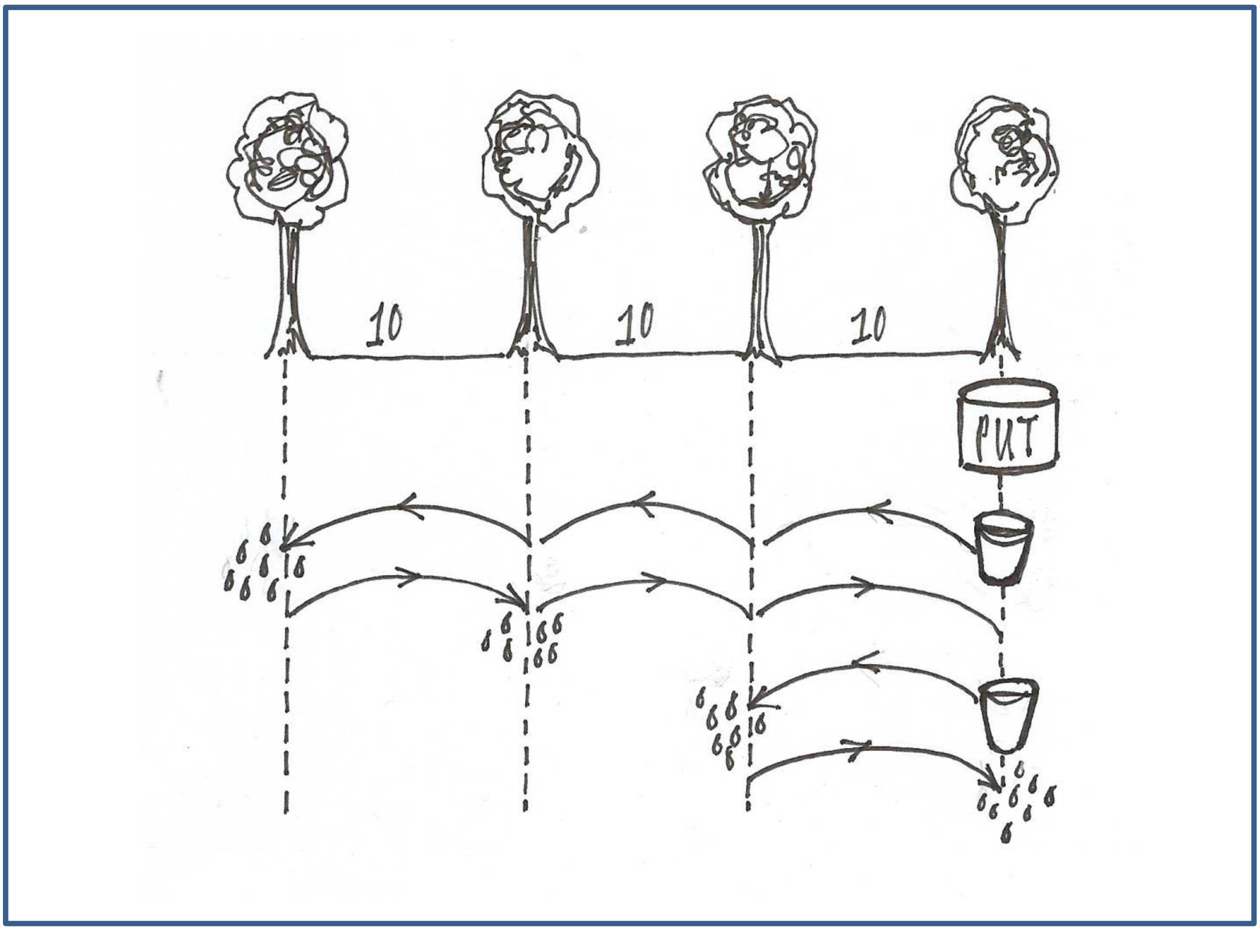
CHAPTER 8

CONCLUDING REMARKS



Four young trees were set out in a row 10 meters apart. A well was situated beside the last tree. A bucket of water is needed to water two trees. How far would a gardener have to walk altogether if he had to water the four trees using only one bucket?

Introduction

The research presented in this thesis focused on the component processes and skills that underlie the successful comprehension of word problems. The studies conducted as part of this research investigated both students' use of visual representations and the quality of these representations. Specifically, we were interested in the extent to which different types of visual representation increase or decrease the chance of solving a word problem correctly.

The first objective of this thesis was to examine students' performances, notably the extent to which students use different types of visual representations. Furthermore, we examined the role that students' spatial and semantic-linguistic skills played in the solving of routine and non-routine word problems in early (second) and later (sixth) grades of elementary school.

The second objective of this thesis was to investigate how teachers implemented an innovative instructional approach – based on the didactical use of visual-schematic representations – in their own classroom teaching practice. This instructional approach required teachers to use visual-schematic representations that visualized the problem structure in a diverse and flexible way. Moreover, they were required to vary the kinds of visual representations in a way that suited the problem characteristics and students' individual needs. This approach made it necessary for teachers to model the representation process transparently, correctly and completely, as well as to construct visual representations that correctly and completely depicted the relations between all the components relevant to the solution of the problem.

Both of the objectives of this thesis, which were stated in Chapter 1 (General Introduction), were met by the research. The concluding remarks in this final chapter are focused on reflecting on the findings. In addition, I will examine possible implications of these findings for educational practice. On the basis of the findings described in the six studies that were included in this thesis, several observations and their consequences merit particular discussion and each will be addressed in due course.

First, I will discuss the findings with regard to the following: (a) the underlying processes of word problem solving; (b) the role of different types of visual representations; (c) the importance of semantic-linguistic and visual-spatial skills; and, (d) the didactical use of visual representations by elementary school teachers. Secondly, I will draw implications for teacher training and teacher professionalization. Finally, I will conclude by reflecting on the reasons why the implementation of instructional innovations in mainstream educational practice is a challenging matter. This will also include an examination of the best course that is open to us if we are to bridge the existing gap between educational research and educational practice.

The underlying processes of word problem solving

In line with previous studies (see e.g., Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Cummins, Kintsch, Reusser, & Weimer, 1988; Krawec, 2010; Van der Schoot, Bakker-Arkema, Horsley, & Van Lieshout, 2009), this thesis revealed that comprehension of a word problem text is a determining factor with regard to students' performance on non-routine and routine word problems. In addition, we have gained insight into the component skills and abilities that underlie the comprehension of word problems.

On the basis of the results from Chapter 2, we can conclude that comprehension of non-routine word problems consists of the following elements: 1) the identification of relevant numerical and linguistic components, and of the relations between these components; 2) the visual representation of these components and relations in a complete and coherent way. While the identification of linguistic components and the relations between these components is semantic-linguistic in nature, the visual representation of these components and relations lies in the visual-spatial processing domain. Furthermore, the key basic ability in the semantic-linguistic domain is reading comprehension; whereas in the visual-spatial domain spatial ability is key.

In many previous studies these two processing domains were investigated separately. The importance of semantic-linguistic factors was mostly investigated in relation to routine word problem solving (Pape, 2003; Van der Schoot et al., 2009), whereas the use of visual representations was often examined in relation to non-routine word problem solving (Hegarty & Kozhevnikov, 1999; Van Garderen, 2006). However, the findings of our research reveal that both processing domains play a prominent role in non-routine word problem solving in the later grades of elementary school. Moreover, our findings make it clear that the identification of numerical and linguistic components and the visual representation of these relations are not separate processes, but seem to develop parallel to each other.

The production of different types of visual representations

When we looked more closely at the importance of the visual-spatial elements in word problem solving, by specifically examining the production of visual representations, our findings revealed that in most cases (sixth) grade students did not make use of a visual representation to solve a word problem. Furthermore, when a student did decide to use a visual representation, he/she did not always have the ability to construct a visual-schematic representation that could be called accurate. That is, a visual-schematic representation that contained the correct relations between all the elements relevant to the solution, and gave a coherent and complete view of the problem structure. The study described in Chapter 4 showed, for example, that students did construct pictorial representations and provide images of a specific element (i.e., object or person) of the word problem text when solving non-routine word problems. Furthermore, several students who made use of visual-schematic representations included incorrect relations in these types of representations, or produced incomplete visual-schematic representations (i.e., solution-relevant relations were missing). This finding showed that only making a distinction between pictorial and visual-schematic representations, as was the case in previous research (Hegarty & Kozhevnikov, 1999; Van Garderen, 2006; Van Garderen & Montague, 2003), is too limited. Our findings suggested that introducing a third type of visual representations, namely inaccurate visual-schematic representations, is justified.

Furthermore, because we used item-level analysis, we were better able to examine the extent to which these different types of visual representations contributed to students' word problem solving performance. We can conclude that only accurate visual-schematic representations increase the chance of solving a word problem correctly, and that our approach offers a more comprehensive view with respect to the importance of different types of visual representations than previous studies, which were based on test-level analysis.

The importance of visual-spatial and semantic-linguistic skills in routine word problems

Besides underscoring the importance of both the visual-spatial and semantic-linguistic processing domains in non-routine word problems (as shown in Chapter 2 and Chapter 4), our research also provided more insight into the role played by visual-spatial and semantic-linguistic skills in solving a specific type of routine word problems, namely compare word problems.

Based on the findings described in Chapter 3, we can conclude that sixth grade students who built a high quality mental representation of the problem structure experienced less difficulties with solving inconsistent compare problems that contained an unmarked relational term (i.e., 'more than'). The relational term in an inconsistent compare problem primed an inappropriate mathematical operation (e.g., 'more than' when the required operation is subtraction). However, translating a marked relational term like 'less than' into an addition operation was found to be closely associated with a student's performance on a general measure of semantic-linguistic skills, namely reading comprehension. In other words, reading comprehension skills together with sophisticated visual representation skills were found to be essential in dealing with semantically complex word problems.

The difficulties with solving compare word problems were, however, already visible in the early grades of elementary school. In line with Cummins et al., (1988), the findings of the study described in Chapter 6 showed that second grade students experienced more difficulties solving this type of routine word problem compared to combine and change word problems. Thus, semantic-linguistic skills do not only play a role in the solution of non-routine word problems, but are also important in routine word problems that have a complex semantic structure or, as in the case of students in higher grades of elementary school, in routine problems that contain semantically complex text elements (like relational terms).

In short, the findings of this research showed that both younger and older elementary school students experience difficulties with solving word problems. These difficulties were clearly caused by difficulties with the comprehension of the text of a word problem. Moreover, our findings also indicate that students often lacked a sophisticated visual representation strategy and/or were not able to deal with the semantic complexities of certain types of word problems.

Teachers' didactical use of visual representations

As teachers' behavior in the classroom (i.e., what teachers do in the classroom and how they interact with their students) plays a prominent role in the development of students (Kyriakides, Christoforou, & Charalambous, 2013; Kyriakides & Creemers, 2008), it can be assumed that teachers might perpetuate or even cause the difficulties that students experience.

On the basis of the findings of Chapter 3 for example, we can question whether enough attention is paid to the training of semantic-linguistic skills in educational practice. These findings showed that students who performed well on a standardized math test experienced a lot of difficulties with the solution of compare problems that asked for semantic-linguistic skills (i.e., inconsistent marked compare problems). If indeed these semantic-linguistic skills are not, or inadequately, trained in elementary school, this may lead to problems in secondary education where word problems contain more verbal information and become more complex.

When we examined the attention paid to the visual-spatial domain of word problem solving specifically, the findings of Chapter 7 showed that several mainstream teachers, like their students, experienced difficulties using accurate visual-schematic representations during word problem solving instruction in their own lessons. Significantly, even teachers who had followed a training on visual-schematic representation and who indicated that they felt competent, were not able to construct these visual-schematic representations in an accurate way in all the word problems they modeled. Moreover, we saw that teachers use mathematical representations instead of visual-schematic representations in a lot of situations. This is understandable when we take into account that math text books in contemporary math education and teacher training predominantly offer and teach student teachers to use mathematical representations. However, these specific types of visual representations play an important role only during the solution phase (i.e., they support the calculation process of mathematical operations and not the comprehension process).

These findings highlight the fact that the importance of accurate visual-schematic representations in the comprehension phase of word problem solving is still largely unrecognized both by those who set the curriculum and by teacher educators. Changing this situation is of the utmost importance, since the research presented in the present thesis indicates that comprehension of word problems causes the most difficulties in students and that these difficulties can be overcome by teaching the use of accurate visual-schematic representations.

Finally, the findings of the study in Chapter 7 revealed that when teachers did use accurate visual-schematic representations, in most cases a *bar model* was used. Other forms of visual-schematic representations that could have been used to elucidate the structure of the word problem, like *number lines*, *pie charts* or *own constructions,* were used only to a limited extent. Therefore, in general the level of diversity of forms of visual representations used by teachers was low. Moreover, the visual representations that teachers used did not always suit the problem characteristics and/or meet the individual needs of the students.

These findings also suggest that mainstream teachers who do not receive any training in word problem solving, and specifically in the use of visual-schematic representations as part of the solution process, experience even more difficulties. Evidently, if teachers are not able to construct accurate visual-schematic representations themselves, and, in addition, are not able to construct these representations in a diverse and flexible way, there is little reason to expect that their students will learn to use these types of visual representations. Therefore, in the next paragraph I will focus specifically on the implications of our findings for teacher professionalization and teacher training.

Implications for teacher professionalization and teacher training

The results of the studies described in this thesis, and of the study presented in Chapter 7 in particular, have clear implications for teacher professionalization and teacher training. These are summarized in the following list of recommendations:

- ✓ School teachers and their students should be competent at constructing accurate visual-schematic representations as an aid in word problem solving, and the development of this competence should have a prominent place in the math curriculum of regular classrooms. (Student) teachers should have knowledge about the purpose of accurate visual-schematic representations and be trained in the construction and proper use of these types of visual representations.
- ✓ Teacher professionalization and training should focus on the correct use of different forms of visual-schematic representations while solving a word problem. (Student) teachers should learn how to construct *number lines, pie charts, own constructions* and other appropriate forms of visual-schematic representations.
- ✓ There should be a particular emphasis on the construction process of these types of visual representation. Teachers should be able to use the construction process in a transparent, correct, and complete manner. (Student) teachers should learn to make their reasoning transparent by explaining which elements of the problem should be represented, and how the representation can be used to solve the problem. This reasoning process should also be correct (e.g., naming and using visual representations correctly), as well as complete (e.g., indicating why and how a visual representation can be used).
- ✓ Finally, teacher training should pay particular attention to teaching how to identify the characteristics of word problems. (Student) teachers should know the distinction between

routine and non-routine word problems, and the role that accurate visual-schematic representations play in these word problem types. Namely, in routine word problems (like combine, change and compare problems) the use of only one type of visual representation can suffice, because the problem structure of each of these types of word problems is identical. However, the problem structure of non-routine word problems varies, which makes it inappropriate to offer only one kind of accurate visual-schematic representation. Hence, (student) teachers should learn to use visual-schematic representations in a way that is both diverse (i.e., demonstrating a varied use of visual representations) and flexible (i.e., offering different visual representations to solve one word problem). Moreover, these visual representations should be functional and suit the specific characteristics of the word problem (e.g., the use of a pie chart while solving word problems involving percentages).

Besides their importance in teacher professionalization and teacher training, the recommendations listed above provide interesting aspects for further research about the importance of visual representations in word problem solving. Based on our findings the focus of future studies should initially be on teachers' own competence and didactical use of visual representations during word problem solving instruction. Once teachers have more knowledge about the importance of visualization in the word problem solving process, they can use this knowledge to help their students successfully overcome the difficulties that they are experiencing (Antoniou & Kyriakides, 2013; Penuel, Fishman, Yamaguchi, & Gallagher, 2007).

Bridging the gap between educational research and educational practice

The research presented in this thesis adds valuable new insights into the processes, factors and skills that influence performance in word problem solving to a well-established discussion in the field of educational research. Moreover, throughout the years several innovative word problem solving instructions have been developed which were meant to implement findings of educational research into practice (e.g., Jitendra & Star 2012; Jitendra et al., 2009; Montague, 2003; Montague, Warger, & Morgan, 2000). Nevertheless, based on the observations of teaching practice reported in Chapter 7, and on the difficulties experienced by students in early and later grades of elementary school reported in Chapters 3 and 6, we can conclude that it is hard for these innovative instructions to find their way into actual classroom practice.

These findings contribute to the discussion of an important issue that has been frequently debated in the last decades, namely the gap between educational research and educational practice (e.g., Broekkamp & Van Hout-Wolters, 2007; McIntyre, 2005; VanderLinde & Van Braak, 2010). Doubts have been sometimes raised about the quality and relevance of educational research, because educational research often does not provide educational professionals with clear, practical answers (Broekkamp & Van Hout-Wolters, 2007; Biesta, 2007). The gap between educational research and educational practice is in this view a result of the existence of two mutually exclusive types of knowledge: on the one hand, research-based knowledge that is published in scientific journals; and on the other hand, pedagogical knowledge that is used by classroom teachers in their day-to-day teaching practice (McIntyre, 2005).

However, to maintain that educational research should be left to academics, and that educational practice should be the sole domain of practicing teachers, could be detrimental to both fields of education (Biesta, 2007). Educational research should be or should become evidence-based, and teaching should be or should become an evidence-based profession, as is already the case in several countries in the world (Biesta, 2007). This approach considers evidence-based teaching practice to be of great value. The role of research in education should to be to tell us 'what works' and a preferred way to discover 'what works' is through experimental studies (Biesta, 2007; Slavin, 2002). Hence, besides the systematic observation, recoding, and analysis of data and the publication of its findings, educational research also has an important role to play in the improvement of educational processes and the evidence-based evaluation of outcomes.

However, in a lot of cases the improvements and innovations proposed by educational researchers only make it to the stage of publication in journals, and never reach teachers and students in the classrooms (Vanderlinde & Van Braak, 2010). Important findings are consequently rarely brought to the attention of teachers. Once research has been published in an academic journal, researchers move on the next study, rather than attempting to relate their findings to the teaching practice (Stevens, 2004). A good line of communication between researchers and practitioners is absent, and practitioners are not encouraged to get actively involved in the research process (Könings, Brand-Gruwel, & Van Merriënboer, 2007). It is therefore necessary that more opportunities are made available to practitioners and researchers to collaborate, disseminate findings, co-construct ideas and set research agendas. Involving practitioners in the design and implementation of research makes it possible to link research and practice. More cooperation between researchers and practitioners goes hand in hand with a change in thought concerning the way that research is disseminated. There still is a traditional top-down model when it comes to the dissemination of educational innovations, in which innovations are developed by the researcher and then transferred to others in oral or written form. This linear way of dissemination should be replaced by a circular model, which emphasizes a two-way flow of information between researchers and practitioners and encourages practitioners to adapt and negotiate research findings within the context of their use (Nutley, Walter, & Davies, 2007).

When educational practitioners are actively involved in the research process it often becomes clear that what works in practice is not always as straightforward as research findings suggest and that success depends on several factors. Also, sometimes it becomes clear that innovations that work are not desirable in educational practice. Biesta (2007) gives a good example of this situation. He concludes that, although we have conclusive empirical evidence that in all cases physical punishment is the most effective way of deterring or controlling disruptive behavior, most societies would find it undesirable to choose an option that involves such a violation of human rights.

Another reason why it is difficult to know whether an innovation works, is that research can tell us what works in a particular situation, but not what will work in any future situation. Furthermore, teachers often perceive innovation as involving more work, time and energy than the traditional well-known methods, to which they may therefore tend to revert (Könings et al., 2007). As the quest for an evidence-based educational practice might not be viable, it might be more suitable to strive for a less stringent approach, such as evidence-informed, evidence-influenced or evidence-aware practice (Biesta, 2007).

Nevertheless, in order to realize an evidence-informed practice and successfully implement educational innovations, close collaboration and transparent communication between researchers and practitioners is vital. It is essential that the content of the innovation fits with practitioners' beliefs and desires about teaching. Research into teachers' adoption of educational innovations showed that their valuation of what they are required to perform (i.e., perceived importance and value of the innovation, and perceived amount of effort required to implement it) could determine the extent to which they act as intended (Bitan-Friedlander, Dreyfus, & Milgrom, 2004). Teachers' 'ownership' of an innovation, and therefore its success in practice, develops only once teachers are prepared to invest mentally or physically in its implementation at classroom level (Ketelaar, Beijaard, Boshuizen, & Den Brok, 2012).

These points have been taken into consideration when conducting the research described in this thesis. We tried to establish a close collaboration between educational research and practice in several ways: by making an inventory of the needs and desires of teachers with regard to word problem solving; by using authentic word problems that arose in the regular classroom practice of teachers and students; by involving teachers in the design of an innovative word problem instruction; and by letting teachers implement the innovation in their own classroom practice.

Our primary goal was to give teachers ownership of the innovation by giving them the opportunity to adapt the innovation to their wishes. We also took into consideration how important it is that teachers get the feeling that they are in control of their own actions when implementing the innovation (i.e., agency; Bijeaard, 2009; Ketelaar et al., 2012). Therefore, teachers were encouraged to use their own explanations and elaborations while implementing the innovative instruction, and to implement the teaching intervention in a way that was compatible with their own teaching approach.

An innovation may be regarded to have been successfully introduced once teachers have adopted it, are able to and willing to implement it in their classes, and are confident in their ability to adapt the innovation to the needs and abilities of their students (Bitan-Friedlander et al., 2004). However, in spite of the close collaboration with the educational practice the implementation of the innovative word problem instruction, which played a central role in Chapter 7, was not completely successful. This was not the consequence of a top-down approach or a lack of collaboration, but because teachers did not completely master the skills necessary to implement the innovation. This was particularly remarkable because the teachers who implemented the innovative instruction had earlier reported that, after they had received training in the use of the innovation, they felt confident and competent in using it. This indicates that teachers might not always be able to critically assess their competence and behavior in their own teaching practice.

This highlights another task that might be considered part of educational researchers' domain, namely teaching (student) teachers and other educational practitioners to evaluate and observe their own skills and teaching behavior in order to become 'self-reflecting' teachers. In the past four years this has been one of my main tasks as a teacher educator; educating student teachers to become teachers who are curious, open-minded and self-critical. Educational researchers who work in teacher training should also make (student) teachers aware of developments in research and help them to evaluate the significance of these developments. In order to do this successfully, educational researchers need to have a thorough understanding of educational practice, and to keep themselves informed about the wishes and desires of teachers and the circumstances in which they do their work.

More cooperation between researchers and practitioners can, for example, be realized by promoting ‘design-based research’ or, maybe even more promising, by establishing ‘professional learning communities’ (PLC; Vescio, Ross, & Adams, 2008). In these PLCs a group of educators and researchers would meet regularly, share expertise, and work collaboratively to improve teaching skills and the academic performance of students. Initiatives such as the establishment of PLCs might be able to bridge the gap between educational research and educational practice. This could be an important step toward successfully fulfilling the main task of both educational researchers and teachers: the improvement of educational practice.