Chapter 5

Second-best road taxes in polycentric networks with distorted labor markets

5.1. Introduction

The taxation of externalities in road networks has received considerable attention in the literature of transport economics during the last decades. To a large extent, this attention was directed to a series of second-best issues arising from the implementation of pricing schemes that leave certain links, routes, or areas of the network untaxed. It has been shown in a variety of stylized settings that this partial taxation pushes the optimal externality taxes below or above their Pigouvian levels. Some earlier contributions in the first- and second-best literature have derived rules for optimal road pricing in a generic static network. For instance, Verhoef (2002a; 2002b) offers a general analytical solution for the second-best problem where not all links of a congested network can be charged; an algorithm based on this analytical solution is then tested on a medium size network. Also, van Dender (2004) shows that constraints in network pricing can cause the optimal toll to deviate in a complex way from the marginal external cost of congestion.

The above partial taxation is unintentional, in the sense that it is always suboptimal to a Pigouvian tax rule, but the regulator cannot impose the latter because of some exogenous hampering factor (e.g. political acceptability, implementation costs etc.). Because no other distortionary tax or subsidy is considered elsewhere in the economy (even in the transport system), as soon as the hampering factor is removed, optimal taxes return to their Pigouvian levels.

The advances in double dividend theory have highlighted another reason for which second-best settings may emerge. This regards the existence of at least one distortionary tax (or subsidy) somewhere in the economy, even inside the transport system (e.g. public transport subsidies). Similar to the case of partial taxation, the presence of a distortionary tax causes

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95 Throughout the entire paper the term Pigouvian level of a tax refers to a tax level that is equal to the marginal external cost of congestion. Also, the term quasi first-best Pigouvian tax refers to an externality tax that is set to its Pigouvian level but its optimal value deviates from that.

96 These contributions assume that both the residence and job location, i.e. the OD pair of the commuter, is fixed.

97 Goulder (1995b) and Bovenberg (1999) provide excellent synopses of the existing literature in the field.
deviations of the optimal tolls from their Pigouvian levels.\footnote{In fact, as in the partial taxation case (see for instance Verhoef \textit{et al.}, 1996) optimal externality taxes may turn out to be negative even when the entire network can be taxed.} But the critical differentiator between the two streams is that the deviation in the latter case may be welfare increasing, something that is not possible in the case of \textit{partial taxation}. This is because the Pigouvian equilibrium in the presence of a distortionary tax is suboptimal (or quasi first-best) to begin with. Parry and Bento (2001) highlighted the case of an optimal negative road tax for a single-link network with exogenous residential and working locations; they juxtaposed this tax against the quasi first-best Pigouvian toll, which was shown to be welfare decreasing.

However, the use of a single-link network by Parry and Bento (2001) did not allow for a real merge of the two literature streams. This merge has been recently explored by Tikoudis \textit{et al.} (2015b), who show that in a serial monocentric network, the introduction of partial taxation (in the form of a \textit{cordon toll} with \textit{lump-sum revenue recycling}) can be welfare improving with the preexisting labor tax and the rest of determining factors set in accordance with the hypotheses made in Parry and Bento (2001).\footnote{In a serial monocentric network, the cordon toll is expressed as two series of untaxed consecutive links. The first extends from the edge of the serial network, \textit{i.e.} the CBD, to some given node \(x\); the second from node \(x + 1\) to the city fringe. All commuters that live further away than node \(x\) make use of the critical road link that connects nodes \(x\) and \(x + 1\) and are charged a uniform fee.} On the other hand, the quasi first-best Pigouvian toll is shown to generate significant welfare losses. The key requirements for the above result to emerge are: (i) that the (positively priced) cordon toll imposed in a certain distance from CBD is taxing eclectically only the subgroup of the population that provides labor relatively inelastically, and (ii) that the elasticity of labor supply falls with distance from the CBD.

This finding gives rise to a new policy-oriented question: \textit{can partial taxation with lump-sum revenue recycling generate welfare gains in a generic polycentric network too?} That is, can a system of cordon tolls be welfare improving without using the revenue to reduce the background labor tax of the commuters? Exactly as in the case of a monocentric city (see Chapter 2), a \textit{positive} answer would imply that Pareto improvements may result without any sophisticated revenue-recycling programs or pricing schemes. This would generalize the finding of Chapter 2 and would, to some extent, favor a general shift of the attention on complex tax reforms (that usually involve multiple levels of government) to simpler, easier-to-implement schemes. On the other hand, a \textit{negative} answer would mean that the design of the road tax (specifically in a polycentric network) may not be separable from the economy-wide labor tax in the background. We investigate this research question in a setting where the critical underlying factor, \textit{i.e.} the (general equilibrium) elasticity of labor supply varies spatially in a fairly complex way. That is, unlike monocentric settings where it is determined entirely by household location (\textit{e.g.} Tikoudis \textit{et al.} 2015b; Verhoef, 2005), in polycentric settings the location of the job, the chosen route and transport mode play a decisive role.

To focus on the above research question, this chapter introduces a stylized \textit{general equilibrium network} model that is strongly related to previous work of Anas and Kim (1996), Anas and Liu (2007), Tscharaktsiew and Hirte (2010) as well as, more loosely, to relevant CGE
models (see for instance Böhringer and Rutherford, 2007). Anas and Liu (2007) introduced a polycentric model for the wider metropolitan area of Chicago, and a recent application in order to evaluate the potential welfare effects of a cordon toll in the area (Anas and Hiramatsu, 2013).100 In their work, the interaction of transport with the markets of housing and labor is captured in a detailed way, since residence and job locations are endogenous. However, the distortions generated in the respective markets are not considered. Subsequently, the marginal social benefit of road use may be understated or overstated, depending on the relative conditions in these markets (taxes, subsidies, regulations). Our framework accounts explicitly for a double dividend setting, with a realistic tax rate on labor income.

The proposed model has a clear geographical reference. The area of Randstad is a polycentric urban conglomeration in western Netherlands, which comprises the country's three largest urban areas (Amsterdam, Rotterdam and the Hague). The region is of considerable economic significance; while it covers only 20% of the country's land area, at least 40% of the population resides there, and half of the national income is generated within its boundaries. Despite being a prosperous region, it has experienced lower productivity growth compared to other regions in the Netherlands and Europe for a series of years (annually 1.7% over the period 1995-2005). It is characterized by large commuting flows between zones and severe congestion during the peak hours. The territorial review by OECD (2007) places heavy congestion and the incoherency of public transport system as the most important drivers of this sluggish growth. Roughly 80% of traffic jams in the Netherlands in 2005 occurred in Randstad. The congestion problem is deteriorated because the regional public transport is relatively fragmented, i.e. the coherence between the multiple operators and facilities is limited, resulting in a suboptimal use of the public transport system. Broersma and van Dijk (2008) use a growth accounting exercise to disentangle the positive contribution of agglomeration from the negative contribution of congestion. Using Dutch regional industry data for the period between 1995 and 2002, they show that the effect of the latter was large enough to outweigh the effect of the former, leading to the observed slow multifactor productivity growth.101

The model is calibrated to fit a series of stylized facts characterizing the behavior of the average household (expenditures shares, allocation of time, etc.) and the characteristics of Randstad region: the general spatial lay-out and network, the population and employment share of each zone, the average commuting speed of modes, modal split, and the relative land rents, housing prices, wages and floor-to-area ratios. We use the model to explore various pricing schemes (systems of uniform and differentiated cordon tolls around the major cities, as well as Pigouvian taxes in the entire network) accompanied by two distinct revenue recycling programs: i) lump-sum transfers and ii) labor tax cuts. In line with more stylized models, a Pigouvian road tax under lump-sum revenue recycling is shown to generate considerable welfare losses.

100 See also Rhee et al. (2014).
101 Other underlying factors identified by OECD (2007) include distortions originating from the labor and, especially, the housing market, where a series of land-use practices (e.g. density regulations) add burden to the social cost of public transport provision.
Surprisingly, Pigouvian tolls are also welfare decreasing even when the road tax revenue is used to finance labor tax cuts. By computing the optimal road tax for each road link, we find that the latter may not only lie far below its Pigouvian level, but it may also be negative in a large part of the network. We establish a clear connection of the above result with the double-dividend theory by approximating the Pigouvian, tax-interaction and revenue recycling effects for each link. The approximations reveal that, at the margin of the no-toll equilibrium, the tax-interaction effect in these links is strong enough to outweigh the combined Pigouvian and revenue-recycling effect.

A system of cordon tolls around the largest destination nodes leaves most of the negatively-priced links untaxed and is found to produce welfare gains, provided its revenue is used to finance a small (and thus politically feasible) downward adjustment of the uniform labor tax rate. However, these gains are found to be persistently small (almost negligible) for plausible parameter values and labor tax rates. This result emerges from the explicit combination of a polycentric setting and a background distortionary tax as it is in sharp contrast to corresponding findings from: (i) studies considering monocentric settings with (Tikoudis et al., 2015b) or without (Mun et al., 2003; Verhoef, 2005) pre-existing distortionary tax and (ii) studies in polycentric settings without any parallel distortion (Mun et al., 2005). Sensitivity analysis with respect to the labor tax reveals, however, that as the background tax distortionary mechanism becomes weaker (i.e. the labor tax shrinks) the relative efficiency of the cordon system converges to numbers similar to those predicted by Mun et al. (2005). Despite generating small gains, the cordon toll system remains preferable to the Pigouvian toll, as the latter reduces welfare for a considerable range of plausible values of the distortionary tax and for parametric constellations that continue reflecting the desired characterization of the benchmark equilibrium.

The findings bear significant policy implications. First, they highlight that non-marginal, revenue-neutral tax swaps between externality and distortionary taxes may be welfare-reducing even if the underlying externality tax scheme is optimal from an environmental point of view. Therefore, two policy components that would be considered to be optimal in two separate settings, i.e. a Pigouvian tax (that would be optimal in a setting with space but without parallel distortions) and a horizontal-by-default labor tax cut (that would the default optimal way to recycle the externality tax revenue in a spaceless world) may turn out to be detrimental when combined in a third, interim setting that includes both space and a parallel distortion. Second, the above may motivate sophisticated policies, whose optimal design is based primarily on complex fiscal (i.e. the distortionary impact of the externality tax adjustment) rather than environmental information (i.e. the abatement effect of that adjustment). These policies are not exhausted in adjusting the levels of the externality taxes (above or below their Pigouvian levels); they may also expand to more exotic revenue-neutral tax swaps, in which the cut of the distortionary tax is adjusted over space to mirror the spatial variation of tax-interaction effects.

Finally, the motivation for such interventions becomes stronger if the efficiency of alternative policies involving partial taxation of externalities (that bear significantly lower design and implementation costs) is low enough to provide a practical remedy to the issue. Combined
with the findings of Chapter 2, the analysis shows that the latter is more likely to happen as the setting becomes more polycentric and as the level of the background distortion increases. In turn, this highlights the way spatial configuration interacts with the pre-existing distortionary mechanisms and the importance of that interaction in the correct design of urban tolls.

The chapter has the following structure: Section 5.2 presents the behavior of the various agents and the mechanics of the general, stochastic user equilibrium. Section 5.3 presents the data used in the calibration of the model. Section 5.4 provides the policy and sensitivity analyses and Section 5.5 concludes. The technical appendices provide the computational details of the solution algorithm (Appendix 5.B), the calibration algorithms (Appendix 5.C) and the approximations of the key double-dividend effects (Appendix 5.F).

5.2. Model

The model proposed in this section is a network-based, polycentric extension of the general equilibrium monocentric city models by Verhoef (2005), Tikoudis et al. (2015a; 2015b), and is in line with the existing General Equilibrium models based on discrete-continuous household optimization. These contributions include (but are not limited to): Anas and Kim, 1996; Anas and Xu, 1999; Anas and Liu, 2007; Tscharaktschiew and Hirte, 2010; Anas and Hiramatsu, 2012; Hirte and Tscharaktsiew, 2013; Dröes and Rietveld, 2015. The commonalities and differences of the current work vis-à-vis those contributions are discussed along the presentation of the model.

5.2.1. Space, network representation and discrete choice

Economic activity takes place in an ordered set of \( J \) zones (each represented by a single node), \( J \).\(^{102}\) Let the ordered subsets \( J_R \) and \( J_W \) denote the locations that host residences and jobs respectively, with \( J = J_R \cup J_W \). Throughout the text, the subscript \( i \) is used to denote an arbitrary zone in the ordered set \( J_R \) that serves as a residential node, i.e. \( i \in J_R \). Similarly, the subscript \( j \) is used to denote an arbitrary zone in the ordered set \( J \) that serves as an employment node, i.e. \( j \in J_W \). Every zone is characterized by mixed land-use, in this case \( J_R \cap J_W = J \). Let the set \( C_{OD} = J_R \times J_W \) denote the Cartesian product of sets \( J_R \) and \( J_W \), i.e. the set that contains all possible pairs of residential and employment locations. Each element \( a_{ij} \in C_{OD} \) is an origin-destination pair (hereafter, OD pair).

Two arbitrary zones, \( s \) and \( e \), are neighboring if there is at least one transport link \( l_{se}^{(m)} \) starting at \( s \) and ending at \( e \), where the subscript \( m \) denotes the type of transport network the

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\(^{102}\) The chapter deals explicitly with discretized space. For a continuous-space model with mixed land-use, the interested reader is referred to the well-known contribution by Lucas and Rossi-Hansberg (2002). Continuous space in this chapter’s setting would introduce additional sensitivity and computational burden to the existing one without providing further insights. Recently, Dong and Ross (2015) identified a source of systematic imprecision in the simulations of Lucas and Rossi-Hansberg (2002). They offered revised simulations consistent with rents patterns presented in earlier literature.
link belongs to (e.g. road, rail, etc.). Links are directed, thus \( l^{(se)}_m \neq l^{(es)}_m \). A route \( q \) is defined as a sequence (i.e. an ordered list) of links such that, for each pair of consecutive links in the sequence, \( l^{(se)}_m \) and \( l^{(s'e')}_{m'} \), it holds that \( e = s' \), although it can be that \( m \neq m' \), i.e. paths can be multimodal. However, an arbitrary path cannot reach the same node twice, i.e. cyclical paths that contain at least two links, \( l^{(se)}_m \) and \( l^{(s'e')}_{m'} \), for which it holds that \( s = s' \) or \( e = e' \) or \( e = s' \) or \( e = e' \) are excluded.

For each OD pair \( a_{ij} \) in \( C_{OD} \) there is a set of corresponding possible routes, which we denote by \( Q(a_{ij}) \). It is straightforward that, if origin zone, \( i \), and destination zone, \( j \), are neighboring, then it holds that any \( l^{(ij)}_m \in Q(a_{ij}) \). An alternative, \( a \), is a set that contains the OD pair \( a_{ij} \) and a route \( q \in Q(a_{ij}) \), i.e. \( a = \{a_{ij}, q\} = \{i, j, q\} \). The choice set, denoted by \( C \), contains all possible alternatives.

5.2.2. Households

Households can locate in any zone \( i \) and supply labor in any zone \( j \). For simplicity, we normalize the exogenous population, \( N \), to one. For each feasible alternative, \( a = \{a_{ij}, q\} \), the household maximizes the quasi-linear utility function:

\[
U_a = \pi_0 y_a + \pi_1 \left( \frac{s^a T^\beta_{F_a}}{\alpha x_a} \right)^\gamma,
\]

(5.1)

where \( y_a \) corresponds to the general consumption of a composite good and \( x_a \) to the consumption of a good composed by housing consumption, \( s_a \), and leisure, \( T_{F_a} \) (hereafter, \( x_a \) is referred to as the lifestyle choice). The marginal utility of income is constant and equal to \( \pi_0 \). Given that the parameters of the Cobb-Douglas subutility function for the lifestyle choice are such that \( \alpha < 1, \beta < 1 \) and \( \alpha + \beta = 1 \), the marginal utility with respect to the residential space and leisure is diminishing for \( \gamma < 1 \). The total time endowment of the household, \( T \), is spent on commuting from \( i \) to \( j \), \( T_{Ca} \), working, \( T_{La} \), and leisure, \( T_{F_a} \):

\[
T = T_{Ca} + T_{La} + T_{F_a}.
\]

(5.2)

Labor supply is inelastic throughout a working day, which is of fixed duration, \( t_L \), independent of working location. The household anticipates every trip to work to require \( t_a \) units of time. This anticipated commuting time equals the endogenously determined expected commuting time, \( \hat{t}_q \), in the stochastic user equilibrium (see below), where the subscript \( q \) refers to the route associated with alternative \( a = \{a_{ij}, q\} \). For \( D_{Wa} \) working days the time constraint becomes:

\[
T = D_{Wa} (t_L + t_a) + T_{F_a}.
\]

(5.3)
Normalizing the duration of the working day, \(t_L\), to 1, the above constraint becomes:

\[
T = D_{Wa} (1 + t_a) + T_{Fa} \iff D_{Wa} = \frac{T - T_{Fa}}{1 + t_a}.
\]

(5.4)

The net wage per working day is defined as the difference between wage in zone \(j\), \(w_j\), the labor tax, \(\tau_L\), and the expected pecuniary cost of commuting under the choice of alternative \(a\), \(i.e. c_a\).

The full income, \(M_a\), of the household that has chosen alternative \(a = \{a_{ij}, q\}\) is the maximum income that can be realized when leisure time is zero. That is:

\[
M_a = B + B_\ell + \left(\frac{(w_j(1 - \tau_L) - c_a)}{1 + t_a}\right) T,
\]

(5.5)

where \(B\) denotes a lump-sum transfer from the government to the household and \(B_\ell\) is the income from land rents, which are returned to households lump-sum. Both \(B\) and \(B_\ell\) are exogenous (from the viewpoint of the household) and independent of any element composing the alternative \(a = \{a_{ij}, q\}\). We refer to this type of redistribution as horizontal revenue recycling.

For simplicity, we assume that intra-zonal travel time and cost is zero, \(i.e. c_a = t_a = 0\) if \(a = \{a_{ij}, q\} = \{i, j, q\}\) is such that \(i = j\). The full income can be used to buy back leisure at the shadow price of time, \(\left(w_j(1 - \tau_L) - c_a\right)/(1 + t_a)\), (hereafter the value of time, denoted by \(v_a\)) and for the consumption of the composite good and residential space. The budget constraint can then be written as:

\[
B + B_\ell + v_a T = v_a T_{Fa} + p y_a + p_{Hi} s_a,
\]

(5.6)

where \(p\) is the (uniform in the entire region) price of the composite good (hereafter normalized to one) and \(p_{Hi}\) the price of housing per unit of space at the zone indexed by \(i\). By definition, both housing consumption and leisure are essential, thus \(s_a > 0\) and \(T_{Fa} > 0\). Furthermore, leisure is upper-bounded by the total time endowment, therefore \(T_{Fa} < T(= T\ if\ \Delta_{Wa}^* = 0)\), and consumption has to be non-negative, \(i.e. y_a \geq 0\). To maximize (5.1) subject to (5.6) and the above non-negativity constraints we set up the Lagrangian:

\[
\mathcal{L} = \pi_0 y_a + \pi_1 \left(s_a^\alpha T_{Fa}^\beta\right)^\gamma - \psi[v_a T_{Fa} + p y_a + p_{Hi} s_a - (B + B_\ell + v_a T)]
\]

\[+ \theta_c y_a + \theta_f (T - T_{Fa}),\]

(5.7)

and solve the system of the three first order conditions equations \(\mathcal{L}_y = 0, \mathcal{L}_s = 0, \mathcal{L}_\ell = 0\) and the budget constraint in (5.6). For an interior optimum the corresponding four unknowns are \(y_a, s_a, T_{Fa}\) and the Lagrangian multiplier, \(\psi\), since the complementary slackness conditions require
that $\vartheta_C = \vartheta^U_C = 0$. Solving the system yields the Marshallian demand functions for housing and leisure time respectively:

$$s^*_a = \left(\frac{p_{Hi} \pi_0}{\alpha y_1}\right)^{\frac{1}{y-1}} \left(\frac{\alpha v_a}{\beta p_{Hi}}\right)^{\frac{\beta y}{y-1}}, \quad (5.8)$$

$$T^*_{Fa} = \left(\frac{p_{Hi} \pi_0}{\alpha y_1}\right)^{\frac{1}{y-1}} \left(\frac{\alpha v_a}{\beta p_{Hi}}\right)^{\frac{\beta y}{y-1}}. \quad (5.9)$$

Inserting (5.9) into (5.4) yields the optimal labor supply for alternative a:

$$D^*_Wa = \frac{T - \left(\frac{p_{Hi} \pi_0}{\alpha y_1}\right)^{\frac{1}{y-1}} \left(\frac{\alpha v_a}{\beta p_{Hi}}\right)^{\frac{\beta y}{y-1}}}{1 + t_a}. \quad (5.10)$$

Optimal consumption, $y^*_a$, can be computed by inserting (5.8) and (5.9) into (5.6). Substituting $y^*_a$, $s^*_a$ and $T^*_{Fa}$ into the objective function and allowing for an alternative-specific constant, $z_a$, yields the indirect utility of alternative a:

$$V^*_a = V(w_j, p_{Hi}, \tau_L, c_a, t_a, B, B_t) = z_a + \pi_0 y^*_a + \pi_1 (s^*_a T^*_{Fa})^\gamma. \quad (5.11)$$

The alternative-specific constant is the sum of: i) a residential-specific constant, $z_H$, that captures the average utility of locational characteristics (e.g. amenities, ambient pollution) not modeled specifically in zone i, ii) an employment-specific constant, $z_f$, that captures the average utility of non-pecuniary or time characteristics of the average job offered in zone j (e.g. prospects for a better future job arrangement due to spatial concentration of jobs), iii) a mode-specific constant, $z_M$, (discussed below) and iv) a sum of link-specific constants, $z_q$, capturing the average utility of non-pecuniary or time characteristics of the links (e.g. the presence of a gas station or other facility) that form the route $q$ involved in alternative $a = \{a_{ij}, q\}$. Therefore:

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103 Corner optima with zero consumption are found by setting $y_a = 0$ and solving the same system for $\vartheta_C$, $s_a$, $T^*_a$ and $\psi$. Similarly, corner optima with zero labor supply are found by setting $T^*_a = 0$ and solving the system for $y_a$, $\vartheta^U_a$, $s_a$, and $\psi$. The requirements for an admissible solution are that the remaining endogenous variables lie in the interior and that the respective multiplier ($\vartheta_C$ in the first case and $\vartheta^U_a$ in the second) is positive. Zero consumption is an artifact of the quasi-linear preference relation in (5.1). However, the chosen parameters rule out the possibility of a corner solution (in which at least one of the conditions $D^*_Wa = 0$ and $y^*_a = 0$ holds) for any given alternative in the choice set. Later on we discuss the rationale behind the choice of a preference relation characterized by constant marginal utility of income.
\[ z_a = z_{II} + z_{JJ} + z_M + \sum_{l_{m}^{(se)} \in q} z_{l_{m}}^{(se)}. \] (5.12)

The *mode-specific constant*, \( z_M \), captures the average (dis)utility of commuting stemming from factors that are not modelled explicitly: waiting times, changing from a private to a public mode and vice versa, in-vehicle-comfort, cruising time, *etc*. More specifically, it is assumed that:

\[ z_M = \sum_m I(q, m) \cdot z_m + I_t(q) \cdot z_t, \] (5.13)

where the indicator function \( I(q, m) \) equals one if route \( q \) makes use of mode \( m \) (zero otherwise), the indicator function \( I_t(q) \) equals one if route \( q \) involves a transit from a private to a public mode (or vice versa, otherwise zero), \( z_m \) is the average disutility inflicted to the individual by the use of mode \( m \) (compared to those that do not have to commute, *i.e.* individuals which choose \( a = \{a_{ij}, q\} \) such that \( i = j \)), and \( z_t \) the average disutility of a mode change. Finally, a stochastic term, \( \varepsilon_a \), which is *i.i.d. extreme value type I* (EV type I) across alternatives in \( \mathcal{C} \), is added to (5.11) in order to capture the rest of the factors that are omitted in the model and may determine the choice of \( a = \{a_{ij}, q\} \). Total utility is, thus:

\[ U_a^* = z_{II} + z_{JJ} + \sum_m I(q, m) z_m + I_t(q) z_t + \sum_{l_{m}^{(se)} \in q} z_{l_{m}}^{(se)} + \pi_0 y_a^* + \pi_1 \left( s_a^{*a} T_{Fa}^{*b} \right)^Y \] (5.14)

\[ + \varepsilon_a. \]

Due to the inclusion of alternative specific constants, the error term has a mean equal to zero by construction and standard deviation equal to \( \lambda \left( \pi / \sqrt{6} \right) \), where \( \lambda \) is the scale parameter of the EV type I distribution. Because exogenous income, \( B + B_t \), does not appear in (5.8) or (5.9) it is straightforward that the marginal (systematic) utility of income is constant and equal to \( \pi_0 \).\(^{104}\)

Because the error component \( \varepsilon_a \) is additive to the systematic utility and follows a *Generalized Extreme Value* distribution, the expectation of the maximum utility (hereafter, \( E_{max} \)) that can be derived when facing the choice set \( \mathcal{C} \) is the well-known *logsum* expression:

\[ E_{max} = \lambda \left[ \mathcal{E} + \log \sum_{a \in \mathcal{C}} \left( \frac{\exp(V_a^*)}{\lambda} \right) \right], \] (5.15)

\(^{104}\) See Chapter 4 for the closed form expression of \( V_a^* \). Differentiating that with respect to the exogenous income yields \( \pi_0 \).
where $\mathcal{E} \approx 0.5772$ is the Euler constant. The resulting logit choice probability for each alternative $a$ in the choice set $\mathcal{C}$ is:

$$p_a = \frac{(\exp(V_a^*)/\lambda)}{\sum_{a \in \mathcal{C}} \exp(V_a^*)/\lambda}.$$  

(5.16)

5.2.3. Firms

A competitive, representative firm is located in each zone $j \in \mathcal{J}$ and produces a zone-specific intermediate output, $Q_j$, under constant returns to scale, using capital ($K$), and labor ($L$):

$$Q_j^S = A_j K^\delta L^{1-\delta},$$  

(5.17)

where $A_j$ denotes the zone-specific total factor productivity. The zero profit condition, implies that the price of the good produced in zone $j$, $p_j$, is equal to the unit cost:

$$p_j = \frac{1}{A_j} \left\{ \frac{\delta}{1-\delta} (1-\delta) + \frac{1-\delta}{\delta} \right\} R^\delta w_j^{1-\delta},$$  

(5.18)

where $w_j$ is the local equilibrium wage, and $R$ the exogenous price of capital. The conditional factor demands for labor, capital and land can be computed using the Shephard’s lemma, i.e. by differentiating (5.18) with respect to the corresponding price of the input and multiplying with the level of output, $Q_j$. Thus, labor demand is:

$$L_j^D = \frac{1}{A_j} \Phi (1-\delta) R^\delta w_j^{-\delta} Q_j^S,$$  

(5.19)

and capital demand is:

$$K_j^D = \frac{1}{A_j} \Phi R^{\delta-1} w_j^{1-\delta} Q_j^S.$$  

(5.20)

An assembly industry combines the $J$ distinct intermediate goods (which are bought from the local firms, each at price $p_j$) to produce the composite good demanded by the consumers and by the rest of the world. The amount of the composite good produced is given by the Cobb-Douglas production function:

$$Y = \prod_{j \in \mathcal{J}} (Q_j^S)^{\zeta_j},$$  

(5.21)

where $\zeta_j$ is the share of intermediate good produced in zone $j$ in the total cost of $Y$, and $\sum_{j \in \mathcal{J}} \zeta_j = 1$. The associated minimum cost function is:
\[ c(Y) = Y \left( \prod_{j \in \mathcal{J}} p_j^{\zeta_j} \right) \left( \sum_{j \in \mathcal{J}} \omega_j \right) \]

where the auxiliary parameter \( \omega \) is:

\[ \omega_j = \frac{(\sum_{k \neq j} \zeta_k)}{\prod_{k \neq j} \zeta_k} \]

The conditional factor demand for each intermediate can be derived using Shephard’s lemma. This is:

\[ Q_j^D = \frac{\partial c(Y)}{\partial p_j} = \zeta_j Y p_j^{\zeta_j-1} \left( \prod_{k \neq j} p_k^{\zeta_k} \right) \left( \sum_{j \in \mathcal{J}} \omega_j \right) \]

Capital and labor are not used in the combining process.

5.2.4. Developers

A competitive, representative developer produces homogenous residential space, \( s \), using capital (\( K \)) and land (\( X \)). We assume a Cobb-Douglas production function:

\[ s_i^S = K^\theta X^{1-\theta}. \]

Just like the ordinary firms, the firms in the construction sector make zero profits in equilibrium. This implies the following housing price per unit of floor space:

\[ p_{Hi} = \left\{ \left( \frac{\theta}{1-\theta} \right)^{1-\theta} + \left( \frac{1-\theta}{\theta} \right)^{\theta} \right\} R^\theta p_{Li}^{1-\theta}. \]

Again, Shephard’s lemma can be used to derive the conditional factor demands for capital:

\[ K_i^{Dd} = \Phi \theta R^{\theta-1} p_{Li}^{1-\theta} s_i^S, \]

and land:

\[ X_i^D = \Phi (1-\theta) R^\theta p_{Li}^{-\theta} s_i^S, \]

where the supply of land in each zone is exogenous and equal to \( X_i \).
5.2.5. **Transport**

The volume delay function is assumed to be linear in road links. That is, the time required to move from node \( s \) to node \( e \) using a private mode, i.e. the road (\( R \)) link \( l_{se}^{(se)} \), is:

\[
t_{se}^{R} = \ell_{se}^{R} (\xi_{0R}^{se} + \xi_{1R}^{se} d_{se}^{R}),
\]

where \( \ell_{se}^{R} \) and \( d_{se}^{R} \) are the length and demand (see below) of link \( l_{se}^{(se)} \) respectively, \( \xi_{0R}^{se} \) is the free-flow travel time per unit of distance (i.e. the inverse of the free-flow speed) and \( \xi_{1R}^{se} \) is the marginal delay caused by an additional unit of (traffic) demand in the link. Public transport (\( P \)) links are not subject to congestion, but free flow travel time is longer:

\[
t_{se}^{P} = \ell_{se}^{P} \xi_{0P}^{se}.
\]

We now turn to **network loading**. Total demand for road link \( l_{se}^{(se)} \) is:

\[
d_{se}^{R} = N \sum_{a \in \mathcal{C}} \{ I(l_{se}^{(se)} | q) P_a D_{Wa} \},
\]

where the indicator function \( I(l_{se}^{(se)} | q) \) equals one if the route \( q \) of alternative \( a = \{a_{ij}, q\} \) contains link \( l_{se}^{(se)} \) (zero if not or if \( a \) does not imply any commuting). The aggregation of (5.29) across all links of a feasible route \( q \) yields the resulting route-specific (as opposed to the anticipated travel time \( t_a \) by households) travel time:

\[
\hat{t}_q = \sum_{(\ell_{se}^{R}, \ell_{se}^{P}) \in q} t_{se}^{m} \quad \text{with} \quad m = (R, P).
\]

Summing the lengths of the road links involved in route \( q \) yields the total distance generated by car in this route:

\[
L_{Rq} = \sum_{\ell_{se}^{R} \in q} \ell_{se}^{R}.
\]

Similarly, the distance covered by the public transport mode in route \( q \) is:

\[
L_{Pq} = \sum_{\ell_{se}^{P} \in q} \ell_{se}^{P}.
\]

\(^{105}\) Therefore, for two links connecting an identical pair of nodes, it is assumed that \( \xi_{0R}^{se} < \xi_{0P}^{se} \).
The pecuniary cost of route $q$ is:

$$\hat{c}_q = \left( p_g L_{Rq} \right) + \left( p_P L_{Pq} \right) + \sum_{\tau_R^{se} \in q} \tau_R^{se},$$  \hspace{1cm} (5.35)

where $p_g$ and $p_P$ is the monetary cost per unit of distance when commuting by car and public transport respectively, and $\tau_R^{se}$ is the road toll imposed to the commuter that uses the road link $l_R^{(se)}$. Both prices, $p_g$ and $p_P$, are equal to their marginal costs, denoted by $\hat{p}_g$ and $\hat{p}_P$ respectively. Weighting commuting expenditure in (5.35) across alternatives yields the total transport expenditure of households in the economy, i.e.:

$$E_T = N \cdot \sum_{a \in C} \{P_a D_w^a \left[ \left( p_g L_{Rq} \right) + \left( p_P L_{Pq} \right) \right] \},$$  \hspace{1cm} (5.36)

where $q$ refers to the commuting route of alternative $a = \{a_{ij}, q\}$. The total cost of transport provision is assumed to be:

$$C_T = F + N \cdot \sum_{a \in C} \{P_a D_w^a \left[ \left( \hat{p}_g L_{Rq} \right) + \left( \hat{p}_P L_{Pq} \right) \right] \},$$  \hspace{1cm} (5.37)

where $F$ is the fixed cost of public transport provision. From (5.36) and (5.37) it can be seen that the transport provision deficit is:

$$D_T = F + N \cdot \sum_{a \in C} \{P_a D_w^a (\hat{p}_P - p_P) L_{Pq} \} + N \cdot \sum_{a \in C} \{P_a D_w^a (\hat{p}_g - p_g) L_{Pq} \}.$$  \hspace{1cm} (5.38)

5.2.6. Government and public budget

The federal government functions as a benevolent planning authority who controls the tax instruments $\tau_L$ and $\tau_R^{se}$, and the redistribution instruments $B$ and $B_\ell$ in order to maximize the expected maximum utility in (5.15). It must be stressed that if the marginal utility of income is not constant, i.e. if (5.1) is replaced by a utility function in which income effects are switched on, the maximization of (5.15) subject to the equilibrium conditions (as described in Sections 5.2.1-5.2.5) cannot be achieved without alternative-specific redistribution instruments that aim to equalize the marginal utility of income across alternatives (for a proof see Anas, 2012). Consequently, with horizontal revenue recycling, optimal externality-correcting taxes (in the context of this model $\tau_R^{se}$) deviate from their Pigouvian levels (see equation (5.54) below) even

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106 Thus, equation (5.35) provides the monetary cost of all alternatives $a = \{i, j, q\}$ that make use of route $q$. 
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in the absence of any failure in the model (e.g. a labor market distortion), reflecting the anonymity of lump-sum revenue recycling.\footnote{Despite this, expected maximum utility still coincides with the logsum expression. The finding by Anas (2012) has been confirmed using alternative specifications of a simplified (monocentric serial network) model in which income effects were on. These specifications were used to test the optimality of the Pigouvian toll with horizontal lump-sum recycling in the case where $\tau_L = 0$, i.e. under congestion as the only failure in the model. In all cases the constrained (to horizontal recycling) optima were Pareto preferred to the Pigouvian toll.} Given the very large number of alternatives considered in the study, the choice to not model income effects renders the computation of the optimal policy (see Section 5.4 and Appendix 5.D) feasible. Furthermore, this choice facilitates welfare analysis, i.e. the computation of compensating variations from a policy change (e.g. a tax reform), which is straightforward with income effects off but has been shown to be a particularly cumbersome task in discrete choice settings with income effects on (see Herriges and Kling, 1999; Dagsvik and Karlström, 2005, for a complete discussion of the issue).

The government is responsible for the recycling of the expected revenue from road tolls and public transport fares, by controlling the public transport. Inserting (5.9) into (5.4) and weighting across the alternatives of the choice set $C$ yields the expected total labor supply. The expected government revenue from the labor tax is, therefore:

$$R_L = \tau_L \cdot N \cdot \sum_{a \in C} \left\{ P_a D^*_W w^*_J \right\}.$$ \hspace{1cm} (5.39)

The total revenue from road taxes is:

$$R_R = N \sum_{a \in C} \left\{ P_a \left( \sum_{\tau^*_R \in \mathcal{R}} \tau^*_R D^*_W \right) \right\}. \hspace{1cm} (5.40)$$

The total revenue from the $J$ land markets is:\footnote{Unlike monocentric city models, in which the opportunity cost of land is determined by the return to agriculture, in polycentric general equilibrium models there is generally no land-use alternative to development. The opportunity cost of land is zero and it is all being developed in the equilibrium.}

$$B_e = \sum_{i \in J} p_{Li} x^D_i.$$ \hspace{1cm} (5.41)

Public budget is balanced, therefore:

$$B = \frac{1}{N} (R_L + R_R - D_T).$$ \hspace{1cm} (5.42)
5.2.7. **General, stochastic user equilibrium**

In equilibrium, labor, housing, land markets at each zone clear, together with the output market. The expected aggregate labor supply to zone $j$ is:

$$E(L_j^S) = N \sum_{a \in C} \{ I(j|a) \, P_a \, D^W_a \}, \quad (5.43)$$

where $I(j|a)$ is an indicator function that takes the value one if the employment zone of alternative $a$ is zone $j$ (zero otherwise). For each of the $J$ labor markets, the clearing condition is:

$$E(L_j^S) - L_j^D = 0, \quad (5.44)$$

where $L_j^D$ is the aggregate labor demand from (5.19). Similarly, the expected aggregate demand for housing in zone $i$ is:

$$E(H_i^D) = N \sum_{a \in C} \{ I(i|a) \, P_a \, s_a^* \}, \quad (5.45)$$

where $I(i|a)$ is an indicator function that takes the value one if the residential zone of alternative $a = \{a_{ij}, q\}$ is zone $i$ (zero otherwise). For each of the $J$ housing markets, the clearing condition is:

$$H_i^S - E(H_i^D) = 0, \quad (5.46)$$

where $H_i^S$ is the aggregate housing supply in the same zone. Land markets also clear, therefore:

$$X_i^D - \bar{X}_i = 0, \quad (5.47)$$

where the aggregate land demand $X_i^D$ coincides with the land demand of the representative developer in (5.28) and $\bar{X}_i$ denotes the total surface available for development in zone $i$. Clearing of the intermediate $J$ markets implies that:

$$Q_j^D - Q_j^S = 0. \quad (5.48)$$

The aggregate expected demand for the composite good in the entire region is given by:

$$E(Y^D) = N \sum_{a \in C} \{ P_a y_a^* \}. \quad (5.49)$$

In order for the model to close properly, a part of the composite output must be used to import the required capital, and to cover the costs associated with the use of private modes and the public transport system. This implies the closure condition:
\[
    p(Y^S - E(Y^D)) = R \left( \sum_{j \in J} K_j^{Df} + \sum_{l \in J} K_l^{Da} \right) + C_T,
\]

where the term on the left hand side is the value of the composite good (with price, \( p \), normalized to one) that is not consumed inside the region but bought by a virtual trader that exports it to the rest of the world (ROW). The trader then buys capital and transport services (demanded by individuals, firms and developers in the region) of equal value from ROW and sells them back to the region.

Because the equilibrium is competitive, the prices of all final and intermediate goods produced in the region equal their marginal cost. For each of the \( J \) intermediates this implies one zero profit condition as in (5.18). Similarly, for each of the \( J \) housing markets this implies a zero profit condition as in (5.26). Because the price of the composite is normalized to one, the corresponding condition for this good is:

\[
    \left( \prod_{j \in J} p_j^\epsilon \right) \left( \sum_{j \in J} \omega_j \right) = 1,
\]

where \( \omega \) has been defined in (5.23).

Finally, the disaggregate labor supply in (5.10), housing demand in (5.8) and consumption (computed from (5.6)-(5.9)) are based on beliefs for the travel time and cost of each alternative \( a \). In equilibrium these beliefs have to be correct. That is, for each alternative \( a \), the commuting time and pecuniary cost used to derive optimal household behavior turns out to be the resulting commuting time and cost given by (5.32) and (5.35) respectively. This is:

\[
    \hat{t}_q - t_q = 0, \tag{5.52}
\]

and

\[
    \hat{c}_q - c_q = 0. \tag{5.53}
\]

However, it is easy to see that the above holds if the expected travel times and pecuniary costs of links in the network are equal to the resulting ones.

This section described a system of 39 blocks of equations in 39 vectors of unknowns. These are equations: (5.6), (5.8), (5.9), (5.10), (5.11), (5.15), (5.16), (5.18), (5.19), (5.20), (5.24) (5.26), (5.27), (5.28), (5.29), (5.30), (5.31), (5.32), (5.33), (5.34), (5.35), (5.36), (5.37), (5.38), (5.39), (5.40), (5.41), (5.42), (5.43), (5.44), (5.45), (5.46), (5.47), (5.48), (5.49), (5.50), (5.51), (5.52) and (5.53) corresponding to the unknown vectors: \( y^*_a, s^*_a, D^*_W, T^*_F, V^*_a, P_a \) (with the size of each vector equal to the number of elements in the choice set, denoted by \( N_c \)), \( p_j, w_j, E(L^S) \), \( L^D, K^D, Q^D, p_{Hi}, K^D, X^D_l, E(H^D_l), H^S, p_{Li} \) and \( Q^S_j \) (each vector of size equal to \( J \)), \( L_{Rq}, L_{pq} \), \( t_q, c_q, \hat{t}_q \) and \( \hat{c}_q \) (each vector of size equal to the number of feasible routes, \( i.e. \ N_Q \)), \( t^S_{Rq} \) and \( d^S_{Rq} \).
(of size equal to the number of road links in the network, \(N_R\)), \(t_e^{se}\) (of size equal to the number of public transport, \(i.e.\) rail, links, \(N_P\)), \(D_T\), \(R_L\), \(R_R\), \(B_e\), \(B\), \(E(Y^D)\), \(Y\), \(E_{max}\), \(E_T\), \(C_T\) and \(p\) (each of size one).

The model uses a network with \(N_R = 52\) road links, \(N_P = 50\) rail links, \(N_Q = 1738\) feasible routes, \(J = 18\) zones and \(N_C = N_Q + J = 1756\) alternatives.\(^{109}\) This implies a non-linear square system of \(N_e = 21325\) equations in \(21325\) unknowns.\(^{110}\) Appendix 5.B provides the pseudocode for a computationally efficient, Newton-based algorithm that solves the system for the general-stochastic user equilibrium.

### 5.3. Application to the area of Randstad: key data and calibration.

The model is calibrated to fit a series of stylized facts characterizing the behavior of the average household (expenditures shares, allocation of time, \(etc.\)) and the characteristics of Randstad region: the general spatial lay-out and network, the population and employment share of each zone, the average commuting speed of modes, modal split, and the relative land rents, housing prices, wages and floor-to-area ratios. In order to ensure that the research questions can be addressed without disturbing the computational tractability of the model, a resolution that comprises \(18\) zones has been chosen. As shown in Figure 5.1, each zone represents a group of municipalities, which share similar commuting patterns.\(^{111}\) The four largest employment and residential centers (Amsterdam & Amstelveen, Utrecht, Rotterdam and the Hague) constitute separate zones.\(^{112}\) The data used for the calibration of the model regard primarily commuting flows between the 18 zones of the model. The \(18\times18\) \textit{origin-destination} (hereafter, \textit{OD}) matrix

\(^{109}\) We consider 12386 non-cyclical routes of which 10648 are excluded: i) due to abnormal travel time/distance compared to the shortest path route or ii) because they violate the rule of a logical mode use. For example, routes that imply car use at two different, non-subsequent trip components: from \(a\) to \(b\) by car, from \(b\) to \(c\) by public mode and from \(c\) to \(d\) by car.

\(^{110}\) That is: \(N_e = (6 \cdot N_C) + (6 \cdot N_Q) + (11 \cdot J) + (2 \cdot N_R) + N_P + 9 = 21325\) equations.

\(^{111}\) An initial selection excluded municipalities with population below 20000 inhabitants. A first grouping of municipalities into clusters (zones) was made in order to merge neighboring municipalities with populations between 20000 and 180000 inhabitants that share similar labor supply patterns (towards municipalities with a population over 180000 inhabitants). Further refinements rived some of these groupings into smaller parts, to account for the fact that some municipalities had access to more than one major highway link and would therefore hold larger monopoly power (ceteris paribus) had they been granted fiscal autonomy to perform road pricing on their own.

\(^{112}\) The included zones are: 1) Amsterdam and Amstelveen, 2) municipalities between Amsterdam and Utrecht across highway A2, 3) eastern suburbs of Amsterdam along highway A1, including Diemen, Muiden, Weesp and Naarden, 4) cluster of municipalities from Bussum, all the way on A1 to the crossing with A27, and across A27 all the way to Utrecht, 5) The cluster of Amersfoort, Soest and Zeist, municipalities located across A1 and A28 to the northeast of Utrecht, 6) Utrecht, 7) west suburbs of Utrecht (Montfoord, Woerden) across A12, 8) South suburbs of Utrecht (IJsselstein, Houten, Nieuwegein, Vianen) across A2 and A27, 9) Almere and Lelystad on A6, 10) northeast suburbs of Rotterdam, built around A12 and A20, 11) Rotterdam, 12) southeast suburbs of Rotterdam around A15, 13) municipalities located between Rotterdam and the Hague (\(e.g.\) Delft, Zoetermeer), 14) the Hague, 15) municipalities located north of the Hague, around Leiden, which are accessed through A4 and A44, 16) municipalities located southwest of Amsterdam (Haarlem, Haarlemmermeer), with the area including Schiphol airport, 17) cluster of municipalities located across A9 from Haarlem up to Alkmaar, and 18) northwest suburbs of Amsterdam (Zaanstad, Purmerend) accessed through parts of A10, A8 and A7.
has been computed using CBS microdata for fully employed workers in year 2012. The variation of labor supply across different OD-pairs is not observed.

The calibration is treated as a series of optimization problems, one at each stage (See Appendix 5.C). In the first stage the preference parameter vector \((\pi_0, \pi_1, \alpha, \beta, \gamma)\) is calibrated in order for the equilibrium shares of consumption, housing and transport expenses in total expenditure to be in rough accordance with the expenditure profile of the average household in Western Europe (benchmark values 0.65, 0.30 and 0.05 respectively). Similarly, assuming a day of 13½ hours (excluding the essential time needed for physical rest and necessary activities of the household from the full 24-hour day), the benchmark labor time share (0.60) corresponds to a working day of approximately eight hours; therefore, the one-way benchmark commuting time (0.10) corresponds to approximately 40 minutes. The rest of the time endowment, i.e. approximately four hours, is spent on leisure activities.

Table 5.1. Stylized facts on which the model is calibrated.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption share of income</td>
<td>0.65</td>
</tr>
<tr>
<td>Housing expenditure share of income</td>
<td>0.30</td>
</tr>
<tr>
<td>Transportation expenditure share of income</td>
<td>0.05</td>
</tr>
<tr>
<td>Daily labor supply (hours)</td>
<td>8.00</td>
</tr>
<tr>
<td>Daily leisure (hours)</td>
<td>4.00</td>
</tr>
<tr>
<td>One way commuting time (minutes)</td>
<td>40.5</td>
</tr>
<tr>
<td>Equilibrium-to-free-flow travel time ratio</td>
<td>2.3</td>
</tr>
<tr>
<td>Free flow speed (km/h)</td>
<td>120.0</td>
</tr>
<tr>
<td>Public transport mode speed (km/h)</td>
<td>60.0</td>
</tr>
<tr>
<td>Car choice probability</td>
<td>0.65</td>
</tr>
<tr>
<td>Labor tax</td>
<td>0.40</td>
</tr>
</tbody>
</table>

In the second stage, the parameter vector \((\beta, \gamma, \xi_1R)\) -which underlies the level of traffic and the average equilibrium speed- is calibrated in order to minimize the difference between the average (across links) equilibrium-to-free-flow travel time ratio and its benchmark value (2.3).\(^\text{113}\)

Assuming a free flow speed of 120 km per hour, the benchmark value implies an average equilibrium speed of 52 kilometers per hour. This speed is plausible if someone takes into account bottlenecks, traffic lights, parking search time and other events not modeled explicitly in this application.\(^\text{114}\) Furthermore, it is not only compatible to Randstad area but also in rough alignment with international commuting speeds in urban areas, for instance the one reported for large US cities in the national household travel survey (Federal Highway Administration, 2004).

\(^\text{113}\) The calibrated ratio \(t_{rel} (v_B)\) is 2.274.

\(^\text{114}\) To facilitate calibration, we have assumed that \(\xi_{0B}\) and \(\xi_{1B}\) are constant across road links. We have fixed the free-flow parameter \(\xi_{0R}\) to 0.25. Multiplying this with 13½ hours (the assumed time endowment) yields approximately 3.375 hours per unit of distance or, equivalently, a speed of 0.296 units of distance per hour. Setting the unit of distance equal to 405.4 km, \(\xi_{0R} = 0.25\) implies a free-flow speed of 120 km per hour.
For the public transport mode, we assume a constant speed of 60 km per hour, i.e. half of the average private mode free-flow speed.

In the third stage, we calibrate the employment and population shares of each zone to fit those observed in data, by adjusting the parameter vector \((z_i, z_j, \zeta_j)\) while keeping the rest of the parameters fixed.\(^\text{115}\) This part of the algorithm increases (decreases) \(z_i\) for residential zones that attract fewer (more) households than those observed in data, and adjusts \(z_j\) in the same manner to bring employment density in alignment with data. However there is an asymmetry between the two adjustments: while increasing the average utility of local amenities, \(z_i\), attracts more residents and presses housing and land prices upwards, increasing the employment-specific constants, \(z_j\), increases labor supply and therefore (ceteris paribus) pushes wages in these zones downwards.\(^\text{116}\) This downward pressure is partially due to the inclusion of a unique type of skill in the model. It is offset by simultaneously adjusting the cost share of the local intermediate on the cost of the composite good, i.e. by adjusting \(\zeta_j\); this ensures that the main labor attractor

\(^{115}\) Throughout the entire calibration process, parameter \(\lambda\) is fixed to 3.0. We have chosen to abstract from agglomeration effects, which are going to be studied thoroughly in a separate contribution. We have therefore fixed total factor productivity, \(A_j\), to 1.0 in each zone.

\(^{116}\) Relative housing prices (base zone 18) are in the range of 0.59 to 1.67. Relative land prices vary between 0.46 and 2.60. The model produces a poor spatial variation in terms of floor-to-area ratios, which varies in the limited range between 1.16 and 1.95. This is mainly due to the equilibrium elasticity of substitution between residential space and consumption that gives rise to large residential consumption in zones with very low population density.

### Table 5.2. Residential and employment percentages in benchmark equilibrium and data.

<table>
<thead>
<tr>
<th>Zone</th>
<th>Residential share (model)</th>
<th>Residential share (data)</th>
<th>Employment share (model)</th>
<th>Employment share (data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amsterdam &amp; Amstelveen</td>
<td>0.145</td>
<td>0.148</td>
<td>0.179</td>
<td>0.182</td>
</tr>
<tr>
<td>southeast Amsterdam suburbs</td>
<td>0.026</td>
<td>0.021</td>
<td>0.016</td>
<td>0.012</td>
</tr>
<tr>
<td>east Amsterdam suburbs</td>
<td>0.015</td>
<td>0.011</td>
<td>0.017</td>
<td>0.015</td>
</tr>
<tr>
<td>northeast Utrecht suburbs</td>
<td>0.039</td>
<td>0.037</td>
<td>0.038</td>
<td>0.037</td>
</tr>
<tr>
<td>east Utrecht suburbs</td>
<td>0.035</td>
<td>0.036</td>
<td>0.037</td>
<td>0.035</td>
</tr>
<tr>
<td>Utrecht</td>
<td>0.050</td>
<td>0.049</td>
<td>0.054</td>
<td>0.061</td>
</tr>
<tr>
<td>southwest Utrecht suburbs</td>
<td>0.016</td>
<td>0.011</td>
<td>0.014</td>
<td>0.009</td>
</tr>
<tr>
<td>southeast Utrecht suburbs</td>
<td>0.031</td>
<td>0.028</td>
<td>0.029</td>
<td>0.025</td>
</tr>
<tr>
<td>Almere &amp; Lelystad</td>
<td>0.035</td>
<td>0.038</td>
<td>0.031</td>
<td>0.027</td>
</tr>
<tr>
<td>northeast Rotterdam suburbs</td>
<td>0.041</td>
<td>0.039</td>
<td>0.038</td>
<td>0.034</td>
</tr>
<tr>
<td>Rotterdam</td>
<td>0.098</td>
<td>0.102</td>
<td>0.117</td>
<td>0.124</td>
</tr>
<tr>
<td>southeast Rotterdam suburbs</td>
<td>0.072</td>
<td>0.073</td>
<td>0.053</td>
<td>0.056</td>
</tr>
<tr>
<td>cluster between Rotterdam and the Hague</td>
<td>0.127</td>
<td>0.124</td>
<td>0.109</td>
<td>0.107</td>
</tr>
<tr>
<td>the Hague</td>
<td>0.077</td>
<td>0.081</td>
<td>0.091</td>
<td>0.094</td>
</tr>
<tr>
<td>Leiden and suburbs</td>
<td>0.050</td>
<td>0.048</td>
<td>0.038</td>
<td>0.038</td>
</tr>
<tr>
<td>cluster around Schiphol airport</td>
<td>0.049</td>
<td>0.053</td>
<td>0.066</td>
<td>0.071</td>
</tr>
<tr>
<td>northwest Amsterdam suburbs</td>
<td>0.046</td>
<td>0.051</td>
<td>0.040</td>
<td>0.039</td>
</tr>
<tr>
<td>northeast Amsterdam suburbs</td>
<td>0.049</td>
<td>0.050</td>
<td>0.033</td>
<td>0.034</td>
</tr>
</tbody>
</table>
zones offer a higher wage in equilibrium, as data suggest.\footnote{This holds because an increased $\zeta_j$ implies (in general) higher demand for the intermediate goods produced in those zones (see equation (5.48)); and this, in turn, implies higher wages.} Table 5.2 juxtaposes the employment and residential shares in the benchmark equilibrium against the shares observed in data.

The fourth stage of the algorithm adjusts the parameter vector $(z_R, z_P, z_t)$, \textit{i.e.} the mode-specific constants and the transit specific constant, in order for the probability of commuting by car to be sufficiently close to 65%. This is in general alignment with the observed rates in the Netherlands. Furthermore, the algorithm renders commutes with both modes unlikely (below 5%).\footnote{See Schwanen \textit{et al.} (2001) for an analysis of the modal split and urban form based on the Dutch National Travel Survey of 1998.}

\textit{Figure 5.1. Spatial configuration of the model: zonal aggregation (left), main highways of the road network (middle) and network representation (right).}

For the rest, we assume a uniform labor income tax rate of 40\% which is roughly the income tax rate faced by the average commuter in the Netherlands and in accordance with the generic tax rates in Western Europe and North America (25-45\%). In equilibrium, the annual pecuniary cost of car use per kilometer is 0.177\% of total disposable (after tax) income. Setting this income to the plausible level of €30000, this annual cost turns out to be slightly above €53. The corresponding annual kilometer cost with public transport is approximately €16.\footnote{Exogenous price of car use per unit of distance is set to 0.05. Dividing by the number of kilometers per unit of distance (\textit{i.e.} 405.4 km), and then by 0.0697 (the equilibrium after-tax income) yields an annual kilometer cost equal to 0.00177 of after tax income.} The fixed cost of the public transport operator is set to zero in benchmark equilibrium. There are no price subsidies in public or private transport thus pricing is efficient. From (5.5) it is
straightforward that the value of time has an upper bound which is the after-tax wage, $w_j (1 - \tau_L)$. The expected value of time is 0.737 of the net, after-tax wage (or 0.442 of the gross wage), with a standard deviation of 0.151 (0.091 for the gross wage VOT). These values are in line with those proposed at several previous studies.

Contrary to highly stylized models, in which the elasticity of labor supply may attain a uniform value (e.g. Parry and Bento, 2001), or monocentric city models, in which it may vary only across residence locations, here we focus on the variation of the equilibrium elasticity of labor supply (with respect to the labor tax) across origin-destination pairs.\footnote{This is approximated numerically using finite differences. That is, first compute the labor supply matrix in the benchmark equilibrium, $L^S_0$, whose element in the $i$-th row and $j$-column is $L^S_{0ij} = \sum_{a \in \mathcal{A}} \{l(i, j|a) P_a D_wa \}$. Then let the tax rate $\tau_L$ increase by $\Delta \tau_L$. Compute the tax rate change (in monetary units) for each employment zone: this is $d\tau_j = \Delta \tau_w w_j$. Solve for the general equilibrium and compute the new labor supply matrix $L^S_1$. The approximation for the elasticity of labor supply for the OD-pair $a_{ij}$ is $\left[ (L^S_{1ij} - L^S_{0ij}) / d\tau_j \right] / \left[ (\tau_L w_j) / L^S_{0ij} \right]$.} This variation is in accordance with values assumed in previous studies (in the plausible range between -0.028 and -0.296). Appendices 5.A and 5.E provide a complete summary of the endogenous, exogenous variables and parameters (including their values) of the model.

5.4. Policy analysis

We now consider a series of interventions that are highly relevant for policy analysis: the Pigouvian toll, partial taxation of the network, uniformly-priced and differentiated cordon tolls around the three largest employment zones of Randstad (Amsterdam, Rotterdam and the Hague). In Section 5.4.1 we assume that revenue recycling takes its simplest form, \textit{i.e.} a lump-sum transfer to the representative household. In Section 5.4.2 we search for realistic revenue recycling strategies that may reverse the findings of Section 5.4.1. Section 5.4.3 offers an extensive sensitivity analysis of the most important findings.

5.4.1. Lump-sum revenue recycling\footnote{Although recycling the labor tax revenues in a lump-sum manner would be a peculiar policy in reality, we choose to model the public budget this way in order to consider the distortion from labor taxation without introducing further inefficiencies in a setting with public good provision. That is, while the inclusion of a public good has provided useful insights in other settings without tax interactions (see for instance Anas and Pines, 2012), here it would complicate the analysis while being peripheral to what we are interested in: the interactions between labor and transport taxes in a polycentric network.} \footnote{To the knowledge of the author, this is the first study to approach the issue using observed data from a general polycentric network.}

While the Pigouvian toll is the most efficient intervention in a setting where road traffic externalities pose the only failure in the economy, a series of contributions have shown that large welfare losses may occur from such policy in the presence of a pre-existing distortionary tax that remains intact (Parry and Bento 2001; Tikoudis et al., 2015b).\footnote{This is approximated numerically using finite differences. That is, first compute the labor supply matrix in the benchmark equilibrium, $L^S_0$, whose element in the $i$-th row and $j$-column is $L^S_{0ij} = \sum_{a \in \mathcal{A}} \{l(i, j|a) P_a D_wa \}$. Then let the tax rate $\tau_L$ increase by $\Delta \tau_L$. Compute the tax rate change (in monetary units) for each employment zone: this is $d\tau_j = \Delta \tau_w w_j$. Solve for the general equilibrium and compute the new labor supply matrix $L^S_1$. The approximation for the elasticity of labor supply for the OD-pair $a_{ij}$ is $\left[ (L^S_{1ij} - L^S_{0ij}) / d\tau_j \right] / \left[ (\tau_L w_j) / L^S_{0ij} \right]$.}

Since road congestion is generated by identical vehicles, the \textit{marginal external congestion cost (mecc)} of an additional unit of traffic is independent of both the origin and
destination of the vehicle that enters the link, as well as of the characteristics of its driver. To compute this cost, traffic demand in each link needs to be decomposed by user type, i.e. by alternative $a = \{a_{ij}, q\}$. Every additional trip that uses link $l^{(se)}_c$ delays all drivers in the same link by $\rho^{se}_R \xi^{se}_{1R}$. That delay is simply the derivative of link travel time in (5.29) with respect to the demand (i.e. the load) in the link. However, this delay is valued differently by each user type, as the value of time varies not only across OD-pairs, but also across routes.\textsuperscript{123}

Therefore, the link-specific marginal external congestion cost ($mecc^{se}_R$) is computed by:

$$mecc^{se}_R = \rho^{se}_R \xi^{se}_{1R} \sum_{a \in \cal{C}} \left\{ 1\left( l^{(se)}_R | q \right) NP_a D^*_W a \nu_a \right\},$$

(5.54)

Using the above formula, we compute the welfare effect of the full-network quasi first-best Pigouvian toll scheme ($FNP-LS$ in Table 5.3). The toll rule is:

$$\tau^{se}_R = mecc^{se}_R \text{ for all } l^{(se)}_m,$$

(5.55)

i.e. it is applied in the entire network, therefore the suboptimality stems entirely from the interaction with the pre-existing labor tax. In fact, the policy is not only suboptimal: the scheme causes considerable welfare losses that account for approximately 1.83\% of the after-tax, disposable labor income, although it is efficient in curbing the externality (i.e. in terms of reducing traffic).\textsuperscript{124} On the other hand, the same toll rule applied in a setting without a labor tax has been confirmed to be first-best.\textsuperscript{125} This finding is in line with Tikoudis et al. (2015b) and corroborates earlier computations of the welfare effects from Pigouvian pricing under pre-existing distortionary labor taxation in the setting of Parry and Bento (2001). In those contributions, the environmental, or Pigouvian effect (i.e. the first dividend) from a road pricing policy, despite positive in itself, falls short of the negative tax interaction effect at the margin of

\textsuperscript{123} There is, therefore, a unique value of time for each alternative, since by definition $a = \{a_{ij}, q\}$.

\textsuperscript{124} Considering an average, annual after tax income of € 30000, this welfare loss is € 549 per capita.

\textsuperscript{125} Setting the labor tax at 0\% leaves the road traffic externalities as the only remaining source of inefficiency in the model. Starting from that first-best Pigouvian equilibrium the researcher can check whether deviations from the toll rule in (5.55) can increase utility. Such an increase is a sign of model misspecification.
the no-toll equilibrium. As a result, the optimal congestion taxes do not only lie below their Pigouvian levels (here expressed in (5.54)), but they are essentially negative.

However, Tikoudis et al. (2015b) illustrate a case in which partial network taxation that takes the form of a cordon toll in a monocentric serial network may reverse the sign of the welfare effect, even if a pre-existing labor tax exceeds 40%. The key drivers behind that result were that: (i) the cordon toll could be imposed in a certain distance from CBD (i.e. the sole destination node of the model) and that (ii) the elasticity of labor supply is monotonically decreasing with distance from it. Together, these drivers ensure that those affected by the cordon toll are the individuals with the most inelastic labor supply, i.e. the population group that generates the weakest tax-interaction effect.

To test if the above result still holds in the polycentric setting of this study, we perform additional computations regarding systems of uniformly- and differentially-priced cordon tolls around the three largest commuting destination nodes of the region: Amsterdam-Amstelveen (indexed by 1), Rotterdam (indexed by 11) and the Hague (indexed by 14), attracting cumulatively 33.1% of workers. For the uniformly-priced cordon toll system (UPC-LS in Table 5.3), the government’s constrained optimization problem is:

\[
\max_{\tau^R_{se} > 0, \ B} \left\{ \lambda \left[ E + \log \sum_{a \in C} \left( \frac{\exp(V_{a}^*)}{\lambda} \right) \right] \right\} \tag{5.56}
\]

subject to equations (5.8)-(5.13), (5.16)-(5.53) and the scheme specification:

\[
\tau^R_{se} = \begin{cases} \bar{\tau}_R & \text{if } e \in \{1, 11, 14\} \\ 0 & \text{otherwise.} \end{cases} \tag{5.57}
\]

For the differentially-priced cordon toll system (DPC-LS in Table 5.3), the problem is given by (5.56) subject to (5.8)-(5.13), (5.16)-(5.53) and the scheme specification:

\[
\tau^R_{se} = \begin{cases} \bar{\tau}_{Re} & \text{if } e \in \{1, 11, 14\} \\ 0 & \text{otherwise.} \end{cases} \tag{5.58}
\]

However, the two policies bear little practical relevance if the resulting optimal charges \(\bar{\tau}_R\) in (5.57) and \(\bar{\tau}_{Re}\) in (5.58) are found to be negative, as it turns out to be the case in the above

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126 The environmental or Pigouvian effect is the welfare gain from a marginal adjustment of the externality tax. When the latter is set at its Pigouvian level, the above effect is zero. The tax interaction effect is the welfare loss from the erosion of the base that corresponds to a distortionary tax, caused by a marginal increase of the environmental tax.

127 To a certain extent, the finding that road pricing under commuting hours is welfare decreasing depends on the way labor supply is modelled. In general, if labor supply is elastic in the extensive margin (number of days) but completely inelastic in the intensive margin (duration of the working day) the resulting negative tax interaction effect is much stronger compared to the case in which labor supply is adjustable through both margins. This becomes clearer when assuming away the availability of public transport. Then, in the former case the bases of the labor and road tax overlap completely; in the latter case the overlap is only partial, since labor supply (in total hours) is not proportional to the number of commuting trips. There is little empirical evidence on the elasticity of labor supply in the two margins. An in depth analysis of the issue is attempted by Hirte and Tscharaktschiew (2015).
context. The negative optimal cordon charges indicate that the finding by Tikoudis et al. (2015b) may be confined to monocentric city settings, as it fails to be confirmed in a less stylized, mixed polycentric network. The key difference is that in a monocentric city a cordon toll can charge eclectically the inelastic suppliers of labor who sort their residence far from the job location, something that may not be possible in a mixed polycentric network.

Complementary sensitivity analysis shows that the threshold labor tax value at which the full-network Pigouvian generates positive welfare gains is 18%, i.e. far from the benchmark labor tax of 40%. Similar sensitivity analysis is employed to find the threshold labor tax at which the uniformly- and differentially-priced cordon systems generate positive welfare gains. These are found to be 24% and 25.5% respectively. The sign of the welfare effect becomes positive in higher labor tax rates in the case of cordon toll for two reasons. The first is that several network links, in which the optimal tax is strongly negative, are left untaxed instead of being taxed at their -strictly positive- marginal external costs. Setting the road tax equal to zero in a large part of the network reduces significantly the private cost of labor supply (and therefore the marginal excess burden of the labor tax), especially for alternatives that involve long commutes. The second reason is that the optimal toll in the links that form the cordons is flexible, unlike the Pigouvian toll.

Table 5.3. Specification and welfare effects of considered policies.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Tax rule</th>
<th>Coverage</th>
<th>Revenue recycling</th>
<th>welfare change</th>
<th>Labor tax</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>No toll</td>
<td>-</td>
<td>Lump-sum</td>
<td>-</td>
<td>0.40</td>
</tr>
<tr>
<td>FNP-LS</td>
<td>As in (5.54) Full network</td>
<td>Lump-sum</td>
<td>- 1.83%</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>UPC-LS</td>
<td>As in (5.57) Cordons</td>
<td>Lump-sum</td>
<td>0%</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>DPC-LS</td>
<td>As in (5.58) Cordons</td>
<td>Lump-sum</td>
<td>0%</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>FNP-TC</td>
<td>As in (5.54) Full network</td>
<td>Tax cuts</td>
<td>- 0.41%</td>
<td>0.3808</td>
<td></td>
</tr>
<tr>
<td>UPC-TC</td>
<td>As in (5.57) Cordons</td>
<td>Tax cuts</td>
<td>+ 0.011%</td>
<td>0.3944</td>
<td></td>
</tr>
<tr>
<td>DPC-TC</td>
<td>As in (5.58) Cordons</td>
<td>Tax cuts</td>
<td>+ 0.012%</td>
<td>0.3974</td>
<td></td>
</tr>
<tr>
<td>FNO-TC</td>
<td>Free Full network</td>
<td>Tax cuts</td>
<td>+ 0.205%</td>
<td>0.3962</td>
<td></td>
</tr>
</tbody>
</table>

Notes: All cordons regard pricing of the links leading to nodes 1, 11 and 14. The welfare changes are the negative of compensating variations as fractions of the expected income in the policy equilibrium.

In a setting where traffic externalities would be the only failure, i.e. when $\tau_L$ is set to zero, the above schemes would be second-best to Pigouvian pricing.128 Since (in this case) tax interaction effects are absent, the latter produces significant welfare gains, accounting for

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128 Here, we keep all parameters fixed to their calibrated values and set $\tau_L$ equal to zero.
approximately 0.33% of the after-tax labor income.\(^\text{129}\) Because a large portion of the total external costs is generated on the links ending on the above three large employment centers, the relative efficiency of the above second-best measures (in absence of the distortionary labor tax) is high: they are found to capture 48.2% and 48.8% of the Pigouvian gains respectively, in general accordance with Mun \textit{et al.} (2005).

### 5.4.2. Labor tax cuts

The general failure of pricing policies with lump-sum revenue recycling leads naturally to the investigation of policies that return the road tax revenue in the form a labor tax cut.\(^\text{130}\) Because the labor tax is distortionary to begin with, this cut generates a positive welfare effect that is widely known in the theory of double dividend as the revenue-recycling effect. When the latter effect is strong enough to offset the tax interaction effect, the second dividend emerges.

The welfare gains, as well as the link charges of the \textit{optimal road tax scheme} are of special interest to the policy maker, because they provide the benchmark against which other second-best interventions can be compared. In the context of this chapter, the above policy leaves the charge in each road link free to acquire any value, even a negative one.\(^\text{131}\) The optimization problem is:

\[
\max_{\tau_R^{se}>0, \tau_L} \left\{ \lambda \left[ E + \log \sum_{a \in e} \left( \frac{\exp(V_a^*)}{\lambda} \right) \right] \right\}
\]

subject to equations (5.8)-(5.13), (5.16)-(5.53) and the horizontal lump-sum transfer, \(B\), fixed to the value it acquires in the benchmark no-toll equilibrium with the initial labor tax rate at 40%.

Computing the optimal tax scheme, we find that link charges exhibit a large variation around their Pigouvian levels.\(^\text{132}\) This deviation (roughly) ranges from \(-580\%\) to \(+60\%\), with 31 of the 52 links considered in the study receiving subsidies (negative charges).\(^\text{133}\) The second dividend

\(^{129}\) Despite being in the range proposed by prior literature (\textit{e.g.} Anas and Hiramatsu, 2013), the reader may find this value relatively low. A key contributing factor to this result is that the model bounds the value of time to a ceiling that equals the nominal wage associated with each alternative.

\(^{130}\) To compute the equilibrium for a policy with the labor tax cut, one has to fix \(B\) to the value it obtains in the benchmark equilibrium, and replace \(B\) with \(\tau_L\) in the vector of endogenous variables, \(\varphi = (w_f, p_{hi}, p_{Li}, p_j, Q_i^5, H_i^5, D_f, R_f, R_h, R_e, B, Y, t_q, c_q)\), as defined in Section 5.2.

\(^{131}\) This necessitates the exclusion of cyclical paths, which has been discussed in Section 5.2.1.

\(^{132}\) The optimal tax has been computed using variants of \textit{Newton-Raphson (NR)} and \textit{BFGS} methods with \textit{line search}. The \textit{NR} algorithms used proceed with sequential updates of the road tax, in each iteration for a set of links that share the same start node \(s\) or the same end-node \(e\). The gradient vector and the Hessian matrix are computed exclusively for this set of links using finite differences. The \textit{BFGS} algorithms used produce updates for the entire network at each iteration, using approximations of the Hessian matrix. In both cases, a line search method is employed to maximize the value of the objective function across the proposed direction. To test for local extrema points we repeat the computations with different starting values.

\(^{133}\) This result is in line with Tikoudis \textit{et al.} (2015b). With a parameterization similar to the one used here, it is shown that the \textit{location-based} optimal road tax scheme in a monocentric city can be non-monotonic when the benchmark labor tax is high. Considering the monocentric city as a serial network with a unique destination implies that the above result can be expressed as a link-based road tax scheme in which the most distant links receive negative charges.
emerges in four links; the remaining links receive a positive charge below the marginal external cost of congestion. Table 5.D.1 in Appendix 5.D provides the optimal fee and the corresponding marginal external congestion cost in each link.

The size of the optimal tax deviation from its Pigouvian value underlies a surprising finding, namely that the quasi first-best Pigouvian pricing scheme is welfare decreasing even when road tax revenue finances labor tax cuts. This policy is found to cause a welfare loss that accounts for 0.41% of income. The result may at first appear counter-intuitive, since a set of Pigouvian tolls replaces part of a distortionary tax in the form of a revenue-neutral tax swap. However, nothing prevents the tax interaction effect -in many of the network links- from being strong enough to offset the sum of the environmental (Pigouvian) and revenue recycling effects at the margin of the no-toll equilibrium. Since the optimal road tax scheme lies far away from the corresponding quasi first-best Pigouvian, the latter may, as it turns out to be the case in this context, be welfare decreasing.

Figure 5.2. Upper left panel: sensitivity analysis of the welfare gains (expressed as percentages of income) with respect to the initial labor tax (which determines the target revenue). Lower left panel: corresponding deviations of the objective functions from the calibration target. Right panel: the evolution of the optimal tax as a fraction of its Pigouvian level (i.e. the marginal external congestion cost) across the different values of the initial labor tax rate.

Apart from the above schemes, we compute the optimal charges and welfare levels for: i) a uniformly-priced and ii) a differentially-priced cordon toll system around zones 1, 11, 14 (i.e. around Amsterdam, Rotterdam and the Hague). Because these schemes leave a large set of links untaxed, especially most of those receiving a negative charge in the optimal scheme, they generate welfare gains. However, these gains are found to be negligible, as Table 5.3 suggests.
The sensitivity analysis that follows provides further insights on the conditions that may render such a reform desirable.

5.4.3. Sensitivity analysis

To establish that the above findings are locally stable, we perform one-way sensitivity analyses. During all following checks, the value of the labor tax ($\tau_L$) as well as the values of the critical parameters that determine choice ($\pi_0, \pi_1, \gamma, \lambda$), congestion ($\xi_0, \xi_1$), as well as output and floor space ($\delta, \theta$) are perturbed one at a time, without recalibrating the model (i.e. without adjusting the rest of the parameters to reproduce the calibration targets discussed in Section 5.3). This allows for a thorough comparative analysis of the three central policies juxtaposed in the previous section, i.e. the differentially priced cordon ($DPC-TC$), the Pigouvian toll ($FNP-TC$) and the optimal tax ($FNO-TC$), while simultaneously controlling for the deviation of the calibration objective functions from the plausible values they attain in the benchmark equilibrium.

*Figure 5.3. Upper panels: sensitivity analysis of the welfare gains (expressed as percentages of income) in the neighborhood of benchmark $\pi_0$ (left) and $\pi_1$ (right) values. Lower panels: corresponding deviations of the objective functions from the calibration target.*
Figure 5.2 displays the results from the perturbation of the initial labor tax ($\tau_L^0$), away from its benchmark value (0.40). The computation of the resulting tolls in the three policies ($DPC$-$TC$, $FNP$-$TC$, $FNO$-$TC$) as well as the welfare gains (or losses) they induce has been repeated for three additional values of $\tau_L^0$: 0.30, 0.20, and 0.15. The upper left panel shows that the main findings of Section 5.4.2 are relatively robust: the Pigouvian toll ($FNP$-$TC$) remains an inferior policy to the set of differentially priced cordons ($DPC$-$TC$) for labor tax levels close to 0.40, despite the former policy recovers quickly and becomes more efficient than the latter, at some threshold $\tau_L^0$ between 0.30 and 0.35. As $\tau_L^0$ is further reduced below that threshold level, the relative (to the full network optimal toll, $FNO$-$TC$) efficiency of the Pigouvian toll converges to one (from 0.45 at $\tau_L^0 = 0.3$ to 0.82 at $\tau_L^0 = 0.15$), reflecting the reduction in the volume of the labor market distortion. In contrast, as the labor market distortion fades the relative efficiency of the cordon toll system stabilizes around 0.4, a value that is in line with previous findings in settings without parallel distortions (Mun et al., 2005) and fairly plausible for a polycentric network like the one considered in this study.

Figure 5.4. Upper panels: sensitivity analysis of the welfare gains (expressed as percentages of income) in the neighborhood of benchmark $\gamma$ (left) and $\lambda$ (right) values. Lower panels: corresponding deviations of the objective functions from the calibration target.

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$134$ This is the tax used in the no-toll equilibrium to determine the level of target revenue, $B(\tau_L^0)$; subsequently, $\tau_L^0$ is adjusted in the computations of the three policies under investigation ($DPC$-$TC$, $FNP$-$TC$, $FNO$-$TC$), to ensure a revenue-neutral tax swap, i.e. to keep $B$ fixed at $B(\tau_L^0)$. 

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At the same time, the optimal toll in any arbitrary network link converges to its Pigouvian level, *i.e.* the link’s marginal external cost of congestion as given in (5.54). Each curve in the right panel of Figure 5.2 displays the evolution of the optimal toll (expressed as a fraction of the evolving marginal external cost of congestion) in one of the network’s links. At any arbitrary level of initial labor tax, $\tau_L^0$, the number of curves lying over the dashed horizontal line (at one) indicates the number of network links in which the double dividend emerges; this number increases as $\tau_L^0$ approaches 0.15. Similarly, the number of curves lying below the solid horizontal line (at zero) indicates the number of network links receiving a negative optimal toll (due to the magnitude of the tax interaction effect); this number decreases as $\tau_L^0$ approaches 0.15 (all links are positively priced for $\tau_L^0$ below a threshold value between 0.15 and 0.20), as all curves converge to the value one (*i.e.* the optimal toll converges to the marginal external cost in all links).

*Figure 5.5. Upper panels: sensitivity analysis of the welfare gains (expressed as percentages of income) in the neighborhood of benchmark $\delta$ (left) and $\theta$ (right) values. Lower panels: corresponding deviations of the objective functions from the calibration target.*

The lower left panel of Figure 5.2 illustrates the deviation from the calibration objectives as $\tau_L^0$ moves away from its benchmark value. To facilitate visualization, the large number of objectives has been grouped into five composite objectives (see Table 5.C.2 in Appendix 5.C for the formulas): objective 1 represents the deviation from the benchmark allocation of expenditure; objective 2 the corresponding deviation from the corresponding allocation of time; objective 3 the deviation from the benchmark equilibrium-to-free flow travel time ratio; objective 4 the (cumulative across zones) deviation from the benchmark employment and residential shares;
objective 5 the deviation from the benchmark modal split. The graph shows that lowering $\tau^0_L$ distorts primarily the resulting population and employment shares of the model and secondarily the resulting allocations of time and expenditure. The results displayed in the upper left panel of Figure 5.2 should be interpreted in conjunction with the lower left panel, as for low levels of $\tau^0_L$ the resulting no-toll equilibrium provides a weak characterization of the local economy it is designed to represent.

*Figure 5.6. Upper panels: sensitivity analysis of the welfare gains (expressed as percentages of income) in the neighborhood of benchmark $\xi_0$ (left) and $\xi_1$ (right) values. Lower panels: corresponding deviations of the objective functions from the calibration target.*

Figure 5.3 summarizes the sensitivity analysis performed with respect to preference parameters $\pi_0$ and $\pi_1$. From equations (5.8), (5.9) and (5.10) it can be inferred that the equilibrium is determined only through the ratio $(\pi_0/\pi_1)$, *i.e.* scaling both parameters by the same factor leaves the equilibrium intact. This is reflected in the sensitivity analyses of the two parameters that mirror each other. The efficiency of the Pigouvian toll exceeds that of the cordon system for high values of $\pi_0$ (or low values of $\pi_1$), but in such values objectives 1 to 3 deviate widely from their benchmark values. Similar results are obtained from sensitivity analysis with respect to parameter $\gamma$ (Figure 5.4). Perturbing $\lambda$ (also in Figure 5.4) has no qualitative effect on the results of interest.

The results from the sensitivity analysis performed with respect to technology parameters $\delta$ and $\theta$ are displayed in Figure 5.5. Increasing the capital share in production (benchmark value 0.3) increases the welfare gains of all pricing regimes and alters the relative efficiency of the three policies in a manner similar to that observed when increasing $\pi_0$ (or decreasing $\pi_1$ and $\gamma$).
However, as it is the case with those perturbations, the benchmark equilibrium produces allocations of time and expenditure, as well as equilibrium-to-free flow travel times that deviate from the assumed values considerably. Finally, perturbing the free flow and congestion parameters ($\xi_0$ and $\xi_1$) disturbs the equilibrium-to-free flow travel time ratio but has no qualitative impact on the results (Figure 5.6).

5.5. Concluding remarks

This chapter derived new insights by incorporating the two core mechanisms that generate the double dividend (i.e. the tax interactions and revenue-recycling effects) in a network setting with multiple externality-generating facilities (i.e. roads), that complement or substitute each other and may be left untaxed (partial taxation). Among others, this setting facilitates the identification of circumstances under which partial taxation of these externalities may be Pareto preferred to the textbook Pigouvian remedy, not only with revenue returned lump-sum, but also in the form of a cut in the distortionary tax. The model used a clear geographical reference, i.e. the polycentric urban conglomeration in the area of Randstad, which comprises the three largest cities of the Netherlands (Amsterdam, Rotterdam and the Hague). We showed that the introduction of a Pigouvian toll in the conglomerate’s highway system could generate considerable welfare losses.

Surprisingly, marginal external cost pricing was found to be welfare decreasing even with the toll revenue used to finance a cut in the uniform labor tax. The computation of the optimal tax scheme revealed that this is underlied by a large deviation of optimal link charges from their Pigouvian levels. In that scheme, a significant fraction of the network links receives considerable subsidies. Approximations of the Pigouvian, revenue-recycling and tax interaction effects in these links show that the latter is typically strong enough to offset the combined effect of the former two. Partial externality taxation, expressed as a system of differentially-priced cordons around the largest employment centers leaves most of these links unpriced and was found to capture only a small part of the considerable welfare gains generated by the optimal tax scheme. Extensive sensitivity analysis revealed that the above findings are locally robust with respect to the parameters supporting the benchmark equilibrium. That is, perturbing these parameters either leaves the effects of these policies qualitatively intact or generates a no-policy equilibrium that no longer reflects the basic facts characterizing the benchmark equilibrium.

The findings bear significant policy implications. First, they highlight (in a real-world case) that optimizing the tax-system from an environmental point of view may not only be suboptimal but also welfare-reducing, even if the revenue from externality taxation is recycled through a revenue-neutral tax swap (in this case to reduce the most distortionary tax in the economy, i.e. this on labor income). Second, partial taxation of the externalities (i.e. charging those generating the smallest tax interaction effect) may be a welfare-improving alternative. However, in contrast to previous findings showing that small (considerable) welfare gains are possible with lump-sum (labor tax cut) revenue recycling in a monocentric city, this study
predicts that the welfare effect from a set of cordon tolls in a polycentric network is negative in the former case and slightly positive in the latter. This result is shown to be persistent in sensitivity analyses and reflects the cordon system’s limited capacity to tax eclectically the commuters that supply labor relatively inelastically. This is to be juxtaposed against monocentric city settings, where the corresponding capacity of a single cordon is much higher. Finally, the efficiency of the cordon toll in both cases increases as the background labor market distortion fades. When the latter disappears completely, the cordon system captures a satisfactory fraction of the gains generated by the optimal tax scheme, corroborating earlier findings in studies considering polycentricity without taking into account the labor market distortion.

The above findings may underlie a need for policies based on more sophisticated tax-subsidy schemes (like the one computed in this chapter) or, alternatively, on non-horizontal, spatially-differentiated revenue recycling mechanisms (for instance location-specific labor tax cuts) in contexts similar to the one considered in this chapter. The study illustrated that, in a polycentric network, the degree of sophistication in the optimal tax scheme (i.e. the degree to which the levels of optimal externality taxes deviate from the corresponding marginal external costs) grows with the extent of the background preexisting distortion. This sophistication is critical since it codetermines the political and technical feasibility of any environmental tax reform. Further investigation of the complexity with respect to the optimal tax scheme and revenue-recycling program across different network configurations goes beyond the scope of this chapter and is left as a topic for future research.
**Appendix 5.A: Notation**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Equation/Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_a$</td>
<td>consumption of a composite good</td>
<td>$Y^S$ supply of composite good (numéraire)</td>
</tr>
<tr>
<td>$s_a$</td>
<td>housing consumption</td>
<td>$Q^D_j$ intermediate demand by assembly industry</td>
</tr>
<tr>
<td>$T_{Fa}$</td>
<td>leisure</td>
<td>$s_i^S$ supply of floor space by local developer</td>
</tr>
<tr>
<td>$D_{Wa}$</td>
<td>labor supply</td>
<td>$K_i^{Da}$ demand for capital by local developer</td>
</tr>
<tr>
<td>$M_a$</td>
<td>full income</td>
<td>$X_i^D$ demand for land by local developer</td>
</tr>
<tr>
<td>$v_a$</td>
<td>value of time</td>
<td>$d_{Re}^{se}$ demand for road link $s\rightarrow e$</td>
</tr>
<tr>
<td>$t_{Re}^{se}$</td>
<td>toll on road link $s\rightarrow e$</td>
<td>$t_{Re}^{se}$ travel time for the road link $s\rightarrow e$</td>
</tr>
<tr>
<td>$t_a$</td>
<td>assumed commuting time</td>
<td>$t_q$ resulting commuting time</td>
</tr>
<tr>
<td>$c_a$</td>
<td>assumed commuting cost</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$P_a$</td>
<td>alternative’s choice probability</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$V^*_a$</td>
<td>maximum (indirect) utility obtained</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$L^D_j$</td>
<td>labor demand by local firm</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$K^{Da}_j$</td>
<td>capital demand by local firm</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$R_t$</td>
<td>total labor tax revenue</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$R_{Le}$</td>
<td>aggregate land rents</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$w_j$</td>
<td>wage at zone $j$</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$p$</td>
<td>price of composite good (numéraire)</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$L_{Pa}$</td>
<td>kilometers generated with public transport under alternative $a = {i,j,q}$</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$\tau_L$</td>
<td>labor income tax rate</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$P_g$</td>
<td>cost of car use inputs (gasoline, vehicle depreciation, etc.) per unit of distance (as faced by households)</td>
<td>$\gamma_a$</td>
</tr>
<tr>
<td>$P_P$</td>
<td>per passenger price for a unit of distance commute with public transport (as faced by households)</td>
<td>$\gamma_a$</td>
</tr>
</tbody>
</table>

Notes: the subscript $a$ denotes that the variable is conditional on the choice of a given alternative. Because every route $q$ corresponds to a unique alternative $a$, subscripts that refer to a specific route $q$ can be replaced by $a$ (e.g. $L_{Ra} = L_{Rq}$).
Table 5.A.2. Network and choice notation

<table>
<thead>
<tr>
<th>J</th>
<th>an order set of J zones</th>
<th>i</th>
<th>an index pointing at the i-th zone of J</th>
</tr>
</thead>
<tbody>
<tr>
<td>j</td>
<td>an index pointing at the j-th zone of J</td>
<td>C_{OD}</td>
<td>a set of all possible origin-destination pairs</td>
</tr>
<tr>
<td>a_{ij}</td>
<td>an arbitrary origin-destination pair</td>
<td>t_m^{(se)}</td>
<td>an arbitrary link from node s to node e with transport mode m</td>
</tr>
<tr>
<td>q</td>
<td>route: a sequence of neighboring links</td>
<td>p_m^{se}</td>
<td>the length of link t_m^{(se)}</td>
</tr>
<tr>
<td>Q(a_{ij})</td>
<td>the set of all routes q that are compatible to a_{ij}, i.e. they depart from the i-th zone and terminate to the j-th zone</td>
<td>a</td>
<td>alternative: an arbitrary origin destination pair, a_{ij}, coupled with a route q \in Q(a_{ij}). Denoted as ( a = {a_{ij}, q} = {i, j, q} )</td>
</tr>
<tr>
<td>C</td>
<td>Choice set containing all possible alternatives</td>
<td>\textbf{I} \left( t_m^{(se)}</td>
<td>q \right)</td>
</tr>
<tr>
<td>I(j</td>
<td>a)</td>
<td>indicator function that equals one if alternative a implies the j-th zone of J as destination</td>
<td>\textbf{I}(i</td>
</tr>
</tbody>
</table>

Appendix 5.B: Solving for the general, stochastic-user equilibrium

5.B.1. Solution

The proposed algorithm separates the systems of equations in a series of subsystems, that is:

**Subsystem 1:**

Equations given by (5.6), (5.8), (5.9), (5.10) and (5.11) in the unknowns: \( y_a^*, s_a^*, D_{Wa}^*, T_{Fa}^* \) and \( V_a^* \).

**Subsystem 2:**

Equations given by (5.19), (5.20), (5.27), (5.28) and (5.24) in the unknowns: \( L_j^D, K_j^{Df}, K_j^{Da}, X_j^D, Q_j^D \).

**Subsystem 3:**

Equations given by (5.16) in the unknown: \( P_a \).

**Subsystem 4:**

Equations given by (5.29), (5.30), (5.31), (5.32), (5.33), (5.34), (5.35), (5.36), (5.37) and (5.38) in the unknowns: \( L_{Rq}, L_{Pq}, \hat{t}_q, \hat{\ell}_q, t_R^{se}, d_R^{se}, t_P^{se}, C_T, E_T \) and \( D_T \).

**Subsystem 5:**

Equations (5.43), (5.45), and (5.49) in the unknowns: \( E(L_j^S), E(H_i^D), E(Y^D) \).

**Core subsystem:**

Equations (5.18), (5.26), (5.44), (5.46), (5.47), (5.48), (5.38), (5.39), (5.40), (5.41), (5.42), (5.50), (5.52), and (5.53) in the unknowns \( \varphi = (w_j, p_{Hi}, p_{Li}, p_j, Q_j^S, H_l^S, D_T, R_L, R_R, B_e, B, Y, t_q, e_q) \).
Consider the following operations:

**Sequential elimination:**

For any arbitrary vector \( \varphi = (w_j, p_{Hi}, p_{Ll}, p_j, Q_j^S, H_i^S, D_T, R_L, R_R, B, Y, t_q, c_q) \), subsystems 1 and 2 can be solved exactly. Then, the endogenous variables of these subsystems can be inserted in the equations of subsystem 3, which can be solved exactly as well. Subsequently, the resulting endogenous choice probabilities of subsystem 3 can be used to solve subsystems 4 and 5 exactly. This sequential substitution is possible because the equations of subsystem 3 use the unknowns of subsystem 1 as inputs and not vice versa; similarly, the equations of subsystems 4 and 5 use the unknowns of subsystem 1 and 3 as inputs and not vice versa. Therefore, by the end of the sequential elimination process subsystems 1-5 have all been solved.

**Euclidean distance \( \|v(\varphi)\| \) of \( \varphi \) from equilibrium:**

Solve subsystems 1-5 using the sequential elimination method. Insert the resulting values of: \( P_a \) and \( D_{Wa}^i \) in (5.39) and (5.40); \( X_i^D \) in (5.41); \( D_T \) in (5.42); \( E(L_j^2) \) and \( L_j^D \) in (5.44); \( E(H_i^D) \) in (5.46) \( X_i^D \) in (5.47), \( Q_j^D \) in (5.48); \( E(Y^D), K_j^D, K_i^D, C_T \) in (5.50); \( \hat{t}_q \) in (5.52); and \( \hat{e}_q \) (5.53). Denote by \( v(\varphi) \) the vector of equations (5.18), (5.26), (5.44), (5.46), (5.47), (5.48), (5.38), (5.39), (5.40) (5.41), (5.42), (5.50), (5.52), and (5.53); then, \( \|v(\varphi)\| \) is the distance from the equilibrium vector.

**Solution:**

Begin by normalizing the system, *i.e.* by setting an endogenous price equal to one and by dropping an equation from the system. Here we chose to set \( p = 1 \) and to drop equation (5.51) from the system. This equation is used as a *model misspecification check* upon solution. That is, equation (5.51) will generally not hold off-the-equilibrium, but has to hold on the equilibrium, otherwise Walras’ law would be violated.

Begin by choosing a starting value of the endogenous vector \( \varphi_0 \) and set the parameters of the model. Compute \( \|v(\varphi_0)\| \), *i.e.* the Euclidean distance of \( \varphi_0 \) from equilibrium, using the above method. As long as this distance in any arbitrary iteration \( k \), \( \|v(\varphi_k)\| \), exceeds a tolerance level, approximate the Jacobian matrix \( \mathbf{J}(\varphi_k) \) using finite differences. Denoting by \( \varphi_k^n \) a vector such that:

\[
\varphi_k^n - \varphi_k = (0, \ldots, \omega, 0, \ldots, 0)
\]

where \( \omega \) is an infinitesimal perturbation, compute \( \|v(\varphi_k^n)\| \). Then, set the n-th column of \( \mathbf{J} \) equal to:
\[ \frac{v(\varrho^n_k) - v(\varrho_k)}{\omega} \]

repeating for all dimensions of the endogenous vector. Iterate using:

\[ \varrho_{k+1} = \varrho_k - \sigma^* J^{-1} v(\varrho_k), \]

where \(\sigma^*\) is the optimal step found by employing a line search method. Compute \(\|v(\varrho_{k+1})\|\). This numerical tâtonnement process is a variant of the Newton method with a line search. Upon convergence at the \(K\)-th iteration, \(\|v(\varrho_k)\|\) has fallen below the tolerance level. Furthermore, subsystems 1-5 are all solved, because to calculate any arbitrary \(v(\varrho_k)\) or \(v(\varrho^n_k)\) requires to solve them first. The Walras’ law holds, confirming that the system has converged to the general, stochastic user equilibrium.

**Figure 5.B.1.** Two dimensional projection of the convergence to a locally unique benchmark equilibrium (light red point) (i) from an arbitrary initial price-quantity-time vector (dark red) and (ii) from the equilibrium price-quantity-time vectors resulting from the various congestion pricing policies considered (black points).

**5.B.2. Local uniqueness**

Local uniqueness tests have been employed to ensure that the benchmark equilibrium vector of prices, quantities, times and choice probabilities, represented by the red point in the two dimensional projection displayed in Figure 5.B.1, is unique at least in a neighborhood large enough to contain the allocation vectors that result from the implementation of the various policies discussed in the chapter. This area is represented by the grey box in the same figure. To test for local uniqueness, the solution for the benchmark (i.e. the no-toll, 40% labor tax) equilibrium is repeated using the equilibrium prices, quantities, times and choice probabilities that result from the application of the various policies (discussed in Section 5.4) as starting values. These starting values are represented by the black dots. What is obtained in all cases is the benchmark equilibrium, providing serious evidence that the latter equilibrium is unique in a
neighborhood large enough to ensure that uniqueness is not an issue in the policy analysis of this study.

Appendix 5.C: Calibration with genetic algorithms

At any arbitrary iteration $k$ of stage $x$ (see stages below), a genetic algorithm draws a population of random preference vectors $\mathbf{v}_x$ from a ball $B(\mathbf{v}_x^{k-1}, r^k) = \{ \mathbf{v}_x : d(\mathbf{v}_x, \mathbf{v}_x^{k-1}) \leq r^k \}$ of radius $r^k$ centered at the survivor preference vector $(\mathbf{v}_x^{k-1})$ from the previous iteration $k-1$, i.e. $\mathbf{v}_A^{k-1}$. At each draw, the model is fully solved for the equilibrium using the algorithm discussed in Appendix 5.B.

Table 5.C.1. Calibration parameters and objectives

<table>
<thead>
<tr>
<th>Stage</th>
<th>Parameters</th>
<th>Objective functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$\mathbf{v}_A = (\pi_0, \pi_1, \alpha, \beta, \gamma)$</td>
<td>$\mathbf{k}<em>{A0} = (E</em>{sh}(\mathbf{v}<em>A, C) - 0.65) + (E</em>{sh}(\mathbf{v}_A, H) - 0.30)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\mathbf{k}<em>{A1} = (T</em>{sh}(\mathbf{v}<em>A, L) - 0.60) + (T</em>{sh}(\mathbf{v}_A, C) - 0.10)$</td>
</tr>
<tr>
<td>$B$</td>
<td>$\mathbf{v}_B = (\beta, \gamma, \xi_1 R)$</td>
<td>$\mathbf{k}<em>B = t</em>{rel}(\mathbf{v}_B) - 2.3$</td>
</tr>
<tr>
<td>$C$</td>
<td>$\mathbf{v}_C = (z_i, z_j, \zeta_j)$</td>
<td>For each residential zone: $\mathbf{k}<em>{Ci} = \overline{POP}</em>{sh}(\mathbf{v}_C, i) - \overline{POP}_i$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>For each employment zone: $\mathbf{k}<em>{Cj} = \overline{EMP}</em>{sh}(\mathbf{v}_C, j) - \overline{EMP}_j$</td>
</tr>
<tr>
<td>$D$</td>
<td>$\mathbf{v}_D = (z_R, z_P, z_t)$</td>
<td>$\mathbf{k}<em>D = sh</em>{car}(\mathbf{v}_D) - 0.65$</td>
</tr>
</tbody>
</table>

Notes: Expenditure shares of consumption, housing and commuting are denoted by $E_{sh}(\mathbf{v}_A, C)$, $E_{sh}(\mathbf{v}_A, H)$ and $E_{sh}(\mathbf{v}_A, T)$ respectively; average time shares on labor, leisure and commuting are denoted by $T_{sh}(\mathbf{v}_A, L)$, $T_{sh}(\mathbf{v}_A, \ell)$, and $T_{sh}(\mathbf{v}_A, C)$; $t_{rel}(\mathbf{v}_B)$ is the average (across links) equilibrium-to-free-flow travel time ratio. $\overline{POP}_{sh}(\mathbf{v}_C, i)$ and $\overline{EMP}_{sh}(\mathbf{v}_C, j)$ denote the share of a given zone in the total population and employment respectively predicted by the model; $\overline{POP}_i$ and $\overline{EMP}_j$ denote the respective shares observed in data; $sh_{car}(\mathbf{v}_D)$ stands for the car choice probability in the model.

Upon solution, the sets of objective functions displayed in the third column of Table 5.C.1 are computed. The vector that survives iteration $k$ is the one with the lowest objective values. As these objective values get closer to zero, the radius $r^k$ that defines the search area around the survivor vector $\mathbf{v}_x^{k-1}$ becomes smaller. If there is more than one objective function, the algorithm terminates when the Pareto frontier is reached, i.e. no vector that can reduce all objectives simultaneously is drawn after a large number of trials.

To reduce the total number of objective functions and facilitate the visualization used in the sensitivity analyses of Section 5.4.3, the set of objectives C in Table 5.C.1 is aggregated using the deviation function 4 in Table 5.C.2. The two objectives in stage A of Table 5.C.1 are visualized separately (see functions 1 and 2 in Table 5.C.2). The objectives of stages B and D are given by functions 3 and 5 in Table 5.C.2.
Table 5.C.2. Deviation-from-the-objective functions

<table>
<thead>
<tr>
<th>Objective</th>
<th>Function value displayed in sensitivity analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\sqrt{(E_{sh}(v_A, C) - 0.65)^2 + (E_{sh}(v_A, H) - 0.30)^2 + (E_{sh}(v_A, T) - 0.05)^2}$</td>
</tr>
<tr>
<td>2</td>
<td>$\sqrt{(T_{sh}(v_A, L) - 0.60)^2 + (T_{sh}(v_A, l) - 0.30)^2 + (T_{sh}(v_A, c) - 0.10)^2}$</td>
</tr>
<tr>
<td>3</td>
<td>$t_{rel}(v_B) - 2.3$</td>
</tr>
<tr>
<td>4</td>
<td>$\sqrt{\sum_i(POP_{sh}(v_C, i) - \overline{POP}<em>i)^2 + \sum_j(EMP</em>{sh}(v_C, j) - \overline{EMP}_j)^2}$</td>
</tr>
<tr>
<td>5</td>
<td>$s_{h, \text{car}}(v_D) - 0.65$</td>
</tr>
</tbody>
</table>

Notes: Expenditure shares of consumption, housing and commuting are denoted by $E_{sh}(v_A, C)$, $E_{sh}(v_A, H)$ and $E_{sh}(v_A, T)$ respectively; average time shares on labor, leisure and commuting are denoted by $T_{sh}(v_A, L)$, $T_{sh}(v_A, l)$, and $T_{sh}(v_A, c)$; $t_{rel}(v_B)$ is the average (across links) equilibrium-to-free-flow travel time ratio. $POP_{sh}(v_C, i)$ and $EMP_{sh}(v_C, j)$ denote the share of a given zone in the total population and employment respectively predicted by the model; $\overline{POP}_i$ and $\overline{EMP}_j$ denote the respective shares observed in data; $s_{h, \text{car}}(v_D)$ stands for the car choice probability in the model.
### Appendix 5.D: optimal tolls against the marginal external cost of congestion

Table 5.D.1. Optimal toll and marginal external congestion cost (by link) in the case of labor tax cut revenue recycling.

<table>
<thead>
<tr>
<th>start node</th>
<th>end node</th>
<th>toll</th>
<th>mecc</th>
<th>start node</th>
<th>end node</th>
<th>toll</th>
<th>mecc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>0.001845</td>
<td>0.00261</td>
<td>3</td>
<td>1</td>
<td>-0.00209</td>
<td>0.002997</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0.002235</td>
<td>0.002428</td>
<td>2</td>
<td>1</td>
<td>-0.00189</td>
<td>0.003122</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>0.00432</td>
<td>0.004144</td>
<td>16</td>
<td>1</td>
<td>0.001272</td>
<td>0.004711</td>
</tr>
<tr>
<td>1</td>
<td>18</td>
<td>0.000125</td>
<td>0.001371</td>
<td>18</td>
<td>1</td>
<td>-0.00467</td>
<td>0.002068</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>-0.00089</td>
<td>0.002729</td>
<td>6</td>
<td>2</td>
<td>0.002846</td>
<td>0.002896</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>-0.001593</td>
<td>0.003181</td>
<td>6</td>
<td>4</td>
<td>-0.00011</td>
<td>0.002827</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>-0.003863</td>
<td>0.001692</td>
<td>6</td>
<td>5</td>
<td>-0.00508</td>
<td>0.001059</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>0.001726</td>
<td>0.001887</td>
<td>8</td>
<td>6</td>
<td>-0.00121</td>
<td>0.001999</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>0.001017</td>
<td>0.002489</td>
<td>7</td>
<td>6</td>
<td>-0.00185</td>
<td>0.002441</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>-0.000735</td>
<td>0.001362</td>
<td>9</td>
<td>3</td>
<td>-0.00621</td>
<td>0.001323</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>-0.000196</td>
<td>0.00325</td>
<td>4</td>
<td>3</td>
<td>0.001692</td>
<td>0.003822</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>-0.000016</td>
<td>0.001854</td>
<td>9</td>
<td>4</td>
<td>-0.00805</td>
<td>0.001721</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>-0.00573</td>
<td>0.001004</td>
<td>5</td>
<td>4</td>
<td>-0.00302</td>
<td>0.001031</td>
</tr>
<tr>
<td>8</td>
<td>100</td>
<td>-0.002062</td>
<td>0.001424</td>
<td>100</td>
<td>8</td>
<td>-0.00025</td>
<td>0.001801</td>
</tr>
<tr>
<td>12</td>
<td>100</td>
<td>-0.000247</td>
<td>0.003427</td>
<td>100</td>
<td>12</td>
<td>-0.00206</td>
<td>0.002709</td>
</tr>
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Notes: Values have been computed using a BFGS algorithm. Different starting values generated by the no-toll and the Pigouvian quasi first-best equilibrium were used, providing a convergence to (roughly) the same optimum.
Appendix 5.E: Replicability of simulation experiments

The following tables provide the calibrated values of parameters used in the simulation experiments, as well as the values of the exogenous variables in the model. A vector of satisfactory initial values of the endogenous variables is available upon request.

Table 5.E.1. Values of parameters and exogenous variables

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<tr>
<th></th>
<th>α</th>
<th>β</th>
<th>γ</th>
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<th>π₁</th>
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<td>0.015</td>
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<td>0.015</td>
</tr>
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</table>

Notes: Capital shares are assumed to be constant across zones for both firms (δ = 0.3) and developers (θ = 0.3). Volume delay function parameters are constant across all (road and rail) links; this is a weak assumption because all links represent large parts of a highway system which, at this level of aggregation, is relatively homogenous. Total factor productivity is uniform over space: differences in factor employment (including job concentration) and output level are generated through non-uniform cost shares (ζ) of the assembly industry. This prevents wages from displaying a spatial variation that would be incompatible with data.

Table 5.E.2. Land endowment (X̅) of each zone

|   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|
|   | .208 | .222 | .068 | .223 | .158 | .095 | .127 | .140 | .363 | .126 | .206 | .233 | .378 | .082 | .204 | .236 | .208 | .187 |

Appendix 5.F: Numerical approximation of the key double-dividend effects

We approximate numerically the double dividend effects discussed in the text. The reader should bear in mind that, given the size, detail and complexity of the model, an analytic decomposition of a total effect (i.e. the total welfare change caused by a marginal increase in the toll of an arbitrary link) in its Pigouvian, Tax Interaction and Revenue Recycling components is not possible.

Instead, what we attempt here is an improvised adoption of the general idea behind decomposition formulas that have appeared with significant variations in the literature of double-
dividend (Bovenberg and de Mooij, 1994; Goulder et al., 1999; Parry and Bento, 2000; Bento and Jacobsen, 2007; Carbone and Smith, 2008; Bento et al., 2011). Define the Pigouvian effect in link \( l_{R}^{(se)} \) as:

\[
p^{se} = (\tau_{R}^{se} - mecc_{R}^{se}) \frac{d(d_{R}^{se})}{dt_{R}^{se}}, \tag{5.F.1}
\]

where the total derivative of the link load (i.e. the total demand for the road link, \( d_{R}^{se} \), as defined in (5.31)) with respect to the link toll is approximated by using finite differences (i.e. by computing \( d_{R}^{se} \) in two general equilibria between which \( \tau_{R}^{se} \) has been increased) and \( mecc_{R}^{se} \) denotes the Pigouvian level of the toll as given by (5.54).

Next, define the total (general equilibrium) derivative of labor tax revenue \( (R_L \text{ as it is expressed in equation (5.39)) as:} \)

\[
\begin{align*}
\frac{dR_L}{d\tau_L} &= \frac{d[\tau_L LS(\tau_L)]}{d\tau_L} = LS(\tau_L) + \tau_L \frac{dLS(\tau_L)}{d\tau_L} \tag{5.F.2}
\end{align*}
\]

where \( \frac{dLS(\tau_L)}{d\tau_L} \) is approximated again using finite differences, and set:

\[
M = -\frac{\left\{ \tau_L \frac{dLS(\tau_L)}{d\tau_L} \right\}}{\left\{ LS(\tau_L) + \tau_L \frac{dLS(\tau_L)}{d\tau_L} \right\}} \tag{5.F.3}
\]

Finally, we compute the tax interaction effect in the link \( l_{R}^{(se)} \) as:

\[
TI^{se} = (1 + M) \frac{dR_L}{dt_{R}^{se}}, \tag{5.F.4}
\]

and the revenue recycling effect in the same link as:

\[
RR^{se} = M \frac{dR_R}{dt_{R}^{se}}. \tag{5.F.5}
\]

Table 5.F.1 presents the sum of \( P^{se}, TI^{se} \) and \( RR^{se} \) computed for each link in the base, no-toll equilibrium. In general, the sign of the total effect is in accordance with the sign and the magnitude of the optimal toll (toll characterization), although the correspondence is imperfect (the sign is opposite in a few cases). The slight differences can be attributed to three, non-mutually-exclusive factors: i) local (instead of global) optimum reached by the optimization algorithm (in this case most probably the global optimum lies in the close neighborhood of the local optimum, since the difference in this case is hard to detect by heuristic tests using different starting values in the optimization algorithm), ii) the above approximation is incomplete, in the
sense that residual effects are missing or analytic formulas differ slightly from the above, and iii) numerical errors in the above approximations.

Table 5.F.1. Double dividend (DD) approximation of the total effect in the untolled (base) equilibrium versus optimal link toll characterizations: double dividend emerges (++) , positive without double dividend (+), approximately zero (≈0), negative (-).

<table>
<thead>
<tr>
<th>startnode $\rightarrow$ endnode</th>
<th>toll characterization</th>
<th>total effect-approximation</th>
<th>startnode $\rightarrow$ endnode</th>
<th>toll characterization</th>
<th>total effect-approximation</th>
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</thead>
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<td>1→2</td>
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<td>2→1</td>
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<tr>
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<td>(-)</td>
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</tr>
<tr>
<td>1→16</td>
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<td>0.00321</td>
<td>16→1</td>
<td>(+)</td>
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<tr>
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<tr>
<td>2→6</td>
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<td>8E-05</td>
<td>6→2</td>
<td>(+)</td>
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<tr>
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<td>6→5</td>
<td>(-)</td>
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<tr>
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<td>6→8</td>
<td>(+)</td>
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<tr>
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Notes: We classify a link toll as being approximately zero (≈0) if the absolute value of the optimal toll is below 10% of the marginal external congestion cost in this link.