Chapter 2

Fiscal and monetary policy coordination, macroeconomic stability and sovereign risk premia*

THE Great Recession has forced governments worldwide to engage in massive fiscal expansions in order to keep their economies afloat, especially since monetary instruments have been effectively depleted due to the zero lower bound. In some cases, these Keynesian-style fiscal policies have resulted in a surge of government indebtedness and widening sovereign bond yields, mostly reflecting concerns regarding the sustainability of public finances. In more critical cases, the fiscal response to the crisis did more bad than good, as the risk of sovereign default became too high and eventually led to a collapse of sovereign bond markets. This begs the question: how does the use of fiscal policy in stabilising macroeconomic conditions affect the ability to ensure a sustainable path for sovereign debt?

We address this question through the lens of the canonical New Keynesian closed economy model. A fiscal authority (or 'government') follows a feedback rule that relates the primary balance to changes in government debt and aggregate output. Leeper (1991) has already shown that, to obtain a stable equilibrium, the response of the primary balance to a given change in debt should

*This chapter is based on Bonam and Lukkezen (2014a).

be strong enough to maintain debt growth below the long-run real interest rate. In order to achieve determinacy, this 'passive fiscal policy' must then be paired with an 'active monetary policy', which ensures that the nominal interest rate moves by more than one-for-one with inflation.[1] Inflation-targeting is performed by a monetary authority (or 'central bank') who follows a standard Taylor rule (Taylor, 1993). Furthermore, in order to capture the recently observed changes in the demand elasticity for government bonds, we introduce a sovereign risk premium which forms a wedge between the bond rate and risk-free rate. The risk premium rises with government indebtedness when agents are concerned with fiscal solvency and remains constant otherwise.

Our main contribution is to show, both analytically and numerically, that the well-established stability and uniqueness requirements from Leeper (1991) change markedly under a debt-elastic sovereign risk premium, and depend strongly on the government's endogenous fiscal response to the business cycle. In particular, when the government maintains a countercyclical fiscal stance, such that output contractions are met by reductions in the primary surplus, a stable and unique equilibrium might be unable to obtain, even if fiscal policy is passive and monetary policy active. Intuitively, when a random shock adversely hits the economy and the primary surplus falls, the stock of government debt rises which drives up the sovereign risk premium. Consequently, the interest rate rises which prompts agents to save more and consume less, causing output and inflation to fall. This *crowding-out* effect of the sovereign risk premium therefore exacerbates the initial decline in output. Given the countercyclical nature of fiscal policy, the government then automatically raises the deficit further, which again drives up debt and the risk premium, causing consumption and output to fall even more, etc. etc. In order to avoid this self-reinforcing cycle of rising debt and falling output, and keep long-term financing costs low, the government must signal its commitment to debt sustainability by pursuing more aggressive debt consolidation. The more responsive is the sovereign risk premium to changes in government indebtedness, the greater becomes the required effort to consolidate debt.

[1] According to the Fiscal Theory of the Price Level, stability and uniqueness can also be ensured when fiscal policy is insufficient to deliver long-run debt sustainability, yet only if the central bank allows the price level to keep the real value of government debt outstanding consistent with the intertemporal government budget constraint (see Leeper, 1991, Sims, 1994, Woodford, 1996, and Woodford, 2001, among others).

The required ‘passiveness’ of fiscal policy that delivers equilibrium stability depends, not only on the severity of the sovereign debt crisis, but also on monetary policy. If monetary policy is active, the rise in the sovereign risk premium and consequent fall in inflation is met by a reduction in the policy rate, which partially offsets the crowding-out effect. However, the less responsive is the central bank to changes in inflation, the weaker is this monetary offset. Keeping the risk premium low then requires greater debt reduction by the government. Our results therefore imply that the interdependence between fiscal and monetary policy is much stronger in the presence of debt-elastic sovereign risk premia and warrants closer coordination between the government and central bank than suggested by Leeper (1991).

When the government maintains a pro-cyclical fiscal stance, such that output contractions are met by fiscal contractions, the stability and uniqueness requirements under a debt-elastic sovereign risk premium are *relaxed*. Particularly, when the primary surplus rises following an adverse shock to output, government debt falls, which reduces the risk premium and interest rate, and raises consumption, thereby offsetting the initial output contraction. The pro-cyclical nature of fiscal policy thus not only eases the task of sustaining government debt by keeping the risk premium low, which makes it possible to obtain stability even if fiscal policy is not passive, it also helps control inflation by essentially substituting for monetary policy, which opens up the possibility for determinacy even if monetary policy is not active.

If the sovereign risk premium remains constant over time, regardless of the degree of government indebtedness, the conventional passive fiscal plus active monetary policy combination becomes a necessary and sufficient condition for equilibrium stability and uniqueness. Also, the stability and uniqueness requirements become independent from the government’s cyclical stance. This means that, as long as fiscal policy is passive, the government may pursue countercyclical fiscal policies and run budget deficits from time to time without endangering long-run debt sustainability or price level determinacy. Under a debt-elastic risk premium, however, short-run debt developments are no longer trivial and the government must take immediate action and signal its commitment to sustain debt. If it does not, then the onus of ensuring stable macroeconomic conditions falls entirely upon the central bank. Our results therefore formalise the need for greater measures of fiscal austerity during times of sovereign debt crises and also questions the desirability of fiscal expansions when public finances are critically weak.

The present chapter is closely related to the literature on the relationship between macroeconomic stability and government debt non-neutrality. For instance, Canzoneri and Diba (2005) and Linnemann and Schabert (2010) show that the conditions for equilibrium stability change when government bonds can be used for transactions. In contrast to our study, an increase in the amount of government bonds outstanding would then have a positive effect on output and inflation through a reduction in the liquidity premium. Fiscal policy is therefore able to accommodate monetary policy in controlling inflation, and determinacy can be achieved even when monetary policy is not active. Piergallini (2005) and Leith and von Thadden (2008) show that similar results can be derived in overlapping generations models in which government bonds generate wealth effects. Furthermore, Schabert and van Wijnbergen (2014) investigate the implications of debt non-neutrality arising from sovereign default risk using an otherwise standard New Keynesian model for a small open economy. The authors show that, in the presence of sovereign risk, the government must respond more aggressively to changes in debt to deliver a stable and unique steady state if the central bank actively targets inflation. Although these results are similar to ours, they are driven by a completely different mechanism. Particularly, whereas in our model the sovereign risk premium crowds out consumption and output, sovereign risk is expansionary in Schabert and van Wijnbergen due to its negative effect on the effective real return on government bonds. While we acknowledge the possibility of sovereign risk to have positive effects on inflation (at least in the short run), we believe the adverse effects on financial market conditions associated with rising sovereign risk premia dominate the overall implications for output and inflation dynamics (see also Corsetti et al., 2013a).

The remainder of this chapter is organised as follows. In the following section, we briefly describe the model, its main building blocks and the calibration of the structural parameters. In Section 2.2, we derive the conditions for equilibrium stability and uniqueness, and show the implications of the cyclicality of fiscal policy and the sovereign risk premium. Finally, Section 2.3 concludes and offers directions for future work.

2.1 A closed economy model

For our main analysis, we use the canonical New Keynesian closed economy model, which is presented in this section. We start by focusing on the public sector, which consists of a fiscal authority, or 'government', and a monetary authority, or 'central bank'. Although the government and central bank act independently from each other, the equilibrium properties of the model are determined jointly by fiscal and monetary policy. Furthermore, we assume that the carrying cost of government debt is subject to a sovereign risk premium which itself is a function of government indebtedness. Our aim is to reveal how the presence of such a risk premium affects the feasible set of fiscal and monetary policies that deliver stable and unique equilibria. We close this section with a brief description of the household and firm sector, the market clearing conditions and equilibrium, and the calibration of the model's structural parameters.

2.1.1 The public sector and sovereign risk premium

The government issues nominal, one-period government bonds, B_t, and levies lump-sum taxes, T_t, in order to cover a constant level of public consumption, g, and repay outstanding debt plus interest. Let $R_{g,t}$ denote the gross nominal interest rate on government bonds, P_t the aggregate price index and $s_t \equiv T_t - g$ the real primary budget surplus. Government debt then accumulates due to the difference between gross public debt and the primary surplus:

$$B_t = R_{g,t-1}B_{t-1} - P_t s_t. \tag{2.1}$$

Dividing (2.1) by P_t and then solving forward recursively yields the intertemporal government budget constraint:

$$R_{g,t-1}\frac{B_{t-1}}{P_t} = \sum_{n=0}^{\infty}\left(\prod_{m=0}^{n} r_{g,t+m}^{-1}\right) s_{t+n}, \tag{2.2}$$

where $r_{g,t} \equiv R_{g,t}\left(P_t/P_{t+1}\right)$ denotes the real bond rate and where we have imposed the following transversality condition which prevents the government from rolling-over its debt indefinitely:

$$\lim_{k\to\infty}\left(\prod_{m=0}^{k-1} r_{g,t+m}^{-1}\right)\frac{B_{t+k}}{P_{t+k}} = 0. \tag{2.3}$$

According to the intertemporal government budget constraint given by (2.2), a feasible set of fiscal and monetary policy is one which ensures that real outstanding public liabilities (left-hand side) equals the discounted sum of current and future primary surpluses (right-hand side).

The trajectory of the primary surplus $\{s_{t+k}\}_{k=0}^{\infty}$ would ordinarily be shaped by many economic and political considerations. Rather than developing a full-blown political economy model that features such considerations, we assume that s_t moves endogenously with changes in the real level of government debt outstanding at t, which is denoted by $b_{t-1} \equiv B_{t-1}/P_{t-1}$, and aggregate output, y_t:

$$s_t = \gamma_b\left(b_{t-1} - \bar{b}\right) + \gamma_y\left(y_t - \bar{y}\right), \tag{2.4}$$

where $\bar{b}$ and $\bar{y}$ denote target values for government debt and output, respectively, and are assumed to equal the corresponding steady-state values.[2] The parameter $\gamma_b \geq 0$ reflects the government's effort to consolidate outstanding debt within a given period. In order to facilitate our discussion, we adopt the terminology from Leeper (1991) and introduce the following definition characterising fiscal policy:

Definition 1. *Fiscal policy is called 'passive' if $\gamma_b > r_g - 1$. Otherwise, fiscal policy is said to be 'active'.*

The parameter r_g denotes the steady-state value of the real bond rate. Hence, when fiscal policy is passive, the government prevents debt from growing at a rate that exceeds the cost of borrowing, and thereby ensures that debt gradu-

[2] Fiscal rules similar to (2.4) have been used extensively in the empirical literature to test for the sustainability of public debt and estimate the cyclical stance of fiscal policy in both advanced and emerging market economies (see e.g. Gavin and Perotti, 1997; Galí and Perotti, 2003; Greiner et al., 2007; Mendoza and Ostry, 2008; Debrun and Kapoor, 2010; Ghosh et al., 2013). In the theoretical literature, such fiscal rules are often used to describe government behaviour (e.g. Linnemann, 2006). Although most models restrict the government to respond only to lagged variables (to reflect political constraints that might delay the implementation of fiscal policy), we allow the contemporary level of output to enter the fiscal rule for analytical convenience. However, replacing y_t with y_{t-1} does not alter the main results of this chapter.

ally converges to some sustainable level in the long run. When, on the other hand, fiscal policy is active, an increase in the level of debt will not be met by a sufficient rise in the primary surplus, which opens up the possibility of explosive debt dynamics.

The parameter γ_y reflects the government's stance with regards to the business cycle and therefore indicates the cyclicality of fiscal policy, which we characterise as follows:

Definition 2. *When $\gamma_y > 0$, fiscal policy is called 'countercyclical'. When $\gamma_y < 0$, fiscal policy is called 'pro-cyclical'.*

A countercyclical fiscal stance essentially indicates that the government uses automatic stabilisers to suppress fluctuations in output: during recessions, when output falls below target, the primary surplus falls *automatically*, whereas during economic prosperity, the primary surplus rises. Pro-cyclical fiscal policies, on the other hand, tend to intensify the business cycle as output contractions (expansions) are met by an increase (decrease) in the primary surplus. Whereas existing literature has mostly focused on the effects of fiscal stabilisation policy on short-run dynamics (see e.g. Galí, 1994, Fatás and Mihov, 2001, and Debrun and Kapoor, 2010), we will show later on that the built-in fiscal response to the business cycle can play a pivotal role in the determination of long-run equilibrium outcomes as well.

One of the key ingredients of the model is a *sovereign risk premium* that raises the cost of public borrowing. Before we discuss how the sovereign risk premium is introduced to the model, we need to make clear what we mean by 'sovereign risk'. In this regard, our modelling approach is line with Davig et al. (2011) and Bi (2012). In particular, we assume that sovereign risk may arise due to the presence of a so-called 'fiscal limit' which determines the maximum primary surplus, say $\mathcal{S}$, that is politically (or economically) feasible, and therefore determines the maximum amount of government debt, denoted by $\mathcal{B}$, that can be repaid. The latter is determined by

$$\mathcal{B} = \sum_{n=0}^{\infty} \left(\prod_{m=0}^{n} r_{g,t+m-1}^{-1} \right) \mathcal{S}_{t+n}.$$

When the stock of government debt reaches a level that, by the fiscal rule (2.4), forces the primary surplus to breach the fiscal limit, i.e. when $s_t >$

$\mathcal{S}$, the government (partially) defaults on its outstanding debt, since raising the surplus beyond $\mathcal{S}$ is not feasible. On the other hand, when $s_t < \mathcal{S}$, the government fully honours its debt. Although agents do not know $\mathcal{S}$ prior to entering a sovereign bond contract ($\mathcal{S}$ is observed only when the bond matures), they know its distribution and, since they are forward-looking, form expectations about future sovereign default probabilities. Consequently, even if the economy has not yet reached the fiscal limit, the mere possibility of getting there can affect today's bond price: the higher is the probability of reaching the fiscal limit, the lower is the price.

Since the fiscal limit is stochastic and dependent on the state of the economy, explicitly modelling its conditional distribution can be quite cumbersome. Also, the fiscal limit implies non-linearities within the model, which renders a linear approximation unsuitable to study the model's equilibrium properties. We do not take up the task to solve these problems here, yet instead circumvent them by assuming that $\mathcal{S}$ and $\mathcal{B}$ are determined exogenously (see Corsetti et al., 2013c, Daniel and Shiamptanis, 2013, Locarno et al., 2013, Roeger and in 't Veld, 2013, and Schabert and van Wijnbergen, 2014, for similar treatments of the fiscal limit). Furthermore, we assume that, when b_t is perceived to be close to the threshold level $\mathcal{B}$, a sovereign risk premium emerges, denoted by $\Xi_{g,t}$, which forms a wedge between the bond rate, $R_{g,t}$, and the risk-free policy rate set by the central bank, R_t, i.e.

$$R_{g,t} = \Xi_{g,t} R_t. \tag{2.5}$$

When, however, agents believe b_t to lie far below $\mathcal{B}$, the risk premium disappears and the government simply pays the risk-free rate. This non-linear relationship between the risk premium and government indebtedness has been confirmed by many empirical studies (e.g. Bernoth et al., 2004; Laubach, 2009; Afonso et al., 2012; Jaramillo and Weber, 2013) and is captured here by the following function:

$$\Xi_{g,t} = \exp\left(\chi_g \frac{b_t}{y}\right). \tag{2.6}$$

The coefficient $\chi_g \geq 0$ determines the sensitivity of the sovereign risk premium to changes in government debt. When $\chi_g = 0$, concerns regarding fiscal insolvency are absent and so further increases in government indebtedness do not affect the bond price. Hence, we say that the risk premium is *debt-inelastic*. Positive values of χ_g, on the the other hand, indicate that agents expect the fiscal limit to be breached soon and thus require a higher risk premium when holding government bonds. In that case, the demand curve for government bonds is downward-sloping and we say that the risk premium is *debt-elastic*.[3]

The central bank sets the policy rate in order to target inflation, which is denoted by $\pi_t \equiv P_t/P_{t-1}$, according to the following rule:

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\overline{\pi}}\right)^{\alpha_\pi}, \tag{2.7}$$

where $\alpha_\pi > 0$ measures the aggressiveness with which the central bank responds to deviations of inflation from target, $\overline{\pi}$, which is assumed to equal steady-state inflation. Following Leeper, the monetary stance is characterised by the following definition:

Definition 3. *Monetary policy is called 'active' if $\alpha_\pi > 1$. Otherwise, monetary policy is said to be 'passive'.*

Under active monetary policy, the central bank is able to ensure price level determinacy by moving the *nominal* interest rate by more than one-for-one with inflation, such that any positive shock to inflation is offset by an increase in the *real* interest rate. Generally speaking, an active monetary policy stance satisfies the Taylor-principle.

2.1.2 Households and firms

An infinitely-lived, representative household chooses the optimal level of consumption, c_t, number of hours worked, n_t, and nominal holdings of government bonds in order to maximise expected life-time utility, given by

$$E_t \sum_{k=0}^{\infty} \beta^k \left(\frac{c_{t+k}^{1-\sigma}}{1-\sigma} - \frac{n_{t+k}^{1+\varphi}}{1+\varphi}\right), \tag{2.8}$$

[3] The sovereign risk premium function is deliberately kept simple in order to derive analytical results. Of course, other factors might influence variations in the risk premium. For instance, most empirical studies that estimate equations like (2.6) control for the forward-looking behaviour of investors by including *expected* fiscal variables (see Gale and Orszag, 2003, for an extensive review). We have experimented with similar 'forward-looking' specifications, yet our results did not change qualitatively. Other factors, such as expected output growth and inflation, and persistence in the risk premium have been tested as well, yet they did not alter our results.

with $\beta \in (0, 1)$ the household's discount factor, $1/\sigma > 0$ the elasticity of intertemporal substitution, and $1/\varphi > 0$ the Frisch elasticity of labour supply. The household pays lump-sum taxes to the government and receives labour income, $W_t n_t$ with W_t the nominal wage, and real dividends, ψ_t, from firms. The household's flow budget constraint is given by

$$B_t + P_t c_t + P_t T_t = R_{g,t-1} B_{t-1} + W_t n_t + P_t \psi_t. \tag{2.9}$$

Subject to (2.9) and the transversality condition (2.3), the household maximises (2.8) which delivers the following first-order conditions:

$$\frac{n_t^{\varphi}}{c_t^{-\sigma}} = w_t, \tag{2.10}$$

$$c_t^{-\sigma} = \beta E_t \left[r_{g,t} c_{t+1}^{-\sigma} \right], \tag{2.11}$$

Equation (2.10) determines the household's optimal labour supply decision by relating the marginal rate of substitution between consumption and leisure to the real wage rate $w_t \equiv W_t / P_t$. The Euler equation (2.11) determines the household's optimal savings decision by relating expected consumption growth to the real bond rate.

The firm sector is straightforward (see Appendix 2.A for more details). A representative final good firm, who is a price-taker, assembles differentiated intermediate goods to produce the final good using a standard constant elasticity of substitution production function. Intermediate goods are produced by a large number of monopolistic firms using labour (supplied by households) and a constant returns to scale production technology. Intermediate goods firms can set prices in excess of marginal costs, yet face a price-setting constraint as in Calvo (1983): in every period, only a share, $1 - \theta \in (0, 1]$, of firms can re-set prices, while remaining firms are forced to keep prices unchanged. The optimal price-setting condition is based on a profit-maximisation problem, which is derived in Appendix 2.A.

2.1.3 Market clearing and equilibrium

In equilibrium, the economy's aggregate resource constraint, $y_t = c_t + g$, must be satisfied and the government bond market clears. Furthermore, labour

Table 2.1: Benchmark calibration

β	Discount factor	0.9926
$1/\sigma$	Intertemporal elasticity of substitution	1
$1/\varphi$	Frisch elasticity of labour supply	1/3
θ	Probability of non-price adjustment	0.75
ϵ	Elasticity of substitution between intermediate goods	11
b/y	Steady-state debt to output ratio (annualised)	0.6
c/y	Steady-state private consumption to output ratio	0.8
g/y	Steady-state government consumption to output ratio	0.2
T/y	Steady-state tax revenue to output ratio	0.22

market clearing implies $n_t = \int_0^1 y_t(i)\,di = y_t\mathcal{D}_t$, where $\mathcal{D}_t \equiv \int_0^1 \left(P_t(i)/P_t\right)^{-\epsilon} di$ is a measure of price dispersion.

Equilibrium is then given by a sequence of c_{t+k}, n_{t+k}, y_{t+k}, w_{t+k}, b_{t+k}, π_{t+k}, $\Xi_{g,t+k}$, R_{t+k}, $R_{g,t+k}$ and s_{t+k} that satisfies the public's budget constraint (2.1), the fiscal policy rule (2.4), the bond-pricing equation (2.5), the sovereign risk premium function (2.6), the monetary policy rule (2.7), the household's first-order conditions (2.10) and (2.11), the firm's optimal re-set price condition (see Appendix 2.A) and the market clearing conditions, given exogenous sequences for the fiscal limits $\mathcal{S}$ and $\mathcal{B}$, for all k.

2.1.4 Calibration

The model is calibrated based on a quarterly frequency. For many parameters in the model, we take those values that are most commonly found in the literature. For an overview, see Table 2.1.

We set the discount rate equal to $\beta = 0.9926$, such that the (annual) equilibrium net bond rate equals about $r_g - 1 = 3\%$. As in Galí and Monacelli (2008), the intertemporal elasticity of substitution is assumed to be $\sigma = 1$, such that household utility depends on the log of consumption. Also, we assume $\varphi = 3$, which implies a Frisch elasticity of 1/3, and set $\theta = 0.75$, which is consistent with an average price contract of one year. Following Corsetti et al. (2012a), we set the elasticity of substitution between intermediate goods at $\epsilon = 11$, implying a price mark-up of 10%. Furthermore, the steady-state debt to output ratio is given the value of $b/y = 0.6$ (on an annual basis), while private and public consumption as a share of output in steady state are set to $c/y = 0.8$ and $g/y = 0.2$, which corresponds to long-run data of OECD countries. The government's flow budget constraint (2.1) then implies a steady-state tax revenue to output ratio of around $T/y = 0.22$. Values for the policy parameters, γ_b, γ_y and α_π, are chosen within realistic ranges when we explore the set of feasible fiscal and monetary policies.

The key parameter of our model is χ_g, which governs the sensitivity of the sovereign risk premium to changes in government indebtedness. There exists a breadth of empirical studies on the effects of government debt (and other variables) on sovereign bond spreads; for an extensive overview, see Haugh et al. (2009). Estimated coefficients similar to χ_g vary considerably across (and within) studies, although its sign is usually found to be positive, as is consistent with theory. For our numerical analysis, we choose χ_g to be 0.08 for the case when demand for bonds is elastic, which is in line with estimates based on European data from De Grauwe and Ji (2012), and implies that a 1% increase in the government debt ratio raises the risk premium by 8 basis points. However, we also experiment with alternative values to test for robustness and show that our results hold, even if χ_g is set closer to zero.

2.2 Requirements for equilibrium stability and uniqueness

In this section, we discuss the fiscal and monetary policy requirements for long-run debt sustainability and price level determinacy. We start by reducing a linearised version of the model presented in the previous section to a manageable system of four equations. This allows us to derive analytical expressions for the stability and uniqueness requirements imposed on the parameters γ_b and α_π. We then discuss these requirements for the case in which the sovereign risk premium is debt-inelastic. This case serves as a benchmark and, as shall be shown, reproduces the results of Leeper. We also explain how the government's output stabilisation objective, which plays no role in Leeper's analysis, affects the government's ability to ensure long-run debt sustainability. Finally, we discuss how the requirements change when the sovereign risk premium is debt-elastic.

2.2.1 Dynamics of the model

The model is linearised around a deterministic steady state, in which prices are fully flexible ($\theta \to 0$) and inflation zero ($\pi = 1$), using a first-order Taylor approximation. Let variables without a t subscript denote the corresponding steady-state value and define $\hat{x}_t \equiv (x_t - x)/x$ as the percentage deviation of any generic variable x_t from its steady state. Using the auxilliary variable $\hat{\pi}'_t = \hat{\pi}_t$, the model can then be reduced to the following 4×4 system of linear difference equations (see Appendix 2.B for a brief derivation):

$$A_0 \begin{bmatrix} E_t\hat{\pi}_{t+1} \\ E_t\hat{c}_{t+1} \\ \hat{b}_t \\ \hat{\pi}'_t \end{bmatrix} = A_1 \begin{bmatrix} \hat{\pi}_t \\ \hat{c}_t \\ \hat{b}_{t-1} \\ \hat{\pi}'_{t-1} \end{bmatrix}, \tag{2.12}$$

where

$$A_0 \equiv \begin{bmatrix} 1 & \sigma & -\chi_g \frac{b}{y} & 0 \\ \beta & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_1 \equiv \begin{bmatrix} \alpha_\pi & \sigma & 0 & 0 \\ 1 & -\Psi & 0 & 0 \\ -\frac{1}{\beta} & -\gamma_y \frac{c/y}{b/y} & \frac{1+\chi_g \frac{b}{y}}{\beta} - \gamma_b & \frac{1}{\beta}\alpha_\pi \\ 1 & 0 & 0 & 0 \end{bmatrix},$$

and where $\Psi \equiv (1-\theta)(1-\theta\beta)/\theta(\varphi c/y + \sigma) > 0$.

The system of endogenous variables described by (2.12) consists of two forward-looking (jump) variables, $\hat{c}_t$ and $\hat{\pi}_t$, one pre-determined (state) variable, $\hat{b}_t$, and an auxiliary variable $\hat{\pi}'_t$. As shown by Blanchard and Kahn (1980), the matrix $A \equiv A_0^{-1}A_1$ should then contain exactly one stable eigenvalue (i.e. smaller than modulus one) and two unstable eigenvalues (i.e. larger than modulus one) to guarantee a stable and unique equilibrium (the eigenvalue corresponding to $\hat{\pi}'_t$ equals unity by construction). Too many unstable eigenvalues suggests that the trajectory of government debt is explosive and an equilibrium in which agents are willing to hold government bonds does not exist. If there are too few unstable eigenvalues, it means that the system has infinitely many solutions and admits the possibility of sunspot shocks affecting equilibrium allocations such that the price level sequence is indeterminate.

2.2.2 Reproducing Leeper (1991)

The conditions under which a stable and unique equilibrium can be supported when the sovereign risk premium is debt-inelastic (i.e. when $\chi_g = 0$) are given by the following proposition:

Proposition 1. *Given $\chi_g = 0$, a fiscal rule of the form (2.4) and a monetary rule of the form (2.7), sufficient conditions for a stable and unique rational expectations equilibrium are either*

$$\gamma_b > \frac{1}{\beta} - 1 \quad \textit{and} \quad \alpha_\pi > 1, \tag{2.13}$$

$$\textit{or}$$

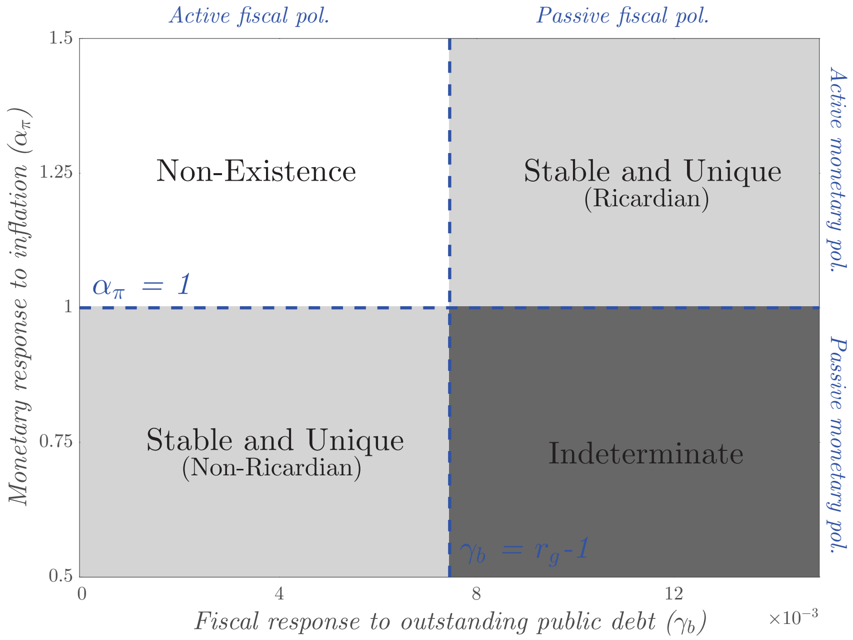
$$\gamma_b < \frac{1}{\beta} - 1 \quad \textit{and} \quad \alpha_\pi < 1. \tag{2.14}$$

provided that $\gamma_b < 1/\beta \min(\Psi/\sigma, 1)$ and $\alpha_\pi > 0$.

Proof. See Appendix 2.C. □

The intuition underlying Proposition 1 is discussed next and illustrated by Figure 2.1, which shows the number of unstable eigenvalues of A as a function of the fiscal and monetary policy stance (which are governed by γ_b and α_π). The vertical dashed line crosses the horizontal axis at $1/\beta - 1$, which is equal to $r_g - 1$ under steady state (see Equation [2.11]), and so partitions the parameter space into active and passive fiscal policy. The horizontal dashed line crosses the vertical axis at 1, such that monetary policy is active above the line and passive below. In the dark-grey area, matrix A has zero unstable eigenvalues, which suggests that expectations are not well-anchored and that the price level is not uniquely determined ('Indeterminate'). In the two light-grey areas, there are two unstable eigenvalues and so equilibrium is both stable and unique ('Stable and Unique'). In the white area, there are three unstable eigenvalues,

Figure 2.1: The 'fiscal-monetary dichotomy'



Notes: The figure displays the model's equilibrium properties as a function of γ_b and α_π for $\chi_g = 0$.

which means that a combination of fiscal and monetary policy within this region is not feasible ('Non-Existence').

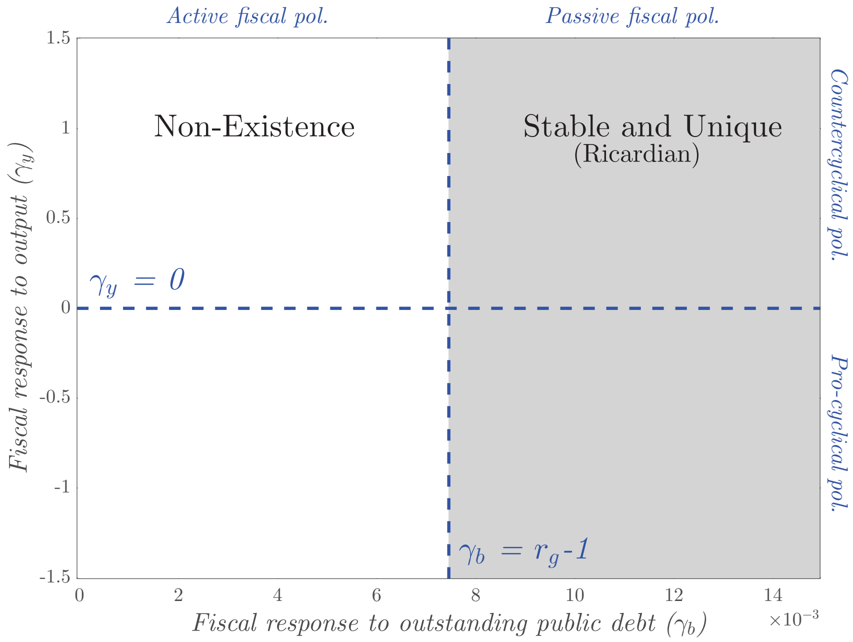
As shown by Figure 2.1, there are two regions in which fiscal and monetary policy deliver a stable and unique equilibrium. In the top-right region, fiscal policy is passive, i.e. $\gamma_b > r_g - 1$, while monetary policy is active, i.e. $\alpha_\pi > 1$. To see how this combination of fiscal and monetary policy delivers stable and unique equilibria, consider a positive shock to output. The rise in output leads to an increase in demand for labour, which raises marginal costs and hence inflation. Given the active stance of monetary policy, the central bank responds by raising the nominal interest rate by more than one-for-one with the change in inflation, causing the real interest rate to go up. By the household's Euler equation (2.11), consumption then falls, which gradually returns output, labour demand and marginal costs back to their respective steady states. Under an active monetary policy stance, the central bank thus successfully pins down expectations and controls inflation. Meanwhile, the higher interest rate drives up public interest expenses, which raises the budget deficit and government debt. Since fiscal policy is passive, the government responds by raising the primary surplus by enough to offset the rise in interest costs and prevent the stock of debt from ever-accumulating. Passive fiscal policy thus achieves long-run debt sustainability. This regime of passive fiscal and active monetary policy, typically referred to as the 'Ricardian regime' (following Woodford, 2001), usually prevails in conventional dynamic macroeconomic models. We therefore refer to the region associated with this policy regime as *Ricardian*.

When fiscal policy is active and insufficient to prevent explosive debt dynamics, i.e. when $\gamma_b \leq r_g - 1$, condition (2.14) of Proposition 1 suggests that a stable and unique equilibrium can still be achieved if the central bank does not actively target inflation, yet adopts a passive monetary policy, i.e. if $\alpha_\pi \leq 1$. This result is shown in the lower-left region of Figure 2.1. Under active fiscal policy, changes in the public's interest expenses are insufficiently offset by the primary surplus and so debt can grow without bounds. To prevent such explosive dynamics, the central bank must allow the price level P_t to jump to whatever level is necessary to keep the real value of outstanding government liabilities consistent with the intertemporal government budget constraint (2.2). This regime of active fiscal and passive monetary policy, which we refer to as *non-Ricardian*, relates to the 'Fiscal Theory of the Price Level' in which the price level is effectively determined by public finances (see Leeper, 1991, Sims, 1994, Woodford, 1996, and Woodford, 2001, among others).

Furthermore, Figure 2.1 shows that, when monetary and fiscal policy are both passive (lower-right region), equilibrium is indeterminate, which indicates that fiscal solvency is obtained for an infinite number of price level sequences. When both policies are active (upper-left region), no equilibrium that supports demand for government bonds exists since the government's debt consolidation policy is too weak to avoid explosive dynamics and monetary policy does not accommodate nominal debt growth.

The policy requirements for equilibrium stability and uniqueness under the benchmark case are well-known and identical to those obtained by Leeper. However, the results hold two lesser known insights. First of all, equilibrium outcomes depend on the mixture of fiscal and monetary policies, yet not on the *relative* policy strengths. For instance, as shown by Figure 2.1, a stable and unique equilibrium is obtained under the Ricardian regime as long as fiscal policy is passive and monetary policy active. This result holds, regardless of the 'passiveness' of fiscal policy or 'activeness' of monetary policy. Policy co-

Figure 2.2: Feasible and unfeasible fiscal policies



Notes: The figure displays the model's equilibrium properties as a function of γ_b and γ_y for $\chi_g = 0$ and $\alpha_\pi > 1$.

ordination between the government and central bank can therefore be kept at a minimum and we refer to this near policy independence as the *fiscal-monetary dichotomy*. Second, note that the parameter γ_y, which determines the cyclicality of fiscal policy, does not enter the stability and uniqueness requirements of Proposition 1. Therefore, the government's systematic response to changes in output, whether it be counter- or pro-cyclical, is *irrelevant* for the determination of the equilibrium outcome.

The irrelevance of the cyclical stance of fiscal policy for the model's equilibrium properties is evidenced by Figure 2.2, which shows the number of unstable eigenvalues as a function of γ_b and γ_y, while assuming that monetary policy is active. The horizontal dashed line now partitions the parameter space into counter- and pro-cyclical fiscal policy. When $\alpha_\pi > 1$, we know that $\gamma_b > r_g - 1$ ensures a stable and unique equilibrium. Figure 2.2 shows that this condition holds regardless of the size and sign of γ_y. Therefore, as long as fiscal policy is passive, and debt sustainability is ensured in the *long run*, can policymakers generate deficits over the *short run* and let automatic stabilisers work fully to absorb adverse aggregate shocks. In other words, both counter- and procyclical policies are feasible under the Ricardian regime of passive fiscal and active monetary policy.

In the following section, we compare the results from the benchmark case to the case in which the sovereign risk premium rises with the stock of government debt.

2.2.3 Implications of the debt-elastic sovereign risk premium

The policy requirements for equilibrium stability and uniqueness in the presence of a debt-elastic sovereign risk premium are given by the following proposition:

Proposition 2. *Given a fiscal rule of the form (2.4) and a monetary rule of the form (2.7), sufficient conditions for a stable and unique rational expectations equilibrium are either*

$$\gamma_b > \frac{1}{\beta} - 1 + \frac{\chi_g \gamma_y \frac{c}{y}(1-\beta)}{\Psi(\alpha_\pi - 1)} \quad \textit{and} \quad \alpha_\pi > 1 + \frac{\chi_g \gamma_y \frac{c}{y}(1-\beta)}{\Psi\left[\gamma_b - \left(\frac{1}{\beta} - 1\right)\right]}, \tag{2.15}$$

or

$$\gamma_b < \frac{1}{\beta} - 1 + \frac{\chi_g \gamma_y \frac{c}{y}(1-\beta)}{\Psi(\alpha_\pi - 1)} \quad \textit{and} \quad \alpha_\pi < 1 + \frac{\chi_g \gamma_y \frac{c}{y}(1-\beta)}{\Psi\left[\gamma_b - \left(\frac{1}{\beta} - 1\right)\right]}. \tag{2.16}$$

provided that $\gamma_b < 1/\beta \min(\Psi/\sigma, 1)$, $\alpha_\pi > 0$ *and (for* $\chi_g > 0$*)* $\gamma_y < (\sigma/\beta)(b/c)$.

Proof. See Appendix 2.C. □

First, note that for $\chi_g = 0$, the requirements stated in Proposition 2 reduce to those of the benchmark case under Proposition 1. Second, for $\chi_g > 0$ and $\gamma_y \neq 0$, the requirements for fiscal policy depend more strongly on monetary policy, and vice versa, as the parameters γ_b and α_π are now a function of each other. Third, the parameter γ_y now enters the requirements for both fiscal and monetary policy, indicating that the cyclical stance of fiscal policy is no longer irrelevant for equilibrium outcomes.

Underlying the change in the stability and uniqueness requirements is a *crowding-out effect* of fiscal policy on consumption that works through the sovereign risk premium $\Xi_{g,t}$. Specifically, when the level of government debt rises, the risk premium goes up by Equation (2.6) which leads to a higher interest rate $R_{g,t}$. In turn, the higher interest rate induces households to save more and consume less by the Euler equation (2.11). The higher is χ_g, the stronger is the rise in the risk premium for a given increase in government debt, and so the greater is the crowding-out effect on consumption. One could think of the crowding-out effect as reflecting the exposure of the banking sector to sovereign default risk. Widespread empirical evidence shows that, due to this exposure, sovereign risk can have significant adverse effects on private interest rates and credit supply to households and firms when the home sovereign runs high budget deficits (see e.g. Borensztein and Panizza, 2009; Balteanu et al., 2011; Panetta et al., 2011; Bofondi et al., 2013; Demirgüç-Kunt and Huizinga, 2013; Zoli, 2013; Popov and Van Horen, 2013; Albertazzi et al., 2014). The parameter χ_g then captures the degree of pass-through of sovereign risk to private borrowing conditions. Furthermore, since a reduction in consumption causes output to decline, inflation falls as well. Hence, Ricardian equivalence breaks down and the ability of the central bank to safeguard price stability is more strongly affected by the choice of fiscal policy and the dynamics of government debt, which in turn depend on the cyclical stance of fiscal policy.

When fiscal policy is countercyclical, that is when $\gamma_y > 0$, the budget deficit rises during economic downturns and falls during economic prosperity. Therefore, when a random shock causes output to contract, the stock of government debt and the risk premium rises which, as explained earlier, crowds out consumption and thereby *exacerbates* the initial contraction in output. Given the countercyclical nature of fiscal policy, the government automatically responds to the decline in output by reducing taxes further, causing government debt and the risk premium to go up even more, resulting in a further decline in consumption, output and inflation, etc. etc. Without an appropriate adjustment in the fiscal policy stance, this vicious cycle of rising levels of government debt and economic decline continues. As agents are forward-looking, an equilibrium that supports the demand for government bonds under these conditions might not exist, even if fiscal policy is passive.

Given the sensitivity of the sovereign risk premium to changes in government indebtedness, the government could avoid the self-reinforcing debt crisis by adopting a more aggressive debt consolidation policy under the Ricardian regime. In fact, according to condition (2.15) of Proposition 2, the higher is χ_g, the greater must be the government's effort to curtail the level of debt back to target, and so the higher must be γ_b. The higher debt coefficient γ_b serves as a signal to holders of government bonds of a commitment by the government to keep a firm grip on public finances. Any surge in government indebtedness would then be met by forceful fiscal contractions to constrain the accumulation of debt and keep the sovereign risk premium at bay.[4]

Note that the required amount of debt consolidation in the presence of a debt-elastic sovereign risk premium depends, not only on χ_g, but also on α_π. Intuitively, under the Ricardian regime, in which monetary policy is active, the central bank responds to the rise in the sovereign risk premium and consequent fall in inflation by reducing the policy rate R_t, which reduces $R_{g,t}$ and raises consumption, and thus partially *offsets* the crowding-out effect.[5] However, the lower is α_π, the less the rise in the sovereign risk premium will be neutralised. Therefore, equilibrium might be indeterminate if monetary policy is not active enough due to the self-reinforcing effects of the sovereign risk premium on consumption and inflation. Preventing the risk premium from damaging the economy would then require more aggressive debt consolidation by the government. In fact, the weaker is the central bank's ability to respond to changes in inflation, i.e. the lower is α_π, the higher must be γ_b. The stronger interdependence between fiscal and monetary policy as compared to the benchmark case indicates that the fiscal-monetary dichotomy breaks down. Consequently, policy coordination between the government and central bank becomes much more relevant in the context of macroeconomic stabilisation.[6]

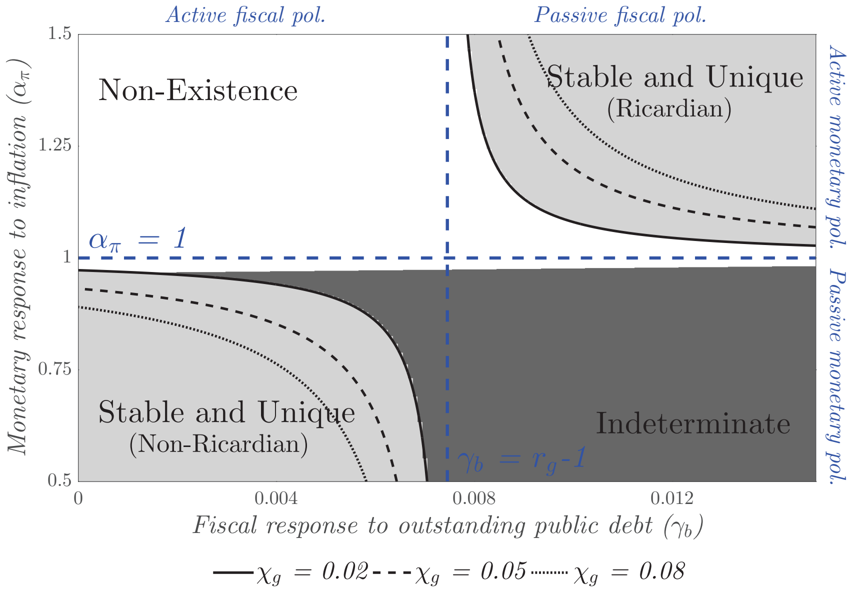
The implications of the debt-elastic sovereign risk premium for the stability

[4]Under the non-Ricardian regime, the government's debt consolidation policy must be relaxed as the sovereign risk premium becomes more responsive to debt changes. Considering that sovereign debt crises typically display more (rather than less) fiscal austerity, we perceive the non-Ricardian regime as less plausible and focus, instead, on the Ricardian regime.

[5]As shown by Attinasi et al. (2011), the European Central Bank's main refinancing operations have contributed to narrowing sovereign risk premia during the recent financial crisis.

[6]The crowding-out effect of sovereign risk also depends on the degree of price stickiness and the sensitivity of the private sector to interest rate shocks. Intuitively, the greater is the share of firms unable to adjust their price (i.e. the higher is θ), the stronger will be the contraction in output for a given fall in aggregate demand. Hence, sticky prices amplify the adverse effects of sovereign risk. Further, the lower is the coefficient of relative risk aversion (denoted by σ), the greater is the consumption response to changes in the real interest rate (see [2.11]) and hence the stronger will be the crowding-out effect. Incidentally, these robustness checks (available upon request) confirm that our results do not hinge on log preferences.

Figure 2.3: Equilibrium outcome under debt-elastic risk premia ($\chi_g > 0$) and countercyclical fiscal policy ($\gamma_y > 0$)



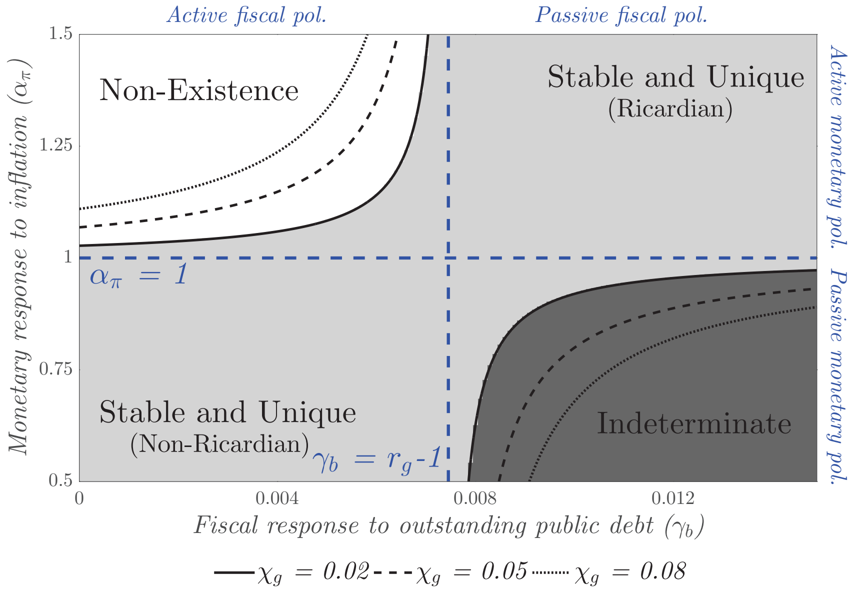
Notes: The figure displays the model's equilibrium properties as a function of γ_b and α_π for $\chi_g > 0$.

and uniqueness requirements under countercyclical fiscal policy are visualised by Figure 2.3, which, as Figure 2.1, shows the properties of the model's steady state as a function of the fiscal and monetary stance. According to the figure, the parameter space that supports a stable and unique equilibrium *contracts* relative to the benchmark case (as reflected by the reduction of the regions related to the Ricardian and non-Ricardian regimes). On the other hand, the likelihood of obtaining either multiple equilibria or non-existence of equilibrium *rises*. The dashed lines that separate active from passive fiscal and monetary policies are again included to facilitate comparison with the benchmark case. In contrast to the results from the previous section, a stable and unique equilibrium might not be feasible, even if fiscal policy is passive and monetary policy active. Furthermore, the figure suggests that higher values for χ_g result in smaller stability and uniqueness regions.[7]

When fiscal policy is pro-cyclical, that is when $\gamma_y < 0$, the stability and uniqueness requirements under a debt-elastic sovereign risk premium are *re-*

[7]Figure 2.6 in Appendix 2.D shows that more positive values for γ_y also contract the stability and uniqueness regions.

Figure 2.4: Equilibrium outcome under debt-elastic risk premia ($\chi_g > 0$) and pro-cyclical fiscal policy ($\gamma_y < 0$)

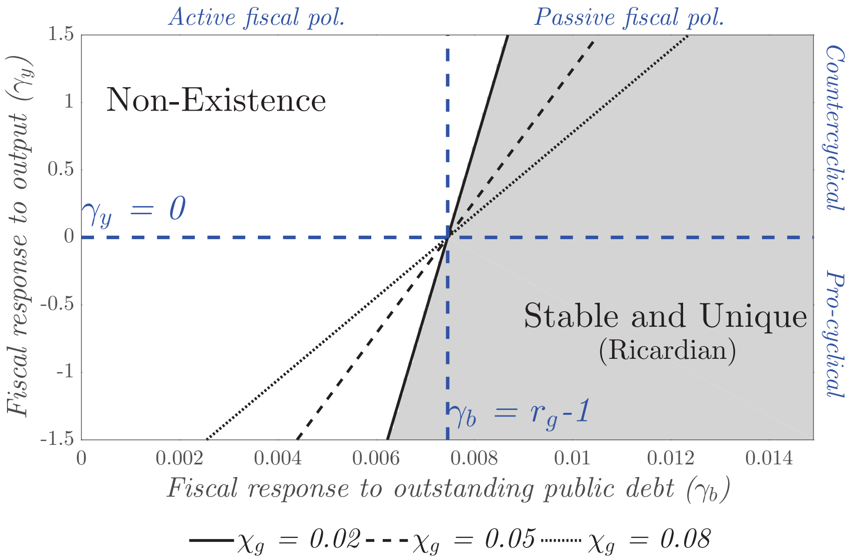


Notes: See notes under Figure 2.3.

laxed, as compared to the benchmark case, for both fiscal and monetary policy. Particularly, when the primary surplus rises following an adverse shock to output, the stock of government debt falls, which reduces the risk premium and interest rate, and raises consumption, thereby offsetting the initial output contraction. The pro-cyclical nature of fiscal policy thus not only eases the task of sustaining government debt by keeping interest costs low, which makes it possible to obtain a stable equilibrium even if fiscal policy is not passive, it also helps pin down household expectations by essentially substituting for monetary policy, which opens up the possibility for equilibrium uniqueness even if the central bank violates the Taylor-principle.

Figure 2.4 again shows the model's equilibrium properties as a function of fiscal and monetary policy in the presence of a debt-elastic sovereign risk premium, yet this time assuming that fiscal policy is pro-cyclical. According to the figure, the stability and uniqueness regions now *expand*, as compared to the benchmark case, whereas the non-existence and indeterminacy regions *contract*. For a given active monetary policy, it is now possible to obtain a stable and unique equilibrium, even if fiscal policy is also active; stability

Figure 2.5: Feasible and unfeasible fiscal policies when $\chi_g > 0$



Notes: The figure displays the model's equilibrium properties as a function of γ_b and γ_y for $\chi_g > 0$ and $\alpha_\pi > 1$.

and uniqueness can also be obtained if the government and central bank both pursue passive policies. Furthermore, the figure shows that the size of the stability regions is larger, the greater is χ_g.[8]

Figures 2.3 and 2.4 imply that the cyclical stance of fiscal policy plays a key role in determining the equilibrium outcome when the sovereign risk premium is debt-elastic. Recall that, when the sovereign risk premium is debt-inelastic, agents are always willing to hold government bonds, regardless of the degree of government indebtedness, provided that government debt is sustainable in the long run. The latter merely requires fiscal policy to be passive and therefore allows the government to let automatic stabilisers work fully, pursue countercyclical fiscal policies and run budget deficits from time to time. However, when agents believe the economy is near its fiscal limit, an increase in the budget deficit *today* raises the probability of breaching the fiscal limit *tomorrow*. Hence, short-run debt developments are no longer trivial and the government may find it more difficult to run budget deficits without upsetting bond markets. Instead, the government must take immediate action

[8]Figure 2.6 in Appendix 2.D shows that more negative values for γ_y also expand the stability and uniqueness regions.

and signal its commitment to sustain debt, which it can achieve through either a pro-cyclical stance or more aggressive debt consolidation.

The importance of the government's attitude towards the business cycle is visualised by Figure 2.5. According to the figure, when monetary policy is active, a passive fiscal stance combined with a countercyclical stance might not be feasible. In fact, the stronger is the countercyclical bent of fiscal policy (i.e. the higher is γ_y), the greater must be the fiscal contraction following any given rise in government debt in order to ensure the existence of equilibrium (i.e. the higher must be γ_b). Thus, compared to Figure 2.2 from the benchmark case, the feasible set encompassing countercyclical policies contracts, especially when the bond market is particularly anxious and χ_g relatively high. On the other hand, the feasible set encompassing pro-cyclical policies expands and a stable equilibrium can be supported, even if fiscal policy is active.

2.3 Concluding remarks

In this chapter, we reviewed the well-established conditions for equilibrium stability and uniqueness from the seminal contribution of Leeper (1991) and showed that these conditions change markedly under debt-elastic sovereign risk premia and depend strongly on the government's endogenous fiscal response to the business cycle. Whereas in Leeper, short-run debt developments do not change the feasible set of fiscal and monetary policies, we show that they can affect the ability to ensure long-run sustainability when demand for government bonds is highly elastic.

Our results suggest that countercyclical policies could come at great cost in terms of widening interest rate spreads and macroeconomic instability. The results therefore provide an interpretation for why some countries, that run large deficits or face severe financial market constraints, tend to pursue pro-cyclical fiscal policies in order to keep interest rate spreads low, as shown empirically by Gavin and Perotti (1997) and Combes et al. (2014). We show that countercyclical fiscal policies can be feasible, provided they are accompanied by sufficiently aggressive debt consolidation policies and/or active inflation-targeting. Alternatively, maintaining the primary balance well below the maximum (politically or economically) feasible level would allow automatic stabilisers to absorb adverse shocks without immediately provoking sharp interest rate hikes.

While our results point towards the importance of fiscal austerity and monetary accommodation during sovereign debt crises, such policies might not always be feasible (or even desirable). For instance, the scope for fiscal austerity might be limited due to political constraints or because the economy has reached the top of its Laffer curve, whereas monetary expansions are constrained by the zero lower bound. Also, the choice between a fiscal or monetary solution to the crisis may have strong implications for welfare (in terms of both the variability and distribution of income). Therefore, a suitable welfare analysis is needed to reveal the optimal mix of fiscal and monetary policy when risk premia are debt-elastic. We would also like to stress that our model does not allow for policymakers to alter their objectives over time. Augmenting the model with regime-switching possibilities might, however, help reveal how 'often' countercyclical policies are permissible and to what extent they contribute to macroeconomic stability. Finally, although our reduced-form specification of the sovereign risk premium is helpful in deriving analytical results, omitting potential non-linear interactions between bond spreads and fiscal fundamentals involves the risk of overlooking important second-order effects (e.g. heightened uncertainty when governments are close to default). Building the sovereign risk premium from micro-foundations would help to account for such effects, yet would most likely merely exaggerate our results. We leave these extensions for future work.

2.A Optimal demand and price-setting

A representative final good firm combines differentiated intermediate goods, $y_t(i)$, purchased from intermediate goods firm $i \in [0,1]$, to produce the final good, y_t, using the following production technology:

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2.17}$$

where $\epsilon > 1$ measures the elasticity of substitution between intermediate goods. Minimising the costs of assembling y_t, subject to (2.17), results in the optimal demand schedule for $y_t(i)$ and an expression for the aggregate price index, P_t:

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} y_t, \quad P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}. \tag{2.18}$$

Intermediate goods firms, which are owned by the households, face the following linear production technology:

$$y_t(i) = n_t(i). \tag{2.19}$$

Subject to (2.19), the firm aims to minimise labour costs, which results in a condition that equates real marginal costs, $mc_t(i)$, to the marginal product of hiring one additional unit of labour:

$$mc_t(i) = w_t. \tag{2.20}$$

Intermediate goods firms set prices with the aim of maximising the discounted sum of current and future profits, conditional on the probability of non-price adjustment (which is governed by θ):

$$E_t \sum_{k=0}^{\infty} \theta^k Q_{t,t+k} \left(\overline{P}_t y_{t,t+k}(i) - W_{t+k} n_{t,t+k}(i)\right),$$

where $\overline{P}_t$ is the optimal re-set price[9] and $Q_{t,t+k} \equiv \beta^k \left(c_{t+k}/c_t\right)^{-\sigma}/\pi_{t+k}$ is the k-step ahead equilibrium pricing-kernel. Profits are distributed as dividends to the households. Subject to the demand schedule (2.18), the production technology (2.19) and the optimality condition for labour demand (2.20), profit maximisation leads to the following optimal re-set price:

$$\overline{P}_t = \mathcal{M} \frac{E_t \sum_{k=0}^{\infty} (\theta\beta)^k P_{t+k}^{\epsilon} c_{t+k}^{-\sigma} y_{t+k} mc_{t+k}}{E_t \sum_{k=0}^{\infty} (\theta\beta)^k P_{t+k}^{\epsilon-1} c_{t+k}^{-\sigma} y_{t+k}}. \tag{2.21}$$

According to (2.21), the optimal re-set price is a mark-up $\mathcal{M} \equiv \epsilon/(\epsilon-1)$ over current and expected real marginal costs. Note that, under flexible prices, $\theta \to 0$ and $\overline{P}_t = P_t$ for all t, such that (2.21) collapses to $mc_t = 1/\mathcal{M}$.

[9]Due to symmetry among firms, we can ignore the i-index.

2.B Reducing the linearised model

The full model from Section 2.1 in linear form is given by:

$$\sigma\hat{c}_t = \sigma E_t\hat{c}_{t+1} - \left(\hat{R}_{g,t} - E_t\hat{\pi}_{t+1}\right), \tag{2.22}$$

$$\varphi\hat{n}_t = \hat{w}_t - \sigma\hat{c}_t, \tag{2.23}$$

$$\hat{y}_t = \frac{c}{y}\hat{c}_t, \tag{2.24}$$

$$\hat{y}_t = \hat{n}_t, \tag{2.25}$$

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\hat{w}_t, \tag{2.26}$$

$$\frac{T}{y}\hat{T}_t = \gamma_b\frac{b}{y}\hat{b}_{t-1} + \gamma_y\hat{y}_t, \tag{2.27}$$

$$\hat{R}_t = \alpha_\pi\hat{\pi}_t, \tag{2.28}$$

$$\hat{b}_t = \frac{1}{\beta}\left(\hat{b}_{t-1} + \hat{R}_{g,t-1} - \hat{\pi}_t\right) - \frac{T/y}{b/y}\hat{T}_t, \tag{2.29}$$

$$\hat{\Xi}_{g,t} = \chi_g\frac{b}{y}\hat{b}_t, \tag{2.30}$$

$$\hat{R}_{g,t} = \hat{R}_t + \hat{\Xi}_{g,t}, \tag{2.31}$$

where $\kappa \equiv (1-\theta)(1-\theta\beta)/\theta$.

To reduce the model, insert (2.25) and (2.24) into (2.23):

$$\hat{w}_t = \left(\varphi\frac{c}{y} + \sigma\right)\hat{c}_t.$$

Insert this new expression for $\hat{w}_t$ into the New Keynesian Phillips curve (2.26):

$$\hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \kappa\left(\varphi\frac{c}{y} + \sigma\right)\hat{c}_t. \tag{2.32}$$

Insert (2.31), (2.28) and (2.30) into (2.22):

$$\sigma\hat{c}_t = \sigma E_t\hat{c}_{t+1} - \left(\alpha_\pi\hat{\pi}_t + \chi_g\frac{b}{y}\hat{b}_t - E_t\hat{\pi}_{t+1}\right). \tag{2.33}$$

Finally, insert (2.31), (2.28), (2.30), (2.24) and (2.27) into (2.29):

$$\hat{b}_t = \frac{1}{\beta}\alpha_\pi\hat{\pi}_{t-1} - \frac{1}{\beta}\hat{\pi}_t - \gamma_y\frac{c/y}{b/y}\hat{c}_t + \left(\frac{1+\chi_g\frac{b}{y}}{\beta} - \gamma_b\right)\hat{b}_{t-1}. \tag{2.34}$$

The reduced system is then given by equations (2.32)-(2.34). Using the auxiliary variable $\hat{\pi}_t' = \hat{\pi}_t$, we obtain the state-space representation (2.12) from the main text.

2.C Proof of Propositions 1 and 2

The 4×4 system of dynamic equations is given by

$$\begin{bmatrix} E_t\hat{\pi}_{t+1} \\ E_t\hat{c}_{t+1} \\ \hat{b}_t \\ \hat{\pi}_t' \end{bmatrix} = \begin{bmatrix} -\frac{\Psi}{\beta} & 0 & \frac{1}{\beta} & 0 \\ 1+\frac{\Psi}{\sigma\beta}-\gamma_y\frac{c}{y}\frac{1}{\sigma}\chi_g & \frac{1}{\sigma}\chi_g\frac{b}{y}\left[\frac{1+\chi_g\frac{b}{y}}{\beta}-\gamma_b\right] & \frac{\alpha_\pi}{\sigma}-\frac{1}{\sigma\beta}-\frac{1}{\sigma\beta}\chi_g\frac{b}{y} & \frac{\alpha_\pi}{\beta\sigma}\chi_g\frac{b}{y} \\ -\gamma_y\frac{c}{b} & \frac{1+\chi_g\frac{b}{y}}{\beta}-\gamma_b & -\frac{1}{\beta} & \frac{\alpha_\pi}{\beta} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{\pi}_t \\ \hat{c}_t \\ \hat{b}_{t-1} \\ \hat{\pi}_{t-1}' \end{bmatrix},$$

where $\Psi = \kappa\left(\varphi c/y+\sigma\right)$. As there is one auxiliary variable, its characteristic polynomial can be written as $H(\lambda) = \lambda G(\lambda)$ with

$$\begin{aligned} G(\lambda) \;=\; & \lambda^3 - \lambda^2\left[1+\frac{\Psi}{\sigma\beta}+\left(1-\gamma_y\frac{c}{y}\frac{1}{\sigma}\chi_g\right)+\left(\frac{1}{\beta}-\gamma_b\right)+\frac{1}{\beta}\chi_g\frac{b}{y}+\left(\frac{1}{\beta}-1\right)\right] \\ & +\lambda\left[\left(\frac{1}{\beta}-\gamma_b\right)\left(1+\frac{1}{\beta}+\frac{\Psi}{\sigma\beta}\right)+\frac{1}{\beta}\chi_g\frac{b}{y}\left(1+\frac{1}{\beta}\right)+\frac{1}{\beta}\left(1-\gamma_y\frac{c}{y}\frac{1}{\sigma}\chi_g\right)+\frac{\Psi}{\sigma\beta}\alpha_\pi\right] \\ & -\left[\left(\frac{1}{\beta}-\gamma_b\right)\left(\frac{1}{\beta}+\frac{\alpha_\pi}{\sigma\beta}\right)+\frac{1}{\beta^2}\chi_g\frac{b}{y}\right]. \end{aligned}$$

Our aim is to derive sufficient conditions such that $G(\lambda) = 0$ twice for $|\lambda| > 1$ and once for $|\lambda| < 1$.

First, note that as $G(-\infty) = -\infty$ and for $\gamma_b < 1/\beta$ and $\alpha_\pi > 0$, $G(0) < 0$ and $dG(\lambda)/d\lambda > 0$ for $\lambda < 0$, $G(\lambda) < 0$ for $\lambda < 0$. So, the characteristic polynomial has no eigenvalues smaller than zero. As it needs to have one eigenvalue smaller than one, $G(1) > 0$, $G(1) < 0$ allows only two or zero eigenvalues smaller than one. $G(1) > 0$ implies

$$\gamma_b > \frac{1}{\beta} - 1 + \frac{\chi_g \gamma_y \frac{c}{y} (1-\beta)}{\Psi (\alpha_\pi - 1)} \quad \text{and} \quad \alpha_\pi > 1 + \frac{\chi_g \gamma_y \frac{c}{y} (1-\beta)}{\Psi \left[\gamma_b - \left(\frac{1}{\beta} - 1\right)\right]}, \tag{2.35}$$

or

$$\gamma_b < \frac{1}{\beta} - 1 + \frac{\chi_g \gamma_y \frac{c}{y} (1-\beta)}{\Psi (\alpha_\pi - 1)} \quad \text{and} \quad \alpha_\pi < 1 + \frac{\chi_g \gamma_y \frac{c}{y} (1-\beta)}{\Psi \left[\gamma_b - \left(\frac{1}{\beta} - 1\right)\right]}. \tag{2.36}$$

To rule out three eigenvalues smaller than one, note that $d^2G(0)/d\lambda^2$ equals the sum of eigenvalues. If it is larger than 3, there can be only one eigenvalue small than one. This implies

$$0 < 2\left(\frac{1}{\beta} - 1\right) + \chi_g \left[\frac{1}{\beta}\frac{b}{y} - \gamma_y \frac{c}{y}\frac{1}{\sigma}\right] + \left(\frac{\Psi}{\sigma\beta} - \gamma_b\right),$$

which holds if $\gamma_b < \Psi/(\sigma\beta)$ and $(\sigma\beta)(b/c) > \gamma_y$. This proves the proposition.

2.D Additional graphs

Figure 2.6: Equilibrium outcome under debt-elastic risk premia ($\chi_g > 0$) and alternative cyclical fiscal stances