. The shape used to describe the decay-time acceptance is

$$a(t) = (1 - e^{-(t-t_{\text{shift}})/\alpha})(1 + \beta t),$$

(4.4.11)
where \( t_{\text{shift}} \), \( \alpha \) and \( \beta \) describe, respectively, the acceptance “turn-on” time, the strength of the turn-on, and the upper acceptance shape factor. This simple description does not accurately describe the data below decay times of 1 ps, hence the nominal fit is only performed in the range \([1, 15]\) ps.

To summarize, the full time-dependence of the signal is described by

\[
P(f, t) = \mathcal{N} \times \sum_i \left( [N(f, t/k_{\text{corr}}^i - t')] \otimes R(t')] k_{\text{corr}}^i F(k_{\text{corr}}^i) \times a(t) \right) \tag{4.4.12}
\]

where \( \otimes R(t') \) is the first, analytic convolution with the decay-length resolution, and \( \mathcal{N} \) is a normalization factor which is left free in the fit. Since the \( B^0 \) lifetime is highly correlated with \( \beta \), the lifetime is fixed to the \( B^0 \) lifetime from Ref. \[94\]. Furthermore, in the fit to data \( A_{\text{det}} \) is fixed from external inputs (see Sec. 4.5), and \( \Delta m_{\text{fit}} \) is fixed to the world-average value \[94\].

### 4.4.4 Simulation studies

The simulated signal samples described in Sec. 4.3 are used to test the signal decay-time model (Eq. 4.4.12). The fit of Eq. 4.4.12 to the corrected decay time in the simulated \( B^0 \rightarrow D^- \mu^+ \nu_\mu X \) sample is performed, and shown in Fig. 4.13. The resulting fit parameters are shown in Table 4.4. No significant bias on the values of \( a_{\text{sl}}^d \) or \( A_P \) is observed.

The correlations between the fitted parameters are shown in Table 4.5. The correlation between the physical parameters \( a_{\text{sl}}^d \) and \( A_P \) is found to be small. The correlation between the acceptance parameters \( t_{\text{shift}} \) and \( \alpha \) is sizeable. Therefore, \( t_{\text{shift}} \) is fixed in the fit to

![Graphs showing fit results](image-url)
4.4. Decay-time model

Table 4.4: Fit results of the time-dependent fit to the simulated $B^0 \to D^- \mu^+ \nu_\mu X$ signal sample.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$0.554 \pm 0.094$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-0.036 \pm 0.003$</td>
</tr>
<tr>
<td>$t_{\text{shift}}$</td>
<td>$0.197 \pm 0.139$</td>
</tr>
<tr>
<td>$a_{\text{sl}}^d$</td>
<td>$0.012 \pm 0.011$</td>
</tr>
<tr>
<td>$A_P$</td>
<td>$-0.006 \pm 0.008$</td>
</tr>
</tbody>
</table>

Table 4.5: Correlation matrix of the time-dependent fit of the simulated $B^0 \to D^- \mu^+ \nu_\mu X$ signal sample.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$t_{\text{shift}}$</th>
<th>$a_{\text{sl}}^d$</th>
<th>$A_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1.00</td>
<td>-0.59</td>
<td>0.96</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.00</td>
<td>-0.45</td>
<td>-0.01</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$t_{\text{shift}}$</td>
<td>1.00</td>
<td>0.00</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_{\text{sl}}^d$</td>
<td></td>
<td>1.00</td>
<td>0.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_P$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

data to the value obtained from the fit to simulation. Finally, the correlation between the acceptance parameters and $a_{\text{sl}}^d$ and $A_P$ is small. This is not surprising, since the acceptance is not expected to be different between the charge-conjugate final states. It also indicates that the choice of the acceptance shape does not have a significant influence on the final result.

4.4.5 Background models

As discussed in Sec. 4.3.2, the background is dominated by $B^+$ decays and combinatorial background originating from real $b$-hadron decays. Contributions from other decays are negligible or treated as a systematic uncertainty. The background due to $B^+$ decays behaves similar to the signal decays; the main difference being that the $B^+$ mesons do not mix. The decay time is described by an exponential decay multiplied with a lower decay-time acceptance function,

$$P_{B^+}(f,t) = N_{B^+} e^{-t/\tau_{B^+}} (1 + A_{\text{det}} + A_{F,B^+})(1 - e^{-(t-t_{\text{shift},B^+})/\alpha_{B^+}}),$$

$$P_{B^+}(\bar{f},t) = N_{B^+} e^{-t/\tau_{B^+}} (1 - A_{\text{det}} - A_{F,B^+})(1 - e^{-(t-t_{\text{shift},B^+})/\alpha_{B^+}}),$$

where the normalization $N_{B^+}$ is fixed to the signal normalization multiplied by the $B^+$ fraction (Eq. 4.3.2). For simplicity of the model, no convolution for the decay-time resolution is made and there is no explicit term for the upper-decay-time acceptance. Instead, these effects are absorbed in the parameter for the lifetime, $\tau_{B^+}$, and the acceptance
parameters $t_{\text{shift},B^+}$ and $\alpha_{B^+}$, which are fixed from a fit to simulated decays. This is motivated by the fact that the lifetime of the $B^+$ meson is not of interest in this analysis. The fit of Eq. (4.4.13) to the simulated $B^+$ sample is shown in Fig. 4.14.

The detection and production asymmetry of the $B^+$ contribution cannot be disentangled due to the lack of mixing, so both are determined from external input. The detection asymmetry is taken to be the same as for the signal data; even though there is at least one additional particle missing in the decay, the kinematic spectra of the reconstructed final-state particles are nearly identical. The production asymmetry is taken from an LHCb measurement using $B^+ \rightarrow J/\psi K^+$ decays, where a charge asymmetry $A_{\text{meas}}(J/\psi K^+) = (-1.3 \pm 0.1)\%$ is measured [102]. Correcting for the measured direct $CP$ asymmetry $A_{CP}(B^+ \rightarrow J/\psi K^+) = (0.3 \pm 0.6)\%$ [94] and kaon detection asymmetry of $(-1.0 \pm 0.2)\%$ [103], results in

$$A_{P,B^+} = (-0.6 \pm 0.6)\% . \quad (4.4.14)$$

Due to the publication status at the time of this analysis, this is the number that is used. However, it is worth noting that at the time of this thesis an update can be made. The average from Ref. [94] can be updated with a new LHCb measurement of the $B^+$ production asymmetry using $B^+ \rightarrow D^0 \pi^+$ decays. A value of $A_{P,B^+} = (-0.47 \pm 0.29)\%$ is obtained when averaging over the two centre-of-mass energies [104].

The combinatorial background in the $D^0$ and $D^-$ invariant mass-distributions originates largely from real $B^0$ decays (see Sec. 4.3.2). This can be seen in the decay-time distribution and mixing asymmetry of the $D^-$ sidebands, shown in Fig. 4.15. The sideband regions

![Figure 4.14: Fit to the decay-time distribution of the simulated $B^+$ background for the $B^0 \rightarrow D^- \mu^+ \nu_\mu X$ mode. (left) Projection of the decay time, (right) projection of the charge asymmetry. The distribution of the fit residual divided by the error is shown below.](image)
4.4. Decay-time model

are defined as $M(D^-) < 1840 \text{MeV}/c^2$ and $M(D^-) > 1900 \text{MeV}/c^2$. In order to display the mixing asymmetry, the initial state is determined using flavour tagging to distinguish between mixed and unmixed decays. The time-dependent behaviour of the asymmetry between mixed and unmixed decays shows the characteristic oscillation from $B^0$. The model used for the decay time of the combinatorial background (sideband background) is

$$N_{sb}(f, t) = N_{sb}e^{-t/\tau_{sb}}(1 + A_{det, sb} + A_{P, sb} \cos \Delta m_d t)(1 - e^{-(t - t_{shift, sb})/\alpha_{sb}}),$$

$$N_{sb}(\bar{f}, t) = N_{sb}e^{-t/\tau_{sb}}(1 - A_{det, sb} - A_{P, sb} \cos \Delta m_d t)(1 - e^{-(t - t_{shift, sb})/\alpha_{sb}}).$$  (4.4.15)

Since it is unclear what the detection asymmetry of the various contributing modes would be, these decays are not used to determine $a_d^{sl}$. Instead, the parameters for the detection and production asymmetry, $A_{det, sb}$ and $A_{P, sb}$, are left free in the fit. In addition, the lifetime parameter $\tau_{sb}$ is a free parameter in the fit, to effectively take into account effects from the decay-time resolution and upper decay-time acceptance. The fit of Eq. (4.4.15) to the sidebands in the data is shown in Fig. 4.16. The fraction of combinatorial background is determined from the $D^-$ or $\bar{D}^0$ invariant-mass distribution. The invariant-mass model is described in Sec. 4.2.2.

4.4.6 Full model

The model that is fit to the data is the three-dimensional fit in either $D^-$ or $\bar{D}^0$ mass depending on the signal channel, the $B^0$ candidate decay time and the charge of the muon. The full model is the sum of the signal, the $B^{+}$ and the combinatorial background models described above. As discussed in Sec. 4.3.2, the contribution from other decays is either negligible or small enough to have an insignificant effect on the final results.

Figure 4.15: (left) Decay-time distribution of the sidebands in data for unmixed ($B^0 \rightarrow B^0$ and $B^0 \rightarrow \bar{B}^0$) and mixed ($B^0 \rightarrow \bar{B}^0$ and $B^0 \rightarrow B^0$) decays. (right) Their final state mixing asymmetry. The shape used to fit the data (Eq. (4.4.15)) is superimposed.
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Figure 4.16: Fit to the decay-time distribution in sideband data for the $B^0 \to D^- \mu^+ \nu_\mu X$ mode. (left) Projection of the decay time, (right) projection of the charge asymmetry. The distribution of the fit residual divided by the error is shown below.

4.5 Detection asymmetries

This section is dedicated to the detection asymmetries, using the methods from Chapter 3. In the $a^d_3$ analysis they are grouped into three subsections. First, the asymmetry of the $\mu^+ \pi^-$ pair where $\pi^-$ is the pion with the highest momentum, and the asymmetry of the $K^+ \pi^-$ pair where $\pi^-$ is the pion with the lowest momentum are discussed. This is sketched by Fig. 4.17. Finally, the asymmetry from the muon trigger and PID is discussed.

![Figure 4.17: The detection asymmetry is split up into two pairs of opposite-charge tracks, shown here for the $B^0 \to D^- \mu^+ \nu_\mu X$ decay.](image)
4.5. Detection asymmetries

4.5.1 The $\mu^+\pi^-$ asymmetry

Section 3.1 describes two methods to determine the $\mu^+\pi^-$ tracking asymmetry, $A_{\mu\pi}$. The first is the $J/\psi$ tag-and-probe method, which was still under development at the time when the $a_{\text{sl}}^d$ analysis was performed. The other method is the $D^*$ partial-and-full method, which was used in an earlier LHCb analysis of $a_{\text{sl}}^s$ [48]. Using only the latter would result in a large systematic uncertainty. Therefore, an alternative approach to correcting the $\mu^+\pi^-$ asymmetry is chosen. Event weights are assigned such that the kinematic distributions of the highest-momentum pion in the $B^0 \to D^{*+}\mu^+\nu_\mu X$ and $B^0 \to D^-\mu^+\nu_\mu X$ signal samples match those of the signal muon. As motivated in Sec. 3.1 the tracking asymmetry of the charge-neutral $\mu^+\pi^-$ pair is expected to be zero when their kinematic distributions are the same. Figures 4.18 and 4.19 display the kinematic distributions of both particles where the background is subtracted using the $s$Plot method [100]. The pion is softer in momentum than the muon. The weights are obtained by dividing the kinematic distributions of both particles. The weights are then normalized by multiplying them with

$$w_{\text{eff}} = \frac{\sum_i w_i}{\sum_i w_i^2},$$  \hspace{1cm} (4.5.1)

where $w_{\text{eff}}$ gives the statistical reduction of the original sample. In the $B^0 \to D^-\mu^+\nu_\mu X$ sample, a weighting in $p_T$ is found to be sufficient, while in the $B^0 \to D^{*+}\mu^+\nu_\mu X$ sample a two-dimensional weighting in $p_T$ and $\eta$ is used. Figures 4.18 and 4.19 show how well the kinematic distributions overlap before and after weighting. The weighting reduces the effective signal yields by about 35% in the $B^0 \to D^-\mu^+\nu_\mu X$ mode, and 25% in the $B^0 \to D^{*+}\mu^+\nu_\mu X$, as shown in Table 4.2.

The residual tracking asymmetry due to the small remaining difference in the kinematic distributions is determined with the $J/\psi$ tag-and-probe method (Sec. 3.1.1). It is found to be

$$A_{\mu\pi} = (0.00 \pm 0.02)\%,$$  \hspace{1cm} (4.5.2)

and is considered as a systematic error on $a_{\text{sl}}^d$ and $A_P$ in both signal channels.

The potential asymmetry due to the hadronic pion interaction is estimated as described in Sec. 3.1.3. The upper limit resulting from the simulation study is 0.07%, and is assigned as a systematic error.

In addition, the asymmetry of the PID cut on the highest-momentum pion is determined with the method described in Sec. 3.3. The results, split by signal mode and magnet polarity, are shown in Table 4.6.

4.5.2 The $K^+\pi^-$ asymmetry

The asymmetry of the $K^+\pi^-$ pair, $A_{K\pi}$, is determined using calibration charm decays $D^- \to K^+\pi^-\pi^-$ and $D^- \to K_S^0(\to \pi^+\pi^-)\pi^-$ with the method described in Sec. 3.2. It is calculated as $A_{K^+\pi^-\pi^-} - A_{K_S^0\pi^-\pi^-} - A_{K_S^0\pi^-\pi^-}$, where the kinematic distributions of the calibration samples are weighted, and $A_{K_S^0} = (-0.054 \pm 0.014)\%$.  

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![Figure 4.18: Distributions of (left) $p_T$ and (right) pseudorapidity $\eta$ of the muon compared to the highest-momentum pion for $B^0 \rightarrow D^- \mu^+ \nu_\mu X$ data sample collected in 2011, (top) before and (bottom) after weighting the pion $p_T$ distribution to match that of the muon. The distributions in 2012 data are similar.](image)

Table 4.6: Asymmetry [\%] of the PID cut on the highest-$p_T$ pion in the $a^d_{si}$ analysis, for each data-taking year and magnet polarity.

<table>
<thead>
<tr>
<th>$A_{\text{PID}}$ [%]</th>
<th>Magnet up</th>
<th>Magnet down</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011 $D^-$</td>
<td>$-0.05 \pm 0.01$</td>
<td>$0.05 \pm 0.01$</td>
</tr>
<tr>
<td>2011 $D^*$</td>
<td>$-0.27 \pm 0.01$</td>
<td>$0.09 \pm 0.01$</td>
</tr>
<tr>
<td>2012 $D^-$</td>
<td>$-0.03 \pm 0.01$</td>
<td>$0.00 \pm 0.01$</td>
</tr>
<tr>
<td>2012 $D^*$</td>
<td>$-0.04 \pm 0.01$</td>
<td>$-0.04 \pm 0.01$</td>
</tr>
</tbody>
</table>

The $K^+ \pi^- \pi^-$ and $K^0_s \pi^+$ asymmetries are significantly affected by the weighting procedure used to match the kinematic distributions between calibration and signal samples, although the average of the magnet polarities varies only by about 0.1% (0.3%) in the $B^0 \rightarrow D^- \mu^+ \nu_\mu X$ ($B^0 \rightarrow D^{*-} \mu^+ \nu_\mu X$) mode. The resulting asymmetries are displayed in Table 4.6.

In the $B^0 \rightarrow D^{*-} \mu^+ \nu_\mu X$ mode the difference of the $K^+ \pi^-$ asymmetry between the two
magnet polarities is larger than that in the $B^0 \rightarrow D^{*-}\mu^+\nu_{\mu}X$ mode. This is because the pion in the $D^{*-} \rightarrow \bar{D}^{0}\pi^-$ decay is very soft in momentum. This enhances the asymmetry due to PID criteria, as well as the asymmetry due to the material cross section and amount of material traversed. In order to determine the size of possible biases, the weighting procedure is varied as outlined in Sec. 3.2, and the fit models are varied. Any differences are assigned as a systematic error, of which the quadratic sum equates to 0.066% for the $B^0 \rightarrow D^-\mu^+\nu_{\mu}X$ mode, and 0.098% for the $B^0 \rightarrow D^{*-}\mu^+\nu_{\mu}X$ mode. The PID criterium on the kaon in the signal selection is the same as that of the calibration samples, such that no additional PID asymmetry on the $K^+\pi^-$ pair needs to be evaluated.

4.5.3 The muon trigger and PID asymmetry

The detection asymmetry due to the muon trigger and PID, $A_\mu$, is determined using the method outlined in Sec. 3.4. The selection on the muon that is probed is the hardware- and lower software-level trigger, as well as the muon PID criterium. The events in the calibration sample are weighted such that the kinematic distributions match those in the
A summary of the detection asymmetries as well as the choice of fit model, are determined by varying these choices, as discussed in which is indeed the case. For the 2011 data, the look-up-table is applied to remove the

Table 4.7: The weighted asymmetries for the $D^{-} \to K^{+} \pi^{-} \pi^{-}$ and $D^{-} \to K_{S}^{0} \pi^{-}$ modes, and the resulting value of $A_{K_{S}}$ for the $B^{0} \to D^{-} \mu^{+} \nu_{\mu}X$ and $B^{0} \to D^{*-} \mu^{+} \nu_{\mu}X$ modes, where the $K_{S}^{0}$ asymmetry is taken into account.

<table>
<thead>
<tr>
<th>$B^{0} \to D^{-} \mu^{+} \nu_{\mu}X$</th>
<th>$A_{K_{S}}$ [%]</th>
<th>$A_{K_{S}}$ [%]</th>
<th>$A_{K_{S}}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011, Magnet up</td>
<td>2.25 ± 0.06</td>
<td>0.73 ± 0.23</td>
<td>1.58 ± 0.23</td>
</tr>
<tr>
<td>2011, Magnet down</td>
<td>1.61 ± 0.02</td>
<td>0.60 ± 0.19</td>
<td>1.07 ± 0.20</td>
</tr>
<tr>
<td>2012, Magnet up</td>
<td>2.09 ± 0.02</td>
<td>0.87 ± 0.13</td>
<td>1.27 ± 0.13</td>
</tr>
<tr>
<td>2012, Magnet down</td>
<td>1.57 ± 0.03</td>
<td>0.65 ± 0.13</td>
<td>0.97 ± 0.13</td>
</tr>
</tbody>
</table>

signal. The weighted asymmetries are summarized in Table 4.8. The observed difference between data-taking years is described in Sec. 3.4 and the asymmetry averaged over magnet polarities is consistent with zero. Since the kinematic distributions of the muon are identical between the $B^{0} \to D^{-} \mu^{+} \nu_{\mu}X$ mode and $B^{0} \to D^{*-} \mu^{+} \nu_{\mu}X$ modes, the muon asymmetry is expected to be the same within statistical variations of the signal sample, which is indeed the case. For the 2011 data, the look-up-table is applied to remove the bias from the $p_{T}$ estimate of the L0 trigger, as discussed in Sec. 3.4.

The size of potential biases due to the choice of binning in the weighting procedure, as well as the choice of fit model, are determined by varying these choices, as discussed in Sec. 3.4. Any observed difference is assigned as a systematic uncertainty. The quadratic sum of these effects is included in Table 4.8.

Table 4.8: Muon asymmetries in the $a_{d}^{d}$ analysis due to the trigger and muon PID [%]. The first error is the statistical error on the $J/\psi$ samples, and the second is the total systematic error.

<table>
<thead>
<tr>
<th>$\Delta_{\mu}$ [%]</th>
<th>Magnet up</th>
<th>Magnet down</th>
</tr>
</thead>
<tbody>
<tr>
<td>2011 $D^{-}$</td>
<td>0.35 ± 0.08 ± 0.02</td>
<td>-0.23 ± 0.08 ± 0.03</td>
</tr>
<tr>
<td>2011 $D^{*-}$</td>
<td>0.31 ± 0.08 ± 0.03</td>
<td>-0.21 ± 0.07 ± 0.02</td>
</tr>
<tr>
<td>2012 $D^{-}$</td>
<td>-0.04 ± 0.05 ± 0.01</td>
<td>-0.03 ± 0.05 ± 0.01</td>
</tr>
<tr>
<td>2012 $D^{*-}$</td>
<td>-0.06 ± 0.05 ± 0.02</td>
<td>-0.02 ± 0.05 ± 0.01</td>
</tr>
</tbody>
</table>

4.5.4 Summary

A summary of the detection asymmetries $A_{det}$ is shown in Table 4.9. The total effect is calculated as the sum of the individual detection asymmetries, and used as input in the fits to the time-dependent decay rates.
4.6. Results

Taking into account the detection asymmetries as discussed in Sec. 4.5, the simultaneous fit to the $D^− (\bar{D}^0)$ mass, $B^0$ decay time and muon charge, described in Sec. 4.4, is applied to the data. The data is binned in each dimension, as is allowed due to the large size of the data sample. The fits are performed on the data for each magnet polarity and data-taking year, and are shown in Appendix B. Figure 4.20 shows the time-dependent decay rates for both modes, as well as the charge asymmetry, for both data-taking years and magnet polarities combined. The obtained values for $a_{sl}$ and $A_P$ are shown in Table 4.10. The correlation between $a_{sl}$ and $A_P$ in the fits is small.

The combination of the individual values of $a_{sl}$ is made by performing an arithmetic (unweighted) average of the results for each magnet polarity. Then, an average over the two data-taking years is made, weighted by the respective luminosity. Finally, a weighted
average is made over the two signal channels. This results in

\[
B^0 \to D^- \mu^+ \nu_\mu X : a^d_{sl} = (-0.19 \pm 0.21 \pm 0.30)\% \\
B^0 \to D^{*-} \mu^+ \nu_\mu X : a^d_{sl} = (0.77 \pm 0.45 \pm 0.34)\% \\
\text{Combined} : a^d_{sl} = (-0.02 \pm 0.19 \pm 0.30)\% \\
\text{(4.6.1)}
\]

where the first error is the combined statistical error resulting from the fits, and the second error is the systematic error resulting from the studies done in the next section. The systematic error is assumed to be fully correlated between the \(B^0 \to D^- \mu^+ \nu_\mu X\) mode and \(B^0 \to D^{*-} \mu^+ \nu_\mu X\) mode. The production asymmetry is found to be

\[
A_P(7\text{ TeV}) = (0.66 \pm 0.26 \pm 0.22)\% \\
A_P(8\text{ TeV}) = (0.48 \pm 0.15 \pm 0.17)\% ,
\]

where \(A_P\) is shown for each centre-of-mass energy separately, due to the expected dependence. A discussion of these results is given in Chapter 6.

### 4.7 Systematic uncertainties

The largest systematic uncertainty is due to the limited size of the calibration samples used in the determination of \(A_{\text{det}}\). This error is included in the systematic uncertainty on \(a^d_{sl}\). The determination of the statistical error on \(A_{\text{det}}\) is described in Sec. 4.5 along with the systematic error. The propagation of these uncertainties to \(a^d_{sl}\) and \(A_P\) is done by generating a sample using the nominal decay-time model, with enough events to be able to neglect effects due to statistical fluctuations. Fits to this large simulated sample with the same model are then applied, only with the parameter of interest varied by plus or minus its error. The deviation of the obtained values of \(a^d_{sl}\) and \(A_P\) is taken as a systematic uncertainty.

\(^1\)This is in contrast to the \(a^s_{sl}\) analysis in the next chapter, where the statistical uncertainty on \(A_{\text{det}}\) is included in the total statistical uncertainty on \(a^s_{sl}\). This follows from the different approaches made in these analyses.

Table 4.10: Results of the nominal fits to data in [%], for the four subsamples of both signal modes.

<table>
<thead>
<tr>
<th>Data Sample</th>
<th>(B^0 \to D^- \mu^+ \nu_\mu X)</th>
<th>(B^0 \to D^{*-} \mu^+ \nu_\mu X)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a^d_{sl}) (A_P)</td>
<td>(a^d_{sl}) (A_P)</td>
</tr>
<tr>
<td>2011 Magnet Down</td>
<td>0.45 ± 0.54 1.09 ± 0.37</td>
<td>2.63 ± 1.18 0.92 ± 0.80</td>
</tr>
<tr>
<td>2011 Magnet Up</td>
<td>-1.94 ± 0.62 0.25 ± 0.0043</td>
<td>-1.29 ± 1.35 0.28 ± 0.93</td>
</tr>
<tr>
<td>2012 Magnet Down</td>
<td>0.40 ± 0.34 0.78 ± 0.24</td>
<td>1.80 ± 0.74 1.54 ± 0.52</td>
</tr>
<tr>
<td>2012 Magnet Up</td>
<td>-0.39 ± 0.33 0.00 ± 0.23</td>
<td>-0.18 ± 0.72 0.18 ± 0.50</td>
</tr>
<tr>
<td>All</td>
<td>-0.19 ± 0.21 0.47 ± 0.14</td>
<td>0.77 ± 0.45 0.79 ± 0.31</td>
</tr>
</tbody>
</table>
4.7. Systematic uncertainties

Figure 4.20: Decay rate and charge asymmetry versus decay time for (left) the $B^0 \to D^- \mu^+ \nu_\mu X$ sample and (right) the $B^0 \to D^{*-} \mu^+ \nu_\mu X$ sample. The data from the two data-taking years and magnet polarities are combined and the fit results are overlaid. The number of bins in the asymmetry plots is reduced for clarity. The visible asymmetry in these plots is fully attributed to the non-zero detection and production asymmetries (not to a non-zero value of $a_{sl}^d$).

The second largest source of systematic error is due to the modelling of the $B^+$ background, and is determined in Sec. 4.7.1. Errors due to other backgrounds are determined in Sec. 4.7.2. Systematic biases from the $k$-factor approach are studied in Sec. 4.7.3, and other sources of systematic uncertainty in the fit model are determined in Sec. 4.7.4. A breakdown of all systematic errors on $a_{sl}^d$ and $A_P$ is shown in Table 4.11.

4.7.1 $B^+$ background

Since the $B^+$ background is almost indistinguishable from the signal, its fraction and the parameters of its decay-time model are fixed in the nominal fit. The uncertainty on the $B^+$ production asymmetry is obtained from external measurements (see Sec. 4.4.5), and found to be $A_{P,B^+} = (-0.6 \pm 0.6)\%$. Varying $A_{P,B^+}$ within its uncertainty in the fit to the large simulated sample results in a systematic error of 0.12% on $a_{sl}^d$ and 0.06% on $A_P$.

There are a few additional uncertainties regarding the $B^+$ background that are considered. The uncertainty on the fraction of the $B^+$ background is propagated to $a_{sl}^d$ and $A_P$ using the method with the large simulated sample described above, resulting in an uncertainty of 0.03% on $a_{sl}^d$. The parameters of the $B^+$ decay-time acceptance are determined from a fit to simulated events as described in Sec. 4.4.5. The uncertainties on these parameters are due to the simulated sample size, and are varied simultaneously in the fit to the large simulated sample, taking into account the correlations between the parameters. The effect is found to be negligible. The nominal decay-time description of the $B^+$ background is simplified by ignoring the convolutions with the $k$-factors and decay length error, and absorbing these effects in the other parameters. Using a decay-time
model for the $B^+$ background similar to that of the signal is limited by the statistical power of the simulated $B^+$ sample. The $k$-factors obtained from the simulated $B^+$ sample are shown in Fig. 4.21 (left). The fit to signal data using the complete description for the $B^+$ background, including $k$-factor convolution and upper decay-time acceptance, results in a value for $a^d_3$ that differs by 0.02% from the nominal fit. This is taken as a systematic error.

### 4.7.2 Other backgrounds

There are two significant background contributions present in the simulation cocktail of Sec. 4.3 that do not have a separate decay-time model. The first is the contribution of $B_s^0$ decays (for instance from $B_s^0 \rightarrow D^- D^+$, with $D^- \rightarrow \phi \mu^- \nu$ decays), contributing about 2% to the signal. The fast oscillations from the $B_s^0$ mixing are completely washed out by the poor decay-time resolution of the partially reconstructed decays. Therefore, the time-dependent decay rates are similar to that of the $B^+$ component, but without any production asymmetry (see also Appendix A). To assess the effect of this contribution, the high-statistics sample is fitted with an additional component describing the $B_s^0$ contribution, with a fixed fraction of 2%. The assigned systematic error is 0.03%.

The second background to consider are the $B^0 \rightarrow DDX$-type decays where $D$ is any charm meson, which contribute $(0.9 \pm 0.4)$% to the total, and are treated as signal decays. They have a smaller $k$-factor due to the larger number of missing particles. The relatively large uncertainty on this fraction could affect the uncertainty on the $k$-factor distribution. The effect this has on the measurement of $a^d_3$ is determined by varying the fraction of

![Figure 4.21](image-url)
another decay with many missing particles, namely $B^0 \rightarrow D^{*+} \tau X$, by 2% when determining the $k$-factor. The effect on $a_{sl}^d$ is found to be negligible.

A last source of potential backgrounds are baryonic decays of the type $A^0_b \rightarrow D(\pm)\mu^\pm \nu_\mu X$, where $X$ represents any neutral baryonic state. Simulated events of this type of decays were not available at the time of this analysis, and are not included in the cocktail described in Sec. 4.3. Instead, their contribution is estimated using various measurements available. The fraction of background from $A^0_b \rightarrow D(\pm)\mu^\pm \nu_\mu X$ decays is estimated to be roughly 2% using the ratio of $A^0_b$ to $B^0$ production cross sections [105], simulated efficiencies, and the branching ratio of $A^0_b \rightarrow A^+_c \pi^-$ decays [106]. The $A^0_b$ production asymmetry is estimated to be $(-0.9 \pm 1.5)\%$, determined from the raw asymmetry observed in $A^0_b \rightarrow J/\psi pK^- [107]$ and subtracting kaon and proton detection asymmetries. The uncertainty on the $A^0_b$ production asymmetry results in a systematic uncertainty on $a_{sl}^d$ of 0.07%.

### 4.7.3 Uncertainties on the $k$-factor

The decay-time model relies on a good description of the $k$-factors, which are obtained from simulation. A possible mismodelling of the production and/or decay models in the simulation could have an effect on the decay-time fit.

The production model is tested by comparing the momentum spectra of beauty mesons in data and simulation. This is done using the fully reconstructed decay mode $B^+ \rightarrow J/\psi K^+$. Weights are obtained as a function of $p(B^+)$ in order to match the momentum distribution of the simulated $B^+$ mesons to that of the data. These weights are then applied to the $B^0$ signal simulation and new $k$-factors are determined. This is shown in Fig. 4.21 (right). The effect of using this data-weighted variation of the $k$-factor histogram on $a_{sl}^d$ and $A_P$ is found to be negligible.

Concerning the decay model, the contribution of the various decays in the signal simulation cocktail to the $k_{corr}$ histogram is shown in Fig. 4.22. The branching ratios assumed for the various decays in this cocktail have an uncertainty [94]. In the $B^0 \rightarrow D^- \mu^+ \nu_\mu X$ cocktail, about 44% of events originate from $B^0 \rightarrow D^- \mu^+ \nu_\mu$ decays which has an absolute error on the branching fraction of about 2.5%. Another 44% comes from $B^0 \rightarrow D^{*-} \mu^+ \nu_\mu$ decays (where $D^{*-} \rightarrow D^- X$) or higher resonances, of which the error on the branching ratio is about 2%. In order to take into account other yet unknown decays, both fractions are separately increased by 10% in the determination of alternative $k$-factors. The deviation when using these alternative $k$-factors on the value of $a_{sl}^d$ is found to be about 0.02% for the $B^0 \rightarrow D^- \mu^+ \nu_\mu X$ mode, and negligible for the $B^0 \rightarrow D^{*-} \mu^+ \nu_\mu X$ mode.

### 4.7.4 Other fit-related systematic uncertainties

The mixing frequency $\Delta m_d$ is assumed from the world-average value [94], and is fixed in the fit for both the signal mode and the sidebands. The error on the world average is
(c.f. Eq. 4.4.11), as used in studies of values for the effect of mismodelling function with a Gaussian function, as described in Sec. 4.2. Any effect of mismodelling are doubled. Even in this very conservative test, no significant change in the obtained effect on Sec. 4.4.3 is obtained from a triple-Gaussian fit to the simulation. In order to test the be 0 from 1 to 04 ps the acceptance parameter is tested, which is of the form 1 + a possible mismodelling of the lower decay-time acceptance might affect the measurement. The effect is estimated by changing the starting point of the decay time in the fit from 1.0 ps to 0.4 ps in the fit to data. The turn-on effect of the acceptance now plays a more important role. This is reflected in a reduced error for the acceptance turn-on parameters α and t_shift, and the change in their correlation. The value for the upper acceptance parameter β remains unchanged. The sensitivity to a_d^0 is similar, although the error on A_P somewhat decreases. This is because for B^0 decay times close to zero, the measured asymmetry (Eq. 4.1.1) is proportional to A_P + A_{det}. The change in central value of a_d^0 is negligible, but the effect on A_P is 0.07%. In addition, the effect of a quadratic upper-decay-time acceptance model is tested, which is of the form 1 + βt + γt^2 (c.f. Eq. 4.4.11), as used in studies of B_s^0 → J/ψ φ in LHCb [28]. A sample is generated from the signal model using the quadratic shape, where β = 0.003 and γ = −0.002, while a fit is applied with the nominal decay-time acceptance. The deviation of a_d^0 is found to be 0.03%.

The flight-distance resolution used to convolve the decay-time model of the signal in Sec. 4.4.3 is obtained from a triple-Gaussian fit to the simulation. In order to test the effect of an underestimated flight-distance resolution, the widths of the triple-Gaussian are doubled. Even in this very conservative test, no significant change in the obtained values for a_d^0 or A_P is observed.

The model for the fit to the D^- (D^0) mass distribution is the sum of a Crystal Ball function with a Gaussian function, as described in Sec. 4.2. Any effect of mismodelling the mass shape is expected to be similar for both charge-conjugate final states, such that the effect on a_d^0 is negligible. Nevertheless, the impact of the choice of parametrization is
determined by using an alternative fit model using the sum of two Gaussian shapes. The effect on \( a_{s1}^d \) and \( A_P \) is found to be negligible. Furthermore, the choice of binning in decay time and \( D^- \) or \( D^{*-} \) mass is varied, and found to have a negligible effect on \( a_{s1}^d \) and \( A_P \).

Finally, the errors on all fixed fit parameters that are not mentioned above are varied within plus or minus one times their error. Their effect on \( a_{s1}^d \) is determined with the high-statistics method, and listed in Table 4.11.

Table 4.11: Overview of all contributions to the systematic uncertainty on \( a_{s1}^d \) and \( A_P \). Entries marked with “-” are found to be negligible. When different from the \( B^0 \to D^-\mu^+\nu_\mu X \) mode, the value for the \( B^0 \to D^{*-}\mu^+\nu_\mu X \) mode is given in parentheses. The contributions from the individual detection asymmetries are multiplied by two (see Eq. 4.1.1) to estimate the effect on \( a_{s1}^d \), while the effect of the total detection asymmetry on \( a_{s1}^d \) and \( A_P \) is estimated with a high-statistics simulated sample, as explained in the text.

<table>
<thead>
<tr>
<th>Source of uncertainty</th>
<th>( a_{s1}^d ) (7 TeV)</th>
<th>( A_P ) (7 TeV)</th>
<th>( A_P ) (8 TeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^+ ) background:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B^+ ) production asymmetry</td>
<td>0.12</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>( B^+ ) fraction</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( B^+ ) acceptance</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( B^+ ) decay time model</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Other backgrounds:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( B^0 ) component</td>
<td>0.03</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>( A_0 ) component</td>
<td>0.07</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>( B^0 \to D^{(*)-}D^+_\pi ) decays</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( k )-factor distribution</td>
<td>0.02 (0.01)</td>
<td>0.01 (-)</td>
<td>0.01 (-)</td>
</tr>
<tr>
<td>Knowledge of ( \Delta m_d ):</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Other fit related systematics:</td>
<td>0.04 (0.03)</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>Detection asymmetry:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( A_{K\pi} )</td>
<td>0.26 (0.30)</td>
<td>0.20 (0.21)</td>
<td>0.14 (0.17)</td>
</tr>
<tr>
<td>Statistical uncertainty</td>
<td>0.16 (0.18)</td>
<td>0.15 (0.14)</td>
<td>0.09 (0.10)</td>
</tr>
<tr>
<td>Systematic uncertainty</td>
<td>0.13 (0.20)</td>
<td>0.07 (0.10)</td>
<td>0.07 (0.10)</td>
</tr>
<tr>
<td>( A_{\mu} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statistical uncertainty</td>
<td>0.06</td>
<td>0.06</td>
<td>0.03</td>
</tr>
<tr>
<td>Systematic uncertainty</td>
<td>0.06</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>( A_{PID} )</td>
<td>0.07</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>( A_{\mu\pi} )</td>
<td>0.04</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Quadratic sum</td>
<td>0.30 (0.34)</td>
<td>0.22 (0.23)</td>
<td>0.17 (0.20)</td>
</tr>
</tbody>
</table>
4.7.5 Further consistency checks

The results for magnet polarity up and down are expected to be statistically compatible, when taking into account the detection asymmetries. Due to the large correlation of the systematic uncertainties, only the statistical error from the fit and the statistical component of the error on the detection asymmetry are considered, and added in quadrature. The comparison is shown in Fig. 4.23. In the 2011 data, the compatibility of $a_{sl}^d$ is around two standard deviations. This discrepancy reduces to a little over one standard deviation for the larger-statistics 2012 sample. The production asymmetry is compatible in all scenarios. It should be noted that the size of some systematic uncertainties on the detection asymmetries depend on the magnet polarity. An example is the variation of binning in the weighting step of the $K^+\pi^-$ asymmetry. This is not taken into account in this comparison, and might explain the differences between the results for magnet polarity up and down.

![Figure 4.23](image_url)

Figure 4.23: Results for (left) $a_{sl}^d$ and (right) $A_P$ on samples divided by year and magnetic field polarity. In blue is the $B^0 \rightarrow D^-\mu^+\nu_\mu X$ mode, in red the $B^0 \rightarrow D^{*-}\mu^+\nu_\mu X$ mode.

A check is done on the $B^0 \rightarrow D^-\mu^+\nu_\mu X$ sample by placing fiducial cuts where the muon asymmetry is large. Large asymmetries occur in momentum regions where muons of one charge are bent out of the geometrical acceptance, depending on the magnet polarity. In addition, the region where muons of one charge pass through different quadrants of the muon stations (e.g. from left to right) is removed. The regions are defined as

$$|p_x| < 0.317(p - 3400\,\text{MeV/c}),$$

$$|p_x| < 600\,\text{MeV/c} \quad \text{or} \quad |p_x| > 1100\,\text{MeV/c},$$

and remove about 10% of data. The resulting values for $A_P$ are compatible with the nominal results. The values for $a_{sl}^d$ are compatible within their statistical error.

It is possible that the detection asymmetries are affected by changes in software or hardware over periods of time. Therefore, the data are split up into seven run periods over 2011 and 2012. The detection asymmetries are found to be stable over these periods. The nominal fit is repeated for each period, and the values of $a_{sl}^d$ are found to be stable as
well, as shown in Fig. 4.24 (left). Furthermore, in case there is more than one PV in the event, it is possible that the $B$ decay is associated to the wrong PV and the decay time is biased. Therefore, the data are split up into events with one, two and more than two PVs. The decay-time distributions are found to be very similar, and the values of $a^d_{sl}$ and $A_P$ are consistent, as shown in Fig. 4.24 (right). These consistency checks give confidence in the result and show that the systematic uncertainty is correctly estimated.

Figure 4.24: Results of $a^d_{sl}$ using the $B^0 \rightarrow D^- \mu^+ \nu_\mu X$ mode, split up into (left) run blocks and (right) number of PVs.
Chapter 4. \textit{CP} violation in mixing of $B^0$ mesons