6 Towards impact functions for stochastic climate change

Most functions of economic impact assume that climate change is smooth. We here propose impact functions that have stochastic climate change as an input. These functions are identical in shape and have similar parameters as do deterministic impact functions. The mean stochastic impacts are thus similar to deterministic impacts. Welfare effects are larger, and the stochasticity premium is larger than the risk premium. Stochasticity is more important for past impacts than for future impacts.

6.1 Introduction

The literature on the aggregate economic impact of climate change has assumed that global warming will be gradual and smooth (Tol 2009). In reality, of course, there is substantial year-to-year variability in all climate variables. This has two implications. First, the models used to estimate future impacts of climate change cannot readily be compared to the models used to infer past impacts, which have to deal with the stochastic nature of weather and climate (Tol 2013). Second, a risk-averse agent would suffer a greater welfare loss if impacts are stochastic (Dalton 1997). Therefore, this paper proposes impact functions with stochastic climate as an input.

Impact functions for smooth climate change can be conceptualized as follows. Impacts are driven by the expected, say, temperature. A slight change in the actual temperature has little effect on the economy. A slight change in the expected temperature, on the other hand, implies little effect in most years but a large effect in some years. For instance, climate change might increase the probability of a drought from 1 in 20 to 1 in 15 years. Even if there is a minimal impact in 14 out of 15 years, the expected impact of drought

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has increased by 25%. Smooth impact functions represent the change in expected weather, rather than the actual weather.

Impact functions for stochastic climate change, on the other hand, represent the impact of the weather. Because the literature has smooth impact functions, we calibrate the proposed stochastic impact functions to the smooth functions. Specifically, we calibrate the expectation of the stochastic impacts to the smooth impacts.

The paper proceeds as follows. Section 6.2 formalizes the above. Section 6.3 discussed the expected impacts of climate change. Section 6.4 turns to certainty equivalents. Section 6.5 has sensitivity analysis on scenarios. Section 6.6 concludes.

6.2 Methods

A typical impact function (Tol 2009;Tol 2012), assuming deterministic climate change, may look something like this:

$$I_t^D = \alpha T_t + \beta T_t^2 \qquad (6.1)$$

where T_t is the global mean surface air temperature in deviation from its pre-industrial mean. Now assume that temperature varies randomly around its secular trend and follows a normal distribution: $T_t = \mu_t + \varepsilon_t$ with $\varepsilon_t \sim \mathrm{N}(\mu_t,\sigma^2)$. Assume further that the stochastic impact function is specified as:

$$I_t^S = \alpha^* T_t + \beta^* T_t^2 \qquad (6.2)$$

Its mean equals:

$$EI_t^S = \int_T \alpha^* T_t + \beta^* T_t^* dt = \alpha^* \mu_t + \beta^* \mu_t^2 + \beta^* \sigma^2 \qquad (6.3)$$

Now calibrate the stochastic impact function such that its mean (Equation 6.3) equals the deterministic function (Equation 6.1). There are many solutions:

$$\alpha^* = \alpha + \beta T_0 - \beta^* \frac{T_0^2 + \sigma^2}{T_0} \qquad (6.4)$$

Equation (6.4) is a line. Its position depends on the calibration temperature T_0.

An intuitive solution would be α*= α and $\beta^*=2.5^2\beta(2.5^2+\sigma^2)^{-1}$. That is, the linear component is unchanged. The quadratic component absorbs the stochasticity. The quadratic component is calibrated at 2.5°C warming, just like the deterministic impact function. We can of course also calibrate the impact function at a different point, say 1.0°C. Alternatively, let the linear component absorb the stochasticity. That is, $\beta^*=\beta$ and $\alpha^*= \alpha-\beta\sigma^2/2.5$. The standard deviation of the temperature is 0.33°C. Table 6.1 shows the resulting parameters.

Table 6.1. Alternative impact functions

	Deterministic	Stochastic				
		Quad, 2.5	Quad, 1.0	Linear, 2.5	Calibrated[a]	Calibrated[b]
A	4.33	4.33	4.33	4.36	4.53	4.55
B	-1.92	-1.21	-0.78	-1.92	-1.94	-1.95

[a] Calibrated to the synthetic scenario of Section 2.
[b] Calibrated to the 54 GCM runs of Section 3.

The problem with these solutions is that they change that the characteristics of the impact function, particularly the points where the total impact and the marginal impact change sign. Therefore, we find α* and β* by minimizing the squared distance between Equation (6.1), evaluated at the 30-year running mean temperature, and the 30-year running mean impact according to Equation (6.2). The impact of the 30-year running mean is defined as

$$I_t^D = \alpha \overline{T}_t + \beta \overline{T}_t^2 \qquad (6.5)$$

where $\overline{T}_t = \frac{1}{31}\sum_{s=-15}^{s=15} T_{t+s}$

The 30-year running mean impact is defined as

$$\bar{I}_t^S = \frac{1}{31}\sum_{s=-15}^{s=15} I_{t+s}^S \qquad (6.6)$$

Table 6.1 shows the results. Figure 6.1 shows the 30-year running mean impact according to the four alternative specifications of Equation (6.2) as well as the deterministic impact (Equation (6.2)) driven by the 30-year running mean temperature (Equation (6.5)). For 1850-2012, observed temperatures are used. For 2013-2100, an average warming of 0.025°C/year is assumed, and a standard deviation of 0.33°C.

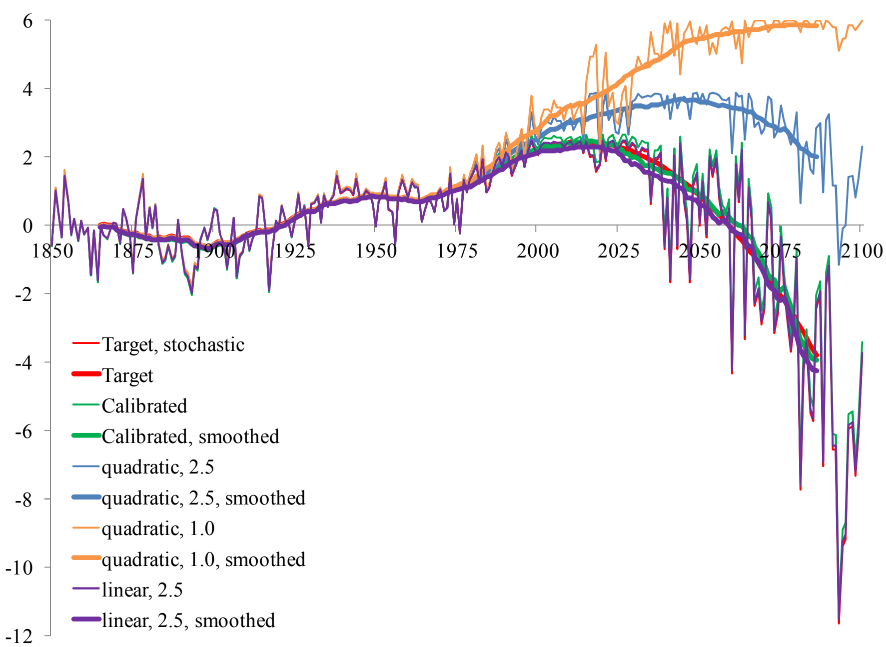

Figure 6.1. The target impact function (defined for smooth inputs) and four alternative calibration of the impact function for stochastic inputs.

The specifications in which the quadratic parameter β absorbs the stochasticity clearly deviate. The specification calibrated by least squares closely tracks the target impact function (6.1), but so does the specification in which the linear parameter α absorbs the stochasticity. Table 6.1 reveals why. If β is adjusted, it changes a lot. If α is adjusted, it changes little. If both are adjusted, they move in opposite directions so that the overall behaviour (within the domain of calibration) is maintained.

Assume there is first-order autoregression in the temperature, that is $T_t \sim N(\mu_t+\rho T_{t-1},\sigma^2)$ The mean of a second order polynomial then equals:

$$EI_t^A = \int_T \alpha^{**} T_t + \beta^{**} T_t^2 dt = \alpha^{**} \frac{\mu_t}{1-\rho} + \beta^{**} \frac{\sigma^2}{1-\rho^2} \tag{6.7}$$

Calibration is as above. This is not a new case.

6.3 Expected impacts of climate change

We have the global mean temperature for the 20th and 21st century according to 54 different runs with General Circulation Models (GCMs), obtained from the World Climate Research Programme's (WCRP's) Coupled Model Intercomparison Project phase 3 (CMIP3) multi-model dataset (Meehl et al. 2007a,b).[16] The runs are based on the A1B emission scenarios (Nakicenovic and Swart 2001). We first recalibrated the model to the least squares fit to the 54 scenarios. There is a minimal difference with the calibration above. See Table 6.1.

The panel a) of Figure 6.2 shows the impact of the 30-year running mean temperature for all 54 GCM simulations (using Equation (6.5)). The panel b) of Figure 6.2 shows the 30-year running mean impact of the temperatures (using Equation (6.6)). For ease of comparison, we added the average of the 54 simulations, and the 95% confidence interval (based on bootstrap with 1,000 replications). The stochastic impact functions are consistent with the deterministic impact function – not just in mean (which is by virtue of calibration) but also in the range.

[16] http://climexp.knmi.nl/selectfield_co2.cgi?id=someone@somewhere.

The panel c) of Figure 6.2 shows the stochastic impacts for the 54 simulations. Unexpectedly, the mean is very similar to the other two panels. Although the individual scenarios reveal a lot of year-to-year variability, the confidence interval does not noticeably widen. The uncertainty is driven by the trend in the temperature, rather than by the variation around that trend.

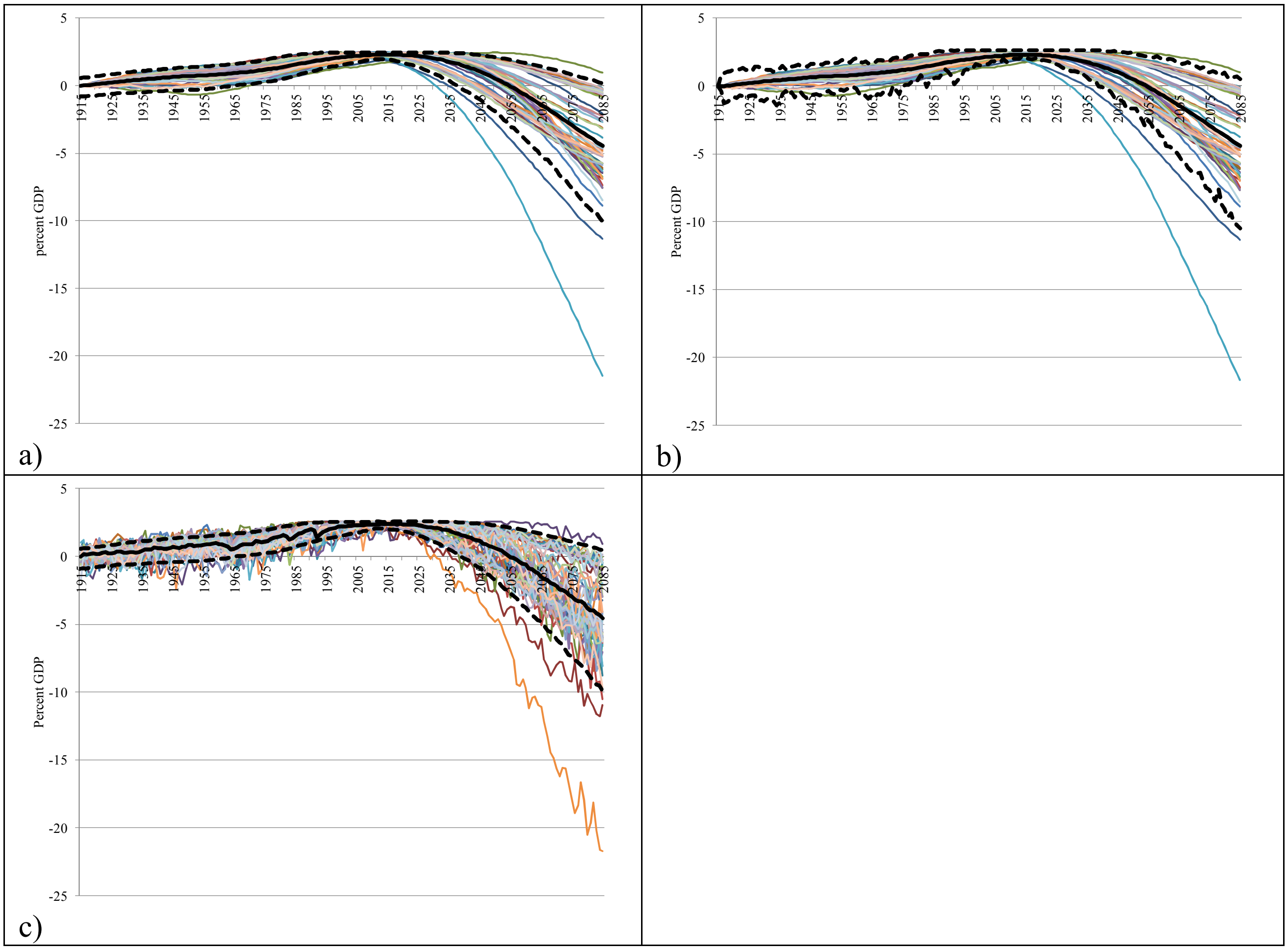

Figure 6.2. The aggregate economic impact according to 54 scenarios; the thick black line is the ensemble average, the dotted lines the average plus and minus twice the standard deviation. Panel a) Impact of 30-year running mean temperature; panel b) 30-year running mean impact; panel c) Stochastic impacts.

6.4 Certainty-equivalent impacts of climate change

Above, we studied the impact of 54 alternative simulations of climate change, and considered its expectation (or average). The top panel of Figure 6.3 shows the expectation again. It also shows the modal impact – here defined as the impact of the temperature averaged over the 54 simulations' 30-year running means. The difference between the mean and mode is substantial: By 2085, the expected impact is about 0.46% of GDP worse that the modal impact. We refer to the difference as the "mean premium".

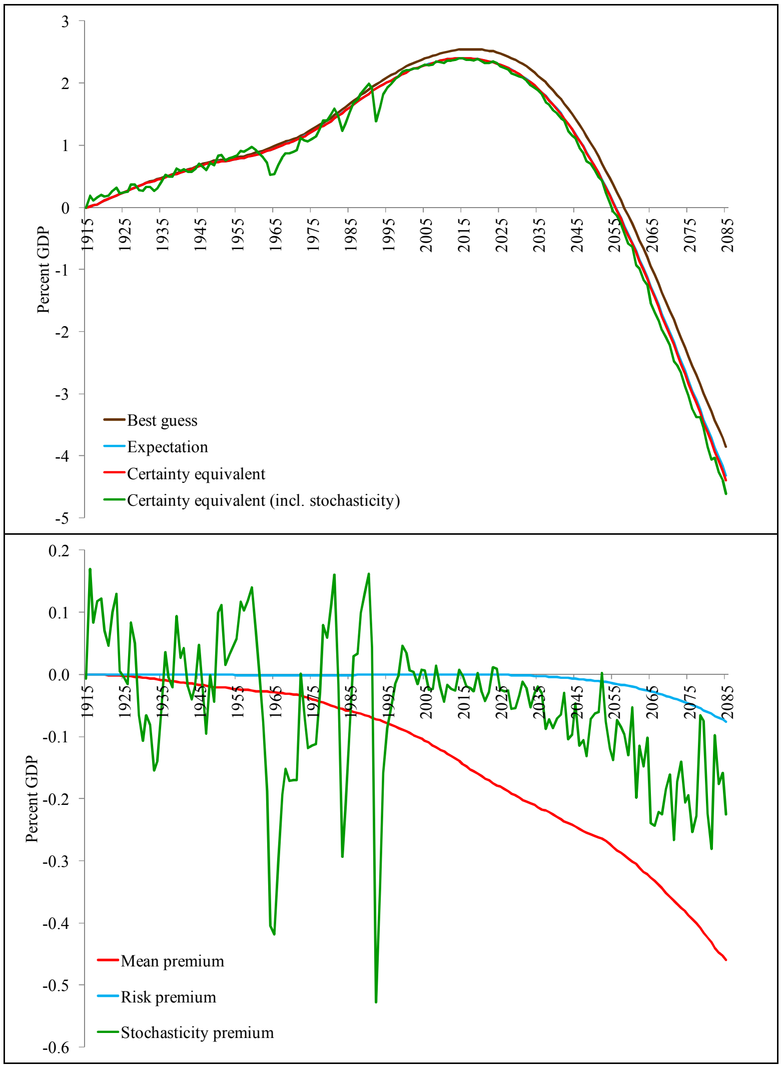

Figure 6.3. The top panel shows best guess impact, its expectation, and its certainty equivalent (with and without stochasticity), the bottom panel the mean premium, risk premium, and stochasticity premium.

The expectation of monetized impact is, of course, an inconsistent welfare measure. The expectation first translates welfare losses into equivalent income losses, and then takes the average. We should compute the certainty equivalent, that is, first take the average of the welfare losses across GCM simulations and then translate into an equivalent income loss. To this end, we need a welfare function – we assume a logarithmic one – and an income scenario. We use per capita income from the WorldBank[17] for 1960-2012. We infrapolate this using (Maddison 2001)'s numbers for 1900-1959. We extrapolate for 2013-2100 using SRES A1B according to IMAGE (IMAGE Team 2001).

[17] http://data.worldbank.org/

The top panel of Figure 6.3 therefore also shows the certainty equivalent impact – first, using the 30-year running mean temperatures and, second, using the annual temperatures. The variation between model simulations appears to be not that important. The risk premium is only about 0.08% of GDP in 2085 – see the bottom panel of Figure 6.3. Stochasticity is almost three times as important. By 2085, the "stochasticity premium" is 0.23% of GDP.

Note that the relative size of the three premia changes over time. See Figure 6.3. The stochasticity premium dominates in the 20th century. Stochasticity is important for understanding the past impacts of climate change. However, in the projections for the 21st century, the signal of climate change grows larger and larger relative to the noise of weather variability. Stochasticity thus becomes relatively less important.[18] Furthermore, as people (are assumed to) grow richer, the risk and stochasticity premia become less important.

The results of Figure 6.3 assume that all simulations are equally likely. This is not the case. Several approaches have been discussed in the literature for assigning probabilities to models' ensembles (Allen 2003; Gay and Estrada 2010; Kinzig et al. 2003; Schneider 2002; Schneider 2001). Here, we reassess the impacts using the models' skills in forecasting the 20th century as weights.[19] Specifically, we compute the sum of squared deviations of the simulated temperature from the actual temperature. The weight of each model is proportional to the exponent of minus half the sum of squared residuals – that is, we assume normality to compute the likelihood of each of the 54 models. Likelihoods are rescaled to sum to one.

Figure 6.4 shows the results. The best guess impacts are 0.76% lower in 2085 than in the case in which all simulations have weight 1/54. That is, the scenarios with more severe climate change in the 21st century do not correspond well with observed warming in the

[18] Note that we assume that GCMs adequately model weather variability. This may be a bad assumption.
[19] Note that skill in reproducing the past does not necessarily imply that the model is better. Reproduction skill may be due to tuning, which would reduce forecast skill.

20th century. The non-linear impact function slightly amplified this for the expected impacts, which fall by 0.79% in 2085. The certainty equivalent impact, however, does not fall as much, as it continues to emphasize the worst scenarios. The risk premium thus goes up, albeit by only a few thousands of a percent. The stochasticity premium increases more, by about 0.04%

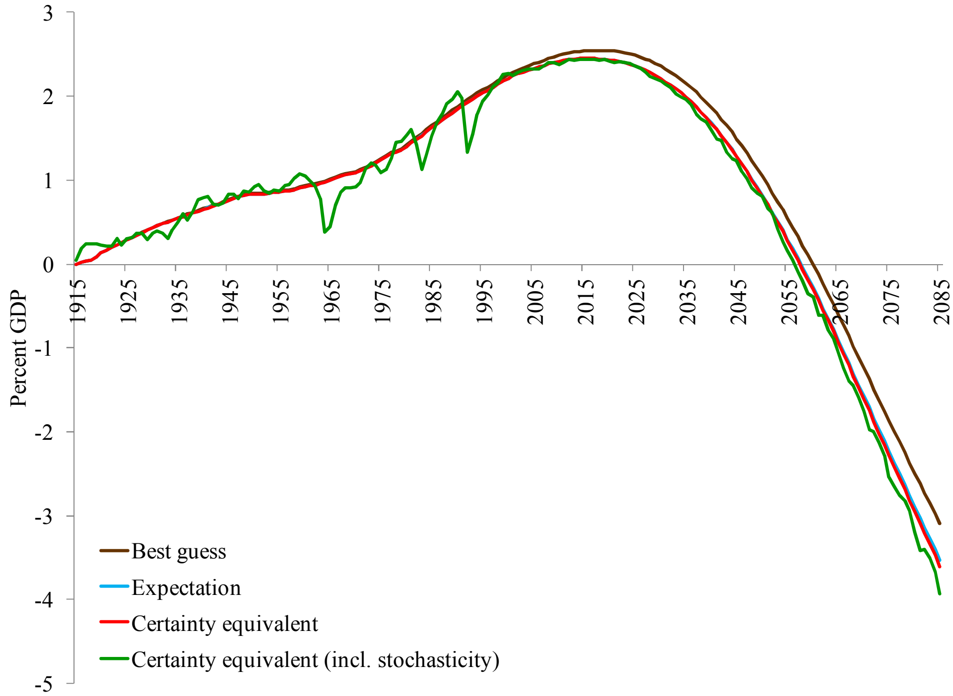

Figure 6.4. The best guess impact, its expectation, and its certainty equivalent (with and without stochasticity) if the GCM runs are weighted according to their ability to reproduce the 20th century temperature record.

6.5 Scenarios

The above results are all for the A1B scenario. We have 54 GCM simulations for the A1B scenario. We have 28 simulations for the A2 scenario, and 18 for the B1 scenario. We repeat some of the above exercises for the two alternative scenarios.

The top panel of Figure 6.5 shows the mean and 95% confidence interval of the impacts for the A2 scenario, and compares it to the A1B scenario (for the subsample of model runs available for both scenarios). The bottom panel of Figure 6.5 does the same for the B1 scenario.

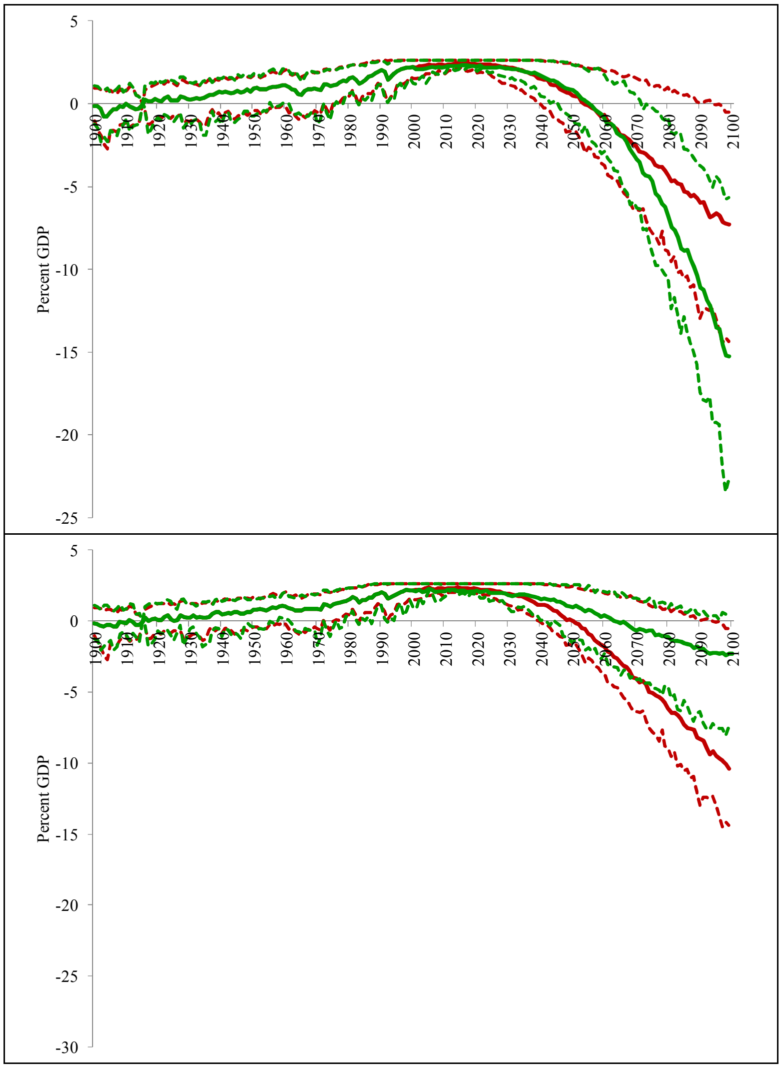

Figure 6.5. The mean and 95% confidence interval of the impacts under A1B (red), A2 (top panel, green, more pessimistic), and B1 (bottom panel, green, more optimistic).

Not surprisingly, impacts are more negative for the A2 scenario. The damage, averaged across simulations, exceed 15% of GDP by 2100, compared to over 7% for A1B. Damages are lower under B1, reaching over 2% of GDP by 2100, compared to over 10% for (the same subsample of models for) A1B.

The confidence intervals, however, overlap. This is partly because the samples are positively correlated. After all, the same models were run for both scenarios. The top panel of Figure 6.6 therefore shows the mean and 95% confidence interval of the difference in impact. The difference between scenarios is not significantly different from zero.

This is largely because the temperature scenarios themselves do not statistically significantly differ from one another. The mean of the A2 scenario only starts to be statistically significantly different from the mean of the A1 scenario from 2095 onwards, whereas the null hypothesis that A1B and B1 are the same cannot be rejected in the 21st century.

The bottom panel of Figure 6.6 repeats the exercise of its top panel, but impacts are now evaluated with the 30 year average temperature. The findings are the same. There is no statistically significant difference between the scenarios. This is not due to stochasticity (which was removed in the bottom panel of Figure 6.6), but rather due to model differences.

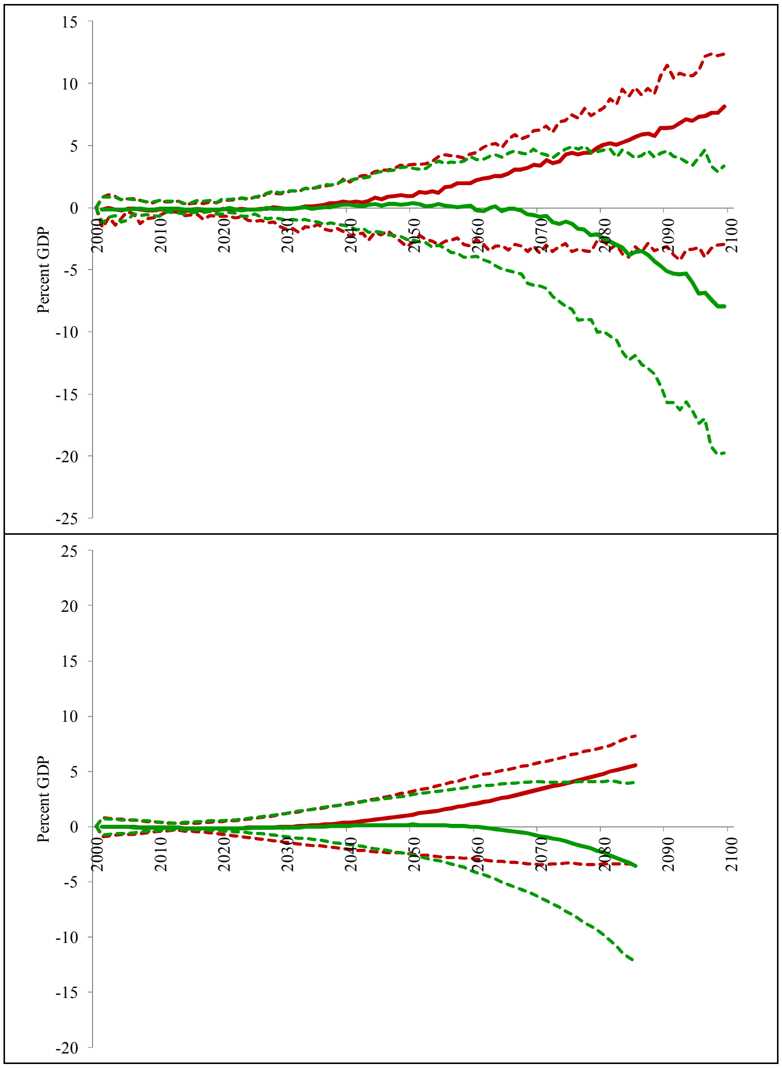

Figure 6.6. The difference between A2 and A1B (green) and B1 and A1B (red) and the 95% confidence interval with stochasticity (top panel) and without (bottom panel).

6.6 Discussion and conclusion

We derive impact functions that have stochastic climate change as an input. We show that these stochastic impact functions are identical in shape and similar in parameterization to deterministic impact functions. As a result, the mean monetary stochastic impact is similar to the deterministic impact. Welfare impacts are larger, however, and the stochasticity premium is larger than the risk premium. Stochasticity is very important for past impacts, but loses its dominance as the signal of climate change becomes larger relative to weather variability.

There are two implications. First, methodologically, stochasticity is more important for understanding and estimating past impacts than it is for projecting future impacts. This is disconcerting because it separates model validation (too rarely done in climate change impact research) from model application. Second, the estimated stochasticity premium increases the estimated welfare impact of climate change and thus strengthens the case for greenhouse gas emission reduction.

The proposed impact functions can readily be used in integrated assessment models that are based on a dynamic stochastic general equilibrium model (Cai et al. 2012;Lontzek and Narita 2011).

There are a number of caveats to the results above. We studied a second-order polynomial. Other impact functions may well lead to different results. We focused on the global mean. Weather variability is much larger at smaller spatial scales. Regionally disaggregated impact functions would thus be more sensitive to stochasticity than our global one, and the stochasticity premium would be larger for a disaggregated analysis. We limit the analysis to temperature. Other climate variables have different stochastic properties. The analysis here can and should be extended to these issues. The impact functions used are a function of climate and climate only. Development changes vulnerability to climate change, including the way societies cope with stochasticity (Yohe and Tol 2002).