Appendix A

Appendices to Chapter 2

A.1 Analysis

A.1.1 Carbon Taxation

In order to obtain analytical results, we make some simplifying assumptions. In particular, we work with a demand function \( D(p_i) \), satisfying \( U'(D(p_i)) = p_i \), \( i = y, x \), with \( p_i \) being the consumer price of the resource or the substitute. Note that the demand functions \( D_i \) and \( D_i^* \), \( i = x, y \), can vary between the two regions. We also suppose that \( r = \rho \), i.e., the world market interest rate equals the rate of time preference, as in Section 2.2.3.

With the possibility of different taxes \( \tau_y \), \( \tau_x \), \( \tau_y^* \) and \( \tau_x^* \) the first-best equilibrium is characterized by six equations, comprising two stock conditions and four transition equations:

\[
\begin{align*}
\int_0^{T_1} D_y^*(\lambda^*_y e^{rt} + c^*_y + \tau^*_y)dt + \int_0^{T_1} D_y(\lambda^*_y e^{rt} + c^*_y + \tau_y)dt &= Y_0^*, \\
\lambda^*_y e^{rT_1} + c^*_y + \tau^*_y &= \lambda^*_x e^{rT_1} + c_x + \tau^*_x, \\
\int_{T_1}^{T_2} D_x^*(\lambda_x e^{rt} + c_x + \tau^*_x)dt + \int_{T_1}^{T_2} D_x(\lambda_x e^{rt} + c_x + \tau_x)dt &= X_0, \\
\lambda^*_x e^{rT_2} + c_x + \tau^*_x &= b^* - \sigma^*, \\
\lambda_x e^{rT_2} + c_x + \tau_x &= b - \sigma.
\end{align*}
\]

Equations (A.2) and (A.3) are equivalent, assuming that \( \tau^*_t = \tau^*_x \) and \( \tau_y = \tau_x \). One of them can thus be dropped. Following Hoel (2011), we differentiate the equation system (A.1) to (A.6) which gives:
\[ M \begin{pmatrix} dT_1 \\ d\lambda_y \\ dT_2 \\ d\lambda_x \end{pmatrix} = \begin{pmatrix} -B_y^* \\ -1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} -B_y \\ -1 \\ 0 \\ 0 \end{pmatrix} d\tau_y + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} d\tau_y + \begin{pmatrix} 0 \end{pmatrix} d\tau_y + \begin{pmatrix} -B_x^* \\ -1 \\ 0 \\ -1 \end{pmatrix} d\tau_x + \begin{pmatrix} 0 \end{pmatrix} d\tau_x \]
\[ + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} d\sigma + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} d\sigma, \quad (A.7) \]

where
\[ B_y^* = \int_0^{T_1} D_y^x(\lambda_y^* e^{rt} + c_y^* + \tau_y^*) dt, \quad B_y = \int_0^{T_1} D_y^x(\lambda_y^* e^{rt} + c_y + \tau_y) dt \]
and
\[ B_x^* = \int_0^{T_2} D_x^x(\lambda_x^* e^{rt} + c_x^* + \tau_x^*) dt, \quad B_x = \int_0^{T_2} D_x^x(\lambda_x e^{rt} + c_x + \tau_x) dt, \]
and:
\[ M = \begin{pmatrix}
D_y(\lambda_y^* e^{T_1} + c_y^* + \tau_y) + D_y(\lambda_y^* e^{T_1} + c_y^* + \tau_y^*) & \int_0^{T_1} e^{rt} D_y^*(\lambda_y^* e^{rt} + c_y^* + \tau_y^*) dt + \int_0^{T_1} e^{rt} D_y(\lambda_y^* e^{rt} + c_y^* + \tau_y) dt \\
(\lambda_y^* - \lambda_x) r e^{r T_1} & -D_x(\lambda_x e^{r T_1} + c_x + \tau_x) - D_x(\lambda_x e^{r T_1} + c_x + \tau_x^*) \\
\end{pmatrix}
\]

\[ D_x(\lambda_x e^{r T_2} + c_x + \tau_x^*) & D_x(\lambda_x e^{r T_2} + c_x + \tau_x) & \int_{T_1}^{T_2} e^{rt} D_x^*(\lambda_x e^{rt} + c_x + \tau_x^*) dt + \int_{T_1}^{T_2} e^{rt} D_x(\lambda_x e^{rt} + c_x + \tau_x) dt \\
D_x(\lambda_x e^{r T_1} + c_x + \tau_x^*) & D_x(\lambda_x e^{r T_1} + c_x + \tau_x) & \int_0^{T_1} e^{rt} D_x^*(\lambda_x e^{rt} + c_x + \tau_x^*) dt + \int_0^{T_1} e^{rt} D_x(\lambda_x e^{rt} + c_x + \tau_x) dt \\
0 & 0 & -e^{r T_1} \\
0 & 0 & -e^{r T_1} \\
\end{pmatrix}
\]

(A.8)
Adjusting this general framework to our baseline case of no taxes and no subsidy in the foreign country \((\tau_x^* = \tau_y^* = \sigma^* = 0)\), a uniform tax in the home country \((\tau_x = \tau_y = \tau > 0)\) and a positive subsidy \((\sigma > 0)\), we get

\[
\int_0^{T_1} D^* (\lambda_y^* e\tau + c_y^*) dt + \int_0^{T_1} D (\lambda_y^* e\tau + c_y^* + \tau_y) dt = Y^*_0,
\]  
(total demand for cheap oil equals supply) \(\lambda_y^* e\tau^{T_1} + c_y^* = \lambda_x^* e\tau^{T_1} + c_x\), \(\text{(A.10)}\)  
(continuous world market price at transition) \(\lambda x^* e\tau^{T_2} + c_x = b^*,\) \(\text{(A.12)}\)

We have then 5 equations in 5 unknowns \((\lambda_y^*, \lambda_x, T_1, T_2^*, T_2)\). From these equations (A.9) to (A.13) we derive the direction of change of our Green Paradox variables of interest.

The effect of an increased tax \(\tau\) on the resource rents is:

\[
\frac{d\lambda_y^*}{d\tau} = \frac{-B_y + B_y \frac{D_y + D_y^*}{\lambda_x} (\lambda_y^* - \lambda_x) - B_y \frac{D_y}{\lambda_x e\tau^2} + B_y \frac{D_y^*}{\lambda_x e\tau^2} + Q (\lambda_y^* - \lambda_x) r \frac{B_y}{D_y + D_y^*} + 1}{A + D_y + D_y^* (\lambda_y^* - \lambda_x) \frac{A}{D_y + D_y^*} + 1 + Q (\lambda_y^* - \lambda_x) r \frac{A}{D_y + D_y^*} + 1}, \]  
(A.14) \[
\frac{d\lambda_x}{d\tau} = \frac{-B_x + B_y \frac{D_x + D_x^*}{\lambda_x} + B_x \frac{D_x}{\lambda_x e\tau^2} - (D_y + D_y^*) \frac{A}{\lambda_x} - D_x - D_x^*}{- (D_y + D_y^*) \frac{A}{\lambda_x} - D_x - D_x^* + Q}, \]  
(A.15)

where:

\[
B_y = \int_0^{T_1} D_y^* (\lambda_y^* e\tau + c_y^* + \tau) dt < 0 \text{ (derivative of the demand function with respect to } \tau),
\]

\[
B_x = \int_{T_1}^{T_2} D_x^* (\lambda_x e\tau + c_x + \tau) dt < 0 \text{ (derivative of the demand function with respect to } \tau),
\]
A.1. ANALYSIS

\[
A = \int_{0}^{T_1} e^{\tau t} D'_y (\lambda^*_y e^{\tau t} + c^*_y + \tau) dt + \int_{0}^{T_2} e^{\tau t} D''_y (\lambda^*_y e^{\tau t} + c^*_y + \tau^*) dt < 0
\]

(derivative of the demand function with respect to \(\lambda^*_y\)),

\[
Q = \int_{T_1}^{T_2} e^{\tau t} D'_x (\lambda^*_x e^{\tau t} + c^*_x + \tau) dt + \int_{T_1}^{T_2} e^{\tau t} D''_x (\lambda^*_x e^{\tau t} + c^*_x + \tau^*) dt < 0
\]

(derivative of the demand function with respect to \(\lambda^*_x\)),

\[
L = A(\lambda^*_y - \lambda^*_x)r - (D_y + D^*_y) < 0,
\]

and abbreviating \(D_y(\lambda^*_y e^{\tau T_1} + c^*_y + \tau)\) with \(D_y\), \(D^*_y(\lambda^*_y e^{\tau T_1} + c^*_y + \tau^*)\) with \(D^*_y\), \(D_x(\lambda^*_x e^{\tau T_1} + c^*_x + \tau)\) with \(D_x\) and \(D_x(\lambda^*_x e^{\tau T_1} + c^*_x + \tau^*)\) with \(D^*_x\).

Equations (A.14) and (A.15) imply that \(\frac{d\lambda^*_y}{d\tau} < 0\) and \(\frac{d\lambda^*_x}{d\tau} < 0\).

The effect of an increase in \(\tau\) on the first switching point \(T_1\) equals:

\[
\frac{dT_1}{d\tau} = \frac{-B_x - \frac{D_x + D^*_x}{r\lambda_x} B_y + \frac{D_x}{r\lambda_x e^{\tau T_1}} + \frac{QB_y}{A}}{-\left(\frac{D_y + D^*_y}{A\lambda_x} - \frac{D_x + D^*_x}{r\lambda_x e^{\tau T_1}} + \frac{QB_y}{A}\right)}, \quad (A.16)
\]

where:

\[
M = D_y + D^*_y.
\]

Whereas the denominator is unambiguously negative, the nominator is also negative but for the first and second terms, i.e., \(-B_x + \frac{D_x}{r\lambda_x e^{\tau T_1}}\) > 0. In order to have \(\frac{dT_1}{d\tau} < 0\), it needs to hold that:

\[
\left| -B_x + \frac{D_x}{e^{\tau T_1} r \lambda_x} \right| > \left| -\frac{(D_x + D^*_x) B_y}{A r \lambda_x} + \frac{QB_y}{A} \right|. \quad (A.17)
\]

The effect of an increase in \(\tau\) on the second switching point of the home region is:

\[
\frac{dT_2}{d\tau} = \frac{-B_x + (D_y + D^*_y) \left(\frac{-B_x + e^{-\tau T_1} A}{K}\right) + \frac{D_x^*}{e^{\tau T_1}} + \frac{Q}{e^{\tau T_1}}}{(D_y + D^*_y) \frac{x^* A}{K} + D_x + D^*_x - Q \tau \lambda_x}, \quad (A.18)
\]

with

\[
K = D_y + D^*_y - r(\lambda^*_y - \lambda^*_x) A > 0.
\]

Whereas the denominator is unambiguously positive, the sign of the nominator depends on the parameter values.
For $\frac{dT_2}{d\tau} > 0$ it must hold that:

$$\left| -B_x + (D_y + D_y^* \left( \frac{-B_y}{K} \right)) \right| > \left| -(D_y + D_y) \left( \frac{A}{Ke^{r T_2}} \right) + \frac{D_y^*}{e^{r T_2}} + \frac{Q}{e^{r T_2}} \right|.$$ 

The effect of an increase in $\tau$ on the second switching point of the foreign region is:

$$\frac{dT_2^*}{d\tau} = -B_x - B_y \frac{D_y + D_y^*}{K} + \frac{B_y^*}{r \lambda_x e^{r T_2}} > 0.$$ (A.19)

This expression is unambiguously positive.

### A.1.2 Proofs of Propositions 1 and 2

**Proof of Proposition 1.**

The home country increases its subsidy $\sigma$. Suppose that the scarcity rent of the low cost resource increased in response, i.e., $\frac{d\lambda_y^*}{d\sigma} > 0$. Then the low cost resource would be depleted later as less resources are consumed at each point in time. The switching point to the high cost resource $T_1$ would have to occur later. If both $T_1$ and $\lambda_y^*$ increase, also $\lambda_x$ needs to increase according to:

$$(\lambda_y^* - \lambda_x)e^{r T_1} = c_x - c_y^* + \tau.$$ (A.20)

An higher $\lambda_x$ results in a shortening of the switching times to the backstop resource, because for $T_2^*$ and $T_2$ the following needs to hold:

$$\lambda_x e^{r T_2} = b - \sigma - c_x - \tau,$$ (A.21)

$$\lambda_x e^{r T_2^*} = b^* - c_x.$$ (A.22)

This, however, is impossible since there would not be enough demand for the high cost resource. Since also the high cost resource needs to be depleted, both $T_1$ and $\lambda_y^*$ need to decrease. Since $(\lambda_y^* - \lambda_x)$ must rise, $\lambda_x$ decreases, too. In this case we know that $T_2$ has to increase (see equation (A.22)). The higher demand for the high cost resource from the part of the foreign region is matched by lower demand in the home region. The home region’s non-renewable resource usage period is shortened and $T_2$ goes down. □

**Proof of Proposition 2.** The home country increases its tax $\tau$. Let us suppose first that the scarcity rent of the high cost resources increased. From (A.21) and (A.22) we know this
would accelerate the switch to using the backstop resource, i.e., \( T_2 \) and \( T^* \) would occur earlier.

Now, \( \lambda_y^* \) could increase or decrease. Let us suppose that it decreases. We see that the low cost resource phase needs to be prolonged from equation (A.20), i.e., \( T_1 \) increases. An increase in \( T_1 \) and a decrease in \( T_2 \) and \( T^* \) result in the high cost resource not being depleted completely. An increase in \( \lambda_x \) and a decrease in \( \lambda_y^* \) is hence not possible in equilibrium.

Suppose now that \( \lambda_y^* \) increases. Then two things can happen: \((\lambda_y^* - \lambda_x)\) can increase or decrease. Firstly, let us suppose it decreases, i.e., the increase of \( \lambda_x \) is bigger than the increase of \( \lambda_y^* \). From (A.20) we know that \( T_1 \) needs to increase. But, as we have already argued before, this cannot be an equilibrium as the high cost resource would not be depleted.

Let us hence examine the case when \((\lambda_y^* - \lambda_x)\) increases, which happens when the increase of \( \lambda_x \) is smaller than the increase of \( \lambda_y^* \). In this case the extraction time of the low cost resource \( T_1 \) shortens. This, however, cannot be an equilibrium since then a part of the low cost resource would be left in the ground.

We conclude hence that an increase in \( \lambda_x \) together with both an increase and a decrease in \( \lambda_y^* \) cannot occur in equilibrium.

Let us suppose now that the high cost resource rent decreases. From (A.22) we know that \( T^* \) is prolonged, whereas \( T_2 \) might be prolonged or shortened. Since \( \lambda_x \) decreases, also \( \lambda_y^* \) needs to fall, otherwise the low cost resource is not fully extracted.

For an equilibrium to exist both scarcity rents have to decrease in response to a tax increase. However, \( T_1 \) can increase or decrease: in case the high cost resource rent decreases more than the low cost rent, i.e., \( \frac{d\lambda_x}{d\tau} > \frac{d\lambda_y^*}{d\tau} \), \( T_1 \) goes down. □

A.2 Change in \( T_1 \) in Response to a Tax Increase

A.2.1 Analytical Example with Linear Demand

In this appendix we demonstrate the ambiguity of \( \frac{dT_1}{dT} \). To that end we assume linear demand, identical across the two countries:
\[ y_c(p) = x_c(p) = \alpha - \beta p, \]
\[ y^*_c(p^*) = x^*_c(p^*) = \alpha - \beta p^*, \]

with \( \alpha > b^* > b \). In an equilibrium we must have that:

\[ \lambda_x e^{rT_1} + c_x = \lambda^*_x e^{rT_1} + c^*_x, \quad (A.23) \]
\[ \lambda_x e^{rT_2} = b - c_x - \tau, \quad (A.24) \]
\[ \lambda_x e^{rT_2^*} = b^* - c_x, \quad (A.25) \]
\[ \int_0^{T_1} (y_c(t) + y^*_c(t)) \, dt = Y^*_0, \quad (A.26) \]
\[ \int_{T_1}^{T_2} x_c(t) \, dt + \int_{T_1}^{T_2^*} x^*_c(t) \, dt = X_0. \quad (A.27) \]

Let us, for the sake of notation, define:

\[ v = rT_2 - rT_1, \quad w = rT_2^* - rT_1, \quad z = rT_1. \]

Then, inserting the linear demand functions into equations (A.26) and (A.27) and solving the integrals, (A.23) to (A.27) boil down to:

\[ (2\alpha - 2\beta c_y^* - \beta\tau)z - 2\beta \left\{ (b - c_x - \tau)e^{-v} + c_x - c_y^* \right\} (1 - e^{-z}) = rY_0, \quad (A.28) \]
\[ (\alpha - \beta c_x - \beta\tau)v - \beta(b - c_x - \tau)(1 - e^{-v}) + (\alpha - \beta c_x)w - \beta(b^* - c_x)(1 - e^{-w}) = rX_0, \quad (A.29) \]
\[ (b - c_x - \tau)e^{-v} = (b^* - c_x)e^{-w}. \quad (A.30) \]

For a given \( \tau \), the unknowns in this system of three equations are \( v, w \) and \( z \).

Next, we compare the outcome under a no-tax regime with a tax regime where the home country never uses its own (expensive) resource.

For \( \tau = 0 \) we find \( \hat{v} \) and \( \hat{w} \) from (A.29) and (A.30) by inserting \( \tau = 0 \). Then \( \hat{z} \) follows from (A.28) upon insertion of \( \hat{v} \).

In the second regime we impose \( \hat{v} = 0 \). Then \( \hat{w} \) follows from (A.29), with \( v = \hat{v} = 0 \). The corresponding tax \( \hat{\tau} \) follows from (A.30). Finally, \( \hat{z} \) is found from (A.28) with \( \hat{\tau} \) and \( \hat{v} = 0 \).
A.2. CHANGE IN $T_1$ IN RESPONSE TO A TAX INCREASE

We now need to compare $\tilde{z}$ and $\hat{z}$. In the zero tax regime we have, using (A.30):

$$ (2\alpha - 2\beta c^*_y)\tilde{z} - 2\beta \left\{ (b^* - c_x)e^{-\tilde{w}} + c_x - c^*_y \right\} (1 - e^{-\tilde{z}}) = rY_0. \tag{A.31} $$

In the tax regime we have, using (A.30):

$$ (2\alpha - 2\beta c^*_y - \beta \hat{\tau})\hat{z} - 2\beta \left\{ (b^* - c_x)e^{-\hat{w}} + c_x - c^*_y \right\} (1 - e^{-\hat{z}}) = rY_0. \tag{A.32} $$

Evidently, it holds that:

$$ 2\alpha - 2\beta c^*_y > 2\alpha - 2\beta c^*_y - \beta \hat{\tau}. $$

It also holds that:

$$ (b^* - c_x)e^{-\tilde{w}} + c_x - c^*_y > (b^* - c_x)e^{-\hat{w}} + c_x - c^*_y, $$

because $\hat{\tau} > 0$ and $\tilde{w} > \hat{w}$. Moreover, $\hat{\tau}, \hat{w}$ and $\tilde{w}$ are determined by (A.29) and (A.30) and do not depend on $Y_0$. Note also that $1 - e^{-z}$ is increasing in $z$. Hence, $\hat{z} < \tilde{z}$ is only possible for small $Y_0^*$, which results in small $z$ and consequently makes $(1 - e^{-z})$ small and the right hand side of (A.31) and (A.32) less important.

For high $Y_0$ is must be the case that $\hat{z} > \tilde{z}$. Hence, for high $Y_0$, $T_1$ increases with a higher tax. However, for low $Y_0$ the effect may be reversed due to (A.32).

The intuition is straightforward: The high cost producers are faced with an immense drop in demand as the home country basically ceases to demand the expensive resource due to $\hat{\tau}$. They have to adjust their scarcity resource more than the low cost resource producers, who still face demand from both regions, even though the home region’s demand is reduced. Hence, $\frac{d\lambda_x}{d\tau} > \frac{d\lambda_y}{d\tau}$ and the switching point $T_1$ occurs earlier.

A.2.2 Numerical Example with Linear Demand

We illustrate the analytical possibility of a weak Green Paradox as a consequence of a tax increase by a numerical example. For the linear demand case presented in the previous Appendix A.2.1, we employ the parameter values in Table A.2.

For the first scenario of Appendix A.2.1, when $\tau = 0$, the resulting switching points amount
Table A.1: Parameter values for the numerical exercise regarding $\frac{dT_1}{dT} < 0$

<table>
<thead>
<tr>
<th>Model’s parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum demand $\alpha$</td>
<td>60</td>
</tr>
<tr>
<td>Demand slope $\beta$</td>
<td>1.1</td>
</tr>
<tr>
<td>World interest rate $r$</td>
<td>0.001</td>
</tr>
<tr>
<td>Extraction costs $c_y^*$</td>
<td>0.005</td>
</tr>
<tr>
<td>$c_x$</td>
<td>0.01</td>
</tr>
<tr>
<td>Backstop production costs $b(b^*)$</td>
<td>45(55)</td>
</tr>
<tr>
<td>Resource stocks $X_0(Y_0^*)$</td>
<td>10000 (0.5)</td>
</tr>
</tbody>
</table>

To $T_1 = 0.01126$, $T_2 = 269.68938$ and $T_2^* = 470.40049$ (i.e., $\tilde{w} = 0.470389$, $\tilde{v} = 0.269678$ and $\tilde{z} = 0.00001126$).

Setting $\nu = 0$ results in a positive $\tilde{\tau} = 16.19972$. Whereas the foreign region switches now much later, namely at $T_2^* = 647.11838$, the first switching point occurs much earlier at $T_1 = 0.004894$ (in this case $\tilde{w} = 0.647113486$ and $\tilde{z} = 0.000004894$).

A Green Paradox is likely to occur if $Y_0^*$ is small and $X_0$ relatively large. Furthermore, the world interest rate has to be relatively low, since it lowers $z$ and hence the importance of the term $(1 - e^{-z})$ in equations (A.31) and (A.32). Also the parameters of the linear demand function play a role: a Green Paradox is likely to occur if $\alpha$ is relatively small, but $\beta$ relatively large. Extraction costs, as long as they are not equal to zero, do not seem to play a major role: for $c_y^* = 0.5$ and $c_x = 1$, the switching points do not change much, only $\tilde{\tau}$ decreases significantly.
A.3 Climate Damage and Social Cost of Carbon

We define $h(t) = \psi(x_c + x^*_c) + \psi^*(y_c + y^*_c)$. The total damage from atmospheric CO$_2$ in both regions and constant marginal damages $\nu$ equals:

$$\int_0^\infty e^{-\rho t} 2\nu(S_1(s) + S_2(s))ds$$  \hspace{1cm} (A.33)

$$= \int_0^\infty 2\nu(S_1(s) + S_2(s))d \left( -\frac{1}{\rho} e^{-\rho s} \right)$$  \hspace{1cm} (A.34)

$$= \frac{2\nu}{-\rho} (S_1(t) + S_2(t)) e^{-\rho t} \bigg|_0^\infty + \int_0^\infty \frac{2\nu}{\rho} e^{-\rho s} \left( S_1(s) + S_2(t) \right)$$  \hspace{1cm} (A.35)

$$= \frac{2\nu}{\rho} (S_1(0) + S_2(0)) + \int_0^\infty \frac{2\nu}{\rho} e^{-\rho s} \left( \alpha h(s) + (1 - \alpha)h(s) - \delta S_2(s) \right)$$  \hspace{1cm} (A.36)

$$= \frac{2\nu}{\rho} (S_{10} + S_{20}) + \int_0^\infty \frac{2\nu}{\rho} e^{-\rho s} h(s)ds - \int_0^\infty \frac{2\nu}{\rho} e^{-\rho s} S_2(s)ds.$$  \hspace{1cm} (A.37)

We need to solve for the last term in equation (A.37):

$$\int_0^\infty e^{-\rho s} S_2(s)ds = \int_0^\infty S_2(s)d \left( -\frac{1}{\rho} e^{-\rho s} \right)$$  \hspace{1cm} (A.38)

$$= -\frac{1}{\rho} S_2(s)e^{-\rho s} \bigg|_0^\infty + \int_0^\infty \frac{1}{\rho} \dot{S}_2(s)e^{-\rho s}ds$$  \hspace{1cm} (A.39)

$$= \frac{1}{\rho} S_2(0) + \int_0^\infty \frac{1}{\rho} e^{-\rho s}((1 - \alpha)h(s) - \delta S_2(s))$$  \hspace{1cm} (A.40)

$$= \frac{1}{\rho} S_{20} + \int_0^\infty \frac{1}{\rho} e^{-\rho s}(1 - \alpha)h(s)ds - \frac{\delta}{\rho} \int_0^\infty e^{-\rho s} S_2(s)ds.$$  \hspace{1cm} (A.41)

Bringing the right term of equation (A.41) to the left of equation (A.39), we solve for $\int_0^\infty e^{-\rho s} S_2(s)ds$:

$$\left(1 + \frac{\delta}{\rho}\right) \int_0^\infty e^{-\rho s} S_2(s)ds = \frac{1}{\rho} S_2(0) + \frac{1}{\rho} \int_0^\infty e^{-\rho s}(1 - \alpha)h(s)ds,$$  \hspace{1cm} (A.42)

$$\int_0^\infty e^{-\rho s} S_2(s)ds = \frac{1}{\rho + \delta} S_2(0) + \frac{1}{\rho + \delta} \int_0^\infty e^{-\rho s}(1 - \alpha)h(s)ds.$$  \hspace{1cm} (A.43)
Inserting this into (A.37), total damages equal:

\[
\frac{2\nu}{\rho} (S_{10} + S_{20}) + \frac{2\nu}{\rho} \int_0^\infty e^{-\rho s} h(s) ds - \frac{2\nu\delta}{\rho} \left\{ \frac{1}{\rho + \delta} S_2(0) + \frac{1}{\rho + \delta} \int_0^\infty e^{-\rho s} (1 - \alpha) h(s) ds. \right\}
\]

(A.44)

= \frac{2\nu}{\rho} (S_{10} + S_{20}) - \frac{2\nu\delta}{\rho} \left\{ \frac{1}{\rho + \delta} S_{20} + \left( \int_0^\infty e^{-\rho s} h(s) ds \right) \left\{ \frac{2\nu}{\rho} - \frac{2\nu\delta}{\rho(\delta + \rho)} (1 - \alpha) \right\} \right\}

(A.45)

Disregarding the given initial carbon stock in the atmosphere, the first two terms of equation (A.45) disappear. The rest can be rewritten as:

\[
\left\{ 2\nu \left( \frac{\alpha}{\rho} + \frac{(1 - \alpha)}{\rho + \delta} \right) \right\} \int_0^\infty e^{-\rho s} h(s) ds.
\]

(A.46)

### A.4 Numerical Analysis and Calibration

We assume the home (foreign) region’s utility function to be of a standard CRRA form:

\[
A \left( \frac{x_c + y_c + z_c}{1 - \gamma} - 1 \right), \quad \left( A^* \left( \frac{x_c^* + y_c^* + z_c^*}{1 - \gamma^*} - 1 \right) \right),
\]

(A.47)

where \(A(A^*)\) is a scaling parameter. From the first order condition we know that the home (foreign) region’s demand equals (with \(i = y, x\)):

\[
D_i(p_i) = \left( \frac{A}{p_i + \tau} \right)^\frac{1}{\gamma} \quad D^*_i(p_i) = \left( \frac{A^*}{p_i + \tau^*} \right)^\frac{1}{\gamma^*}
\]

As noted above, we use logarithmic utility in most of the numerical exercises in Sections 2.2.4 and 2.2.5 and set \(\gamma = \gamma^* = 1\). Yet, the calibration requires us to use the flexibilities provided by the functional forms we assumed. Table A.3 summarizes the parameter values used.
### Table A.2: Parameter values for the numerical exercises

<table>
<thead>
<tr>
<th>Model’s parameters</th>
<th>Section 2.2.4</th>
<th>Section 2.2.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social discount rate</td>
<td>$\rho$</td>
<td>0.05</td>
</tr>
<tr>
<td>Coefficient of Relative risk aversion</td>
<td>$\gamma(\gamma^*)$</td>
<td>1 (1)</td>
</tr>
<tr>
<td>World interest rate</td>
<td>$r$</td>
<td>0.05</td>
</tr>
<tr>
<td>Extraction costs</td>
<td>$c_y^*$</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$c_x$</td>
<td>20</td>
</tr>
<tr>
<td>Backstop production costs</td>
<td>$b(b^*)$</td>
<td>45 (50)</td>
</tr>
<tr>
<td>Emission factor</td>
<td>$\psi(\psi^*)$</td>
<td>1(1)</td>
</tr>
<tr>
<td>Tax rate EP</td>
<td>$\tau$</td>
<td>5</td>
</tr>
<tr>
<td>Subsidy EP</td>
<td>$\sigma$</td>
<td>5</td>
</tr>
<tr>
<td>Social cost of carbon</td>
<td>$2\nu \left( \frac{\alpha}{\rho} + \frac{1-\alpha}{\rho+\delta} \right)$</td>
<td>5</td>
</tr>
</tbody>
</table>

### Table A.3: Parameter values for the calibration exercise

<table>
<thead>
<tr>
<th>Calibration parameters</th>
<th>$\rho$</th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social discount rate (5 years)</td>
<td>$\rho$</td>
<td>0.05</td>
</tr>
<tr>
<td>Coefficient of Relative risk aversion</td>
<td>$\gamma(\gamma^*)$</td>
<td>2</td>
</tr>
<tr>
<td>Scaling parameter</td>
<td>$A^*$</td>
<td>$\lambda_y + c_y$ $\times$ 3.87$^\gamma$</td>
</tr>
<tr>
<td>World interest rate</td>
<td>$r$</td>
<td>0.05</td>
</tr>
<tr>
<td>Extraction costs (in $\text{trillion/ppmv}$)</td>
<td>$c_y^*$</td>
<td>0.1065</td>
</tr>
<tr>
<td></td>
<td>$c_x$</td>
<td>0.852</td>
</tr>
<tr>
<td>Backstop production costs</td>
<td>$b(b^*)$</td>
<td>1.278(1.6614)</td>
</tr>
<tr>
<td>Emission factor</td>
<td>$\psi(\psi^*)$</td>
<td>1(1)</td>
</tr>
<tr>
<td>Social cost of carbon</td>
<td>$2\nu \left( \frac{\alpha}{\rho} + \frac{1-\alpha}{\rho+\delta} \right)$</td>
<td>0.0639</td>
</tr>
</tbody>
</table>