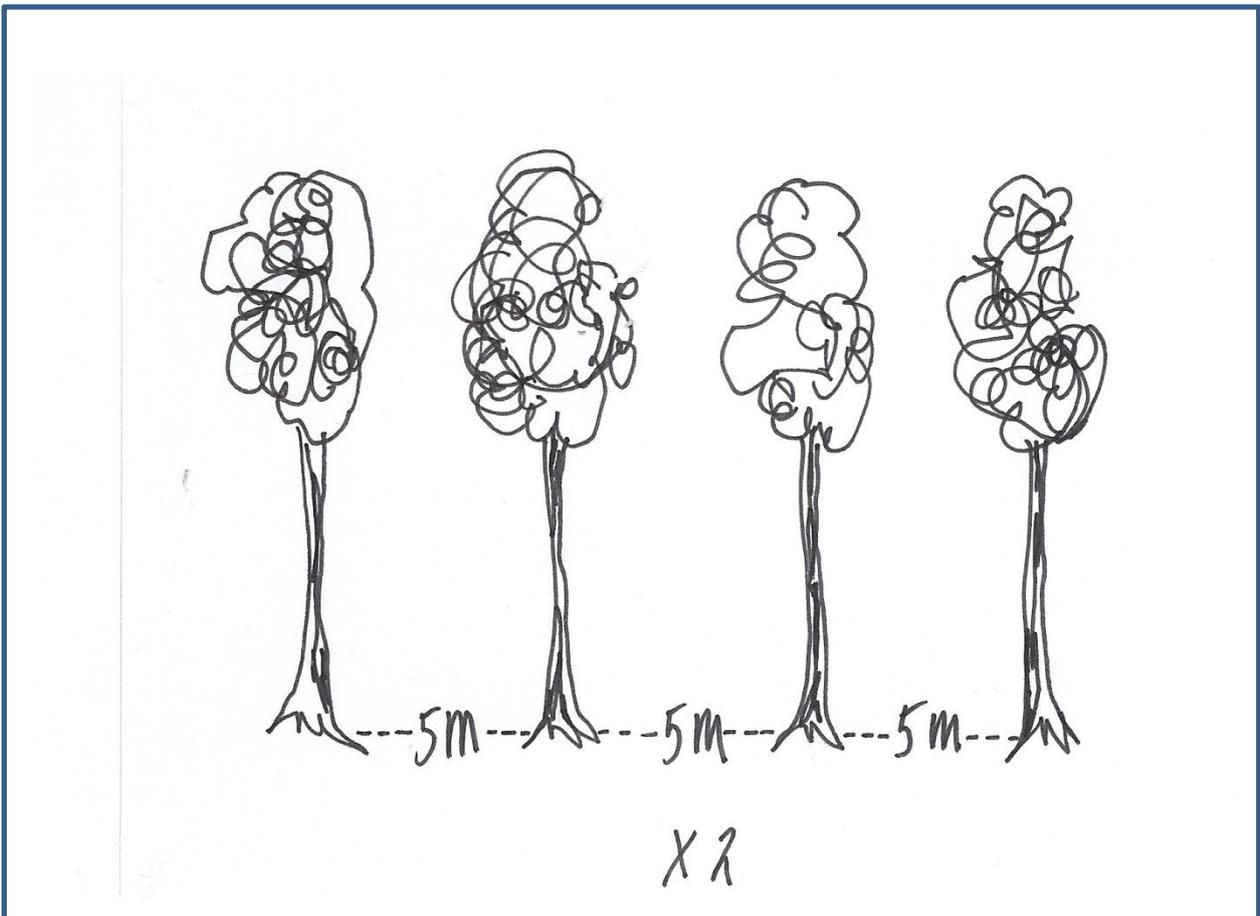


CHAPTER 1

General Introduction



At each of the two ends of a straight path, a man planted a tree and then every 5 meters along the path he planted another tree. The length of the path is 15 meters. How many trees were planted?

Introduction

In the last decades, mathematical word problem solving has gained much attention from both researchers and educational practitioners (Campell, 1992; Depaepe, De Corte, & Verschaffel, 2010; Hegarty, Mayer, & Monk, 1995; Hickendorff, 2011; Moreno, Ozogul, & Reisslein, 2011; Swanson, Lussler, & Orosco, 2013). The main focus of the scientific literature on this subject has generally been the word problem solving performances of elementary, middle school and undergraduate students and their use of superficial and/or sophisticated representation strategies (see Cummins, Kintsch, Reusser, & Weimer, 1988; Hegarty & Kozhevnikov, 1999; Pape, 2003; Van der Schoot, Bakker-Arkema, Horsley, & Van Lieshout, 2009; Verschaffel, De Corte, & Pauwels, 1992). However, previous studies provide limited insight into the exact nature of these representation strategies, and investigate the visual-spatial and semantic-linguistic components skills and abilities underlying word problem solving separately from each other. The research reported in the present thesis has therefore been designed to examine students' representation strategies in more detail, and to investigate the underlying components and skills in conjunction with each other.

The effectiveness of word problem solving instructions has been another main area of focus in past research (Jitendra et al., 2013; Jitendra & Star, 2012; Jitendra et al., 2009; Krawec, 2010, 2012). However, until now instructions have been generally executed by researchers in small groups of low ability students in special education. In contrast to previous studies, the research reported in the current thesis examines the role of the teacher while implementing a word problem solving instruction in his/her own regular classroom practice. This is important, given that mainstream schools are becoming more inclusive, and that a greater number of students attending them have mild to severe learning difficulties (Jitendra & Star, 2012; Sharma, Loreman, & Forlin, 2012).

Background

The findings of the studies that were conducted as part of the research presented in this thesis have implications for the practice of contemporary math education. Drawing clear recommendations

based on these findings could contribute to the improvement of mathematics learning and teaching in schools, particularly when it concerns mathematical word problem solving. Before stating the objectives of this thesis, I will give some background information regarding word problem solving and related factors.

Mathematical word problems and word problem solving

Word problems play a prominent role in both the educational practice of contemporary math approaches and in educational research. The term word problem is used to refer to any math exercise where significant background information on the problem is presented as text rather than in the form of mathematical notation. As word problems often involve a narrative of some sort, they are occasionally also referred to as *story problems* (Verschaffel, Greer, & De Corte, 2000). The literature generally makes a distinction between so-called *routine* and *non-routine* word problems (Pantziara, Gagatsis, & Elia, 2009; Schoenfeld, 1992). Routine word problems have a fixed problem structure and involve the application of routine calculations.

Routine word problems

Combine, change, and compare word problems are routine types of word problem that are commonly offered in elementary school.

In a combine word problem, reflected in the example below, a subset or superset must be computed given the information about two other sets. This type of word problem involves understanding part-whole relationships and knowing that the whole is equal to the sum of its parts (Cummins et al., 1988; Jitendra, 2002, Jitendra, DiPipi, & Perron-Jones, 2002).

[Combine word problem]:

Mary has 4 marbles. John has some marbles. They have 7 marbles altogether. How many marbles does John have?

Change word problems are routine word problems in which a starting set undergoes a transfer-in or transfer-out of items, and the cardinality of a start set, transfer set, or a result set must be computed given information about two of the sets (Cummins et al., 1988; Jitendra et al., 2013). In other words, a change word problem starts with a beginning set in which the object identity and the amount of the object are defined. Then a change occurs to the beginning set that results in an 'ending set' in which the new amount is defined (Jitendra, 2002).

[Change word problem]:

Mary had 8 marbles. Then she gave some marbles to John. Now Mary has 3 marbles. How many marbles did she give to John?

The last type of routine word problem that is investigated in many studies is a compare word problem. In compare word problems the cardinality of one set must be computed by comparing the information given about relative sizes of the other set sizes; one set serves as the comparison set and the other as the referent set. In this type of word problem, students often focus on relational terms like 'more than' or 'less than' to compare the two sets and identify the difference in value between the two sets (Cummins et al., 1988; Hegarty et al., 1995; Pape, 2003; Van der Schoot et al., 2009).

[Compare word problem]:

Mary has 5 marbles. John has 8 marbles. How many marbles does John have more than Mary?

Non-routine word problems

In contrast with routine word problems, non-routine word problems do not have a straightforward solution but require creative thinking and the application of a certain heuristic strategy to understand the problem situation and find a way to solve the problem (Elia, Van den Heuvel-

Panhuizen, & Kolovou, 2009). In other words, it is characteristic for non-routine word problems that they cannot be solved in a prescribed way. Solution strategies of non-routine word problems can, therefore, differ between each word problem that is solved. An example of a non-routine word problem is reported below.

[Non-routine word problem]:

A balloon first rose 200 meters from the ground, then moved 100 meters to the east, then dropped 100 meters. It then traveled 50 meters to the east, and finally dropped straight to the ground. How far was the balloon from its original starting point?

The solution of word problems

Whereas routine word problems like combine, change and compare problems, are frequently offered in the early grades of elementary school, students in the sixth grade are expected to solve a wide variety of non-routine word problems of increasing difficulty. Therefore, the research presented in this thesis is interested in both routine and non-routine word problems and examines students and teachers from early and later grades of elementary school.

Generally, the solution of word problems depends on two major phases: (1) *problem comprehension*, which involves the identification and representation of the problem structure of the word problem; and, (2) *problem solution*, which involves the determination of the used mathematical operations and the execution of these planned computations to solve the problem (Krawec, 2010; Lewis & Mayer, 1987).

A substantial amount of elementary school students has difficulties with solving word problems. This is not because of their inability to execute the planned mathematical computations, but a result of their difficulties with thoroughly understanding and representing the word problem text and distilling the correct mathematical operations to be performed (Carpenter, Corbitt, Kepner,

Lindquist, & Reys, 1981; Cummins et al., 1988; Krawec, 2010; Lewis & Mayer, 1987; Van der Schoot et al., 2009). Hence, mistakes in word problem solutions frequently occur in the problem comprehension phase, rather than in the problem solution phase.

Visualization

The external (i.e., a gesture or drawing with paper and pencil) or internal (i.e., mental) construction of a visual representation is thought to be a powerful tool for overcoming the difficulties in understanding the word problem text. According to Hegarty and Kozhevnikov (1999), two types of visual representations can be distinguished: *pictorial* and *visual-schematic* representations.

Children who create pictorial representations tend to focus on the visual appearance of the given elements in the word problem. These representations consist of vivid and detailed visual images (Hegarty & Kozhevnikov, 1999; Presmeg, 1997, see Figure 1). However, several studies have reported that the production of pictorial representations is negatively related to word problem solving performance (Ahmad, Tarmizi, & Nawawi, 2010; Hegarty & Kozhevnikov, 1999; Kozhevnikov, Hegarty, & Mayer, 2002; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003). An explanation for this finding is that children who make pictorial representations fail to form a coherent visualization of the described problem situation and base their representations solely on a specific element or sentence in the word problem text (Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003).

Visual-schematic representations, on other hand, do contain a coherent image of the problem situation hidden in the word problem, including the relations between the solution-relevant elements (Edens & Potter, 2008; Hegarty & Kozhevnikov, 1999; Kozhevnikov et al., 2002; Van Garderen & Montague, 2003, see Figure 2). This explains why, in contrast to the production of pictorial representations, the production of visual-schematic representations is positively related to word problem solving performance (Hegarty & Kozhevnikov, 1999; Van Garderen, 2006; Van Garderen & Montague, 2003).

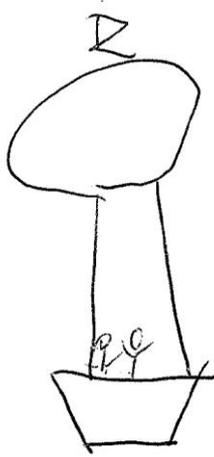


Figure 1. An example of a pictorial representation (of word problem example 2)

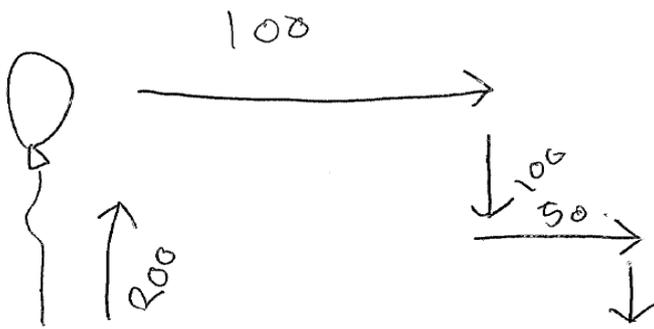


Figure 1. An example of a visual-schematic representation (of word problem example 2)

The importance of visual-spatial skills in word problem solving

The scientific literature shows that spatial ability is a basic ability underlying mathematical word problem solving (e.g., Blatto-Vallee, Kelly, Gaustad, Porter, & Fonzi, 2007; Edens & Potter, 2008; Hegarty & Kozhevnikov, 1999). Spatial ability is related to students' word problem solving performance, as well as to the components/factors that influence this performance. Spatial skills are, for example, closely related to the production of visual-schematic representations, and children with good spatial skills have been found to be better able to make visual-schematic representations than children with poor spatial skills (e.g., Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van Garderen,

2006; Van Garderen & Montague, 2003). These visual-schematic representations are in turn a factor affecting word problem solving performance. Although there are many definitions of what spatial ability is, it is generally accepted to be related to skills involving the retrieval, retention and transformation of visual information in a spatial context (Velez, Silver, & Tremaine, 2005). Especially the involvement of a specific spatial factor, i.e., *spatial visualization*, in making coherent visual-schematic representations has been made clear by several authors (Hegarty & Kozhevnikov, 1999; Krawec, 2010; Van Garderen, 2006; Van Garderen & Montague, 2003).

Besides indications that spatial ability plays an *indirect* role in word problem solving via the production of visual-schematic representations, several authors also report a *direct* relation between spatial ability and word problem solving (Battista, 1990; Blatto-Vallee et al., 2007; Booth & Thomas, 1999; Edens & Potter, 2008; Geary, Saults, Liu, & Howard, 2000). For example Blatto-Vallee et al., (2007), showed that spatial abilities explained almost 20% of unique variance in word problem solving performance.

Spatial ability and students' constructive play activities

Another, somewhat different way in which spatial ability relates to word problem solving is through its role in students' constructive play activities. Constructive play generally involves the manipulation, construction and motion of objects in space (i.e., rotating) (Caldera, Culp, O'Brian, Truglio, Alvarez, & Huston, 1999; Mitchell, 1973; Pomerleau, Malcuit, & Séguin, 1990). Constructive play activities that are related to performance on spatial tasks are Lego, Blocks, and jigsaw puzzles (Caldera et al., 1999; Levine, Ratkiff, Huttenlocher, & Cannon, 2012; Mitchell, 1973; Pomerleau et al., 1990).

However, previous studies have not reported a direct relation between constructive play and students' word problem solving performance. To fill this gap, the research presented in this thesis tries to gain more insight in the specific relation between spatial ability, word problem solving, and constructive play. Specifically, we investigated the mediating role of spatial ability in the relation between constructive play and word problem solving.

The importance of semantic-linguistic skills in word problem solving

Besides (visual-)spatial skills, several previous studies showed that semantic-linguistic skills (i.e., reading comprehension) are also closely related to word problem solving (Pape, 2004; Vilenius-Tuohimaa, Aunola, & Nurmi, 2008). General reading comprehension abilities are found to be important in dealing with semantic-linguistic word problem characteristics, such as the semantic structure of a word problem, the sequence of the known elements in the problem text, and the degree to which the semantic relations between the given and the unknown quantities of the problem are stated explicitly (De Corte, Verschaffel, & De Win, 1985). All these word problem characteristics have been shown to have an effect on children's solution processes (e.g., De Corte et al., 1985; De Corte & Verschaffel, 1987; Sjøvik, Frostrad, & Heggberget, 1999).

Word problem solving instruction

Many studies have concluded that students experience severe difficulties in solving word problems and accentuated the importance of skills that help students identify and represent the word problem text to generate a correct word problem solution (e.g. Carpenter et al., 1981; Cummins et al., 1988; Van Garderen, 2006). In spite of these findings, there is still a lack of instructional programs that take an evidence-based approach to word problem solving and are adapted to the educational practice of contemporary math education.

Cognitive Strategy Instruction (CSI), an instructional approach that focuses on explicit instruction in cognitive and metacognitive strategies that help students identify and represent the problem structure in order to improve their word problem solving performance, seems however promising (e.g., Jitendra, DiPipi, & Perron-Jones, 2002, Jitendra et al., 2013; Jitendra, & Star, 2012; Jitendra et al., 2009; Krawec, 2012; Krawec, Huang, Montague, Kressler, & Melia de Alba, 2013; Montague, Enders, & Dietz, 2011; Montague, Warger, & Morgan, 2000). A commonly investigated example of a CSI is schema-based instruction (SBI, Jitendra et al., 2002, 2009, 2012, 2013). Schema-

based instruction moves away from keywords and superficial problem features and is more focused on helping children find the underlying problem structure. In SBI students are taught to identify and represent the problem structures of certain word problem types (by constructing a visual-schematic representation or diagram), and are encouraged to reflect on the similarities and differences between these problem types. The implementation of SBI in the curriculum of contemporary math education appears, however, to be challenging.

An alternative instructional approach, focused on the use of cognitive strategies, has been developed by Montague (2003) and is known as the *Solve It!* instructional program. The Solve It! program is a more heuristic approach that teaches students how to: (a) read the problem for understanding; (b) paraphrase the problem by putting it into their own words; (c) visualize the problem; (d) set up a plan for solving the problem; (e) compute; and (f) verify the solution of the problem.

However, like SBI also the Solve It! method has some important restrictions. Firstly, the cognitive step in which students are requested to visualize the word problem seems to be defined too generally. Findings show that it is incorrect to assume that a student knows exactly what pictures to draw, when, and under what circumstances, and for which type of problems (Jitendra, Griffin, Haria, Leh, Adams, & Kaduvetoor, 2007; Jitendra et al., 2009). Another problem with this step is that previous research showed that a visual representation should meet certain requirements (i.e., involve the correct relations between solution-relevant elements) and that not all types of visual representations facilitate the solution process of word problems (Krawec, 2010, Van Garderen & Montague, 2003). Secondly, the effectiveness of the Solve it! program has generally been examined in small groups of children with learning and mathematical disabilities (Jitendra et al., 2002, 2013; Krawec, 2010, 2012; Montague et al., 2000), and not in a regular classroom setting. In addition, the Solve It! method has only been implemented by researchers and not by teachers. Surprisingly, there is currently no comparable instructional support available for teachers in mainstream classrooms. This is an important omission, given that mainstream schools are becoming more inclusive, and that

a greater number of students attending them have mild to severe learning difficulties (Jitendra & Star, 2012; Sharma et al., 2012). It would, therefore, help teachers if they had instructional approaches at their disposal that have been designed to teach skills important for word problem solving.

Bearing this state of affairs in mind this thesis examines the introduction of an innovative approach to the instruction of word problem solving in mainstream classrooms, and examines how teachers implemented that approach, with a focus on their use of visual representations.

Thesis outline

Objectives

Research on word problem solving is often focused on the performance of students and not on their comprehension of the word problem text. Difficulties with word problem solving can, however, often be ascribed to problems with the correct understanding of the word problem text. The research presented in this thesis is, therefore, focused on the component processes and skills that underlie the successful comprehension of word problems. In particular, we examined both students' use of visual representations and the quality of these visual representations. Moreover, we were interested in the extent to which different types of visual representation increase or decrease the chance of solving a word problem correctly. To this end, this thesis sets out to achieve the following two objectives.

The first objective is to examine students' performances, notably the extent to which students use different types of visual representations, and the role that spatial and semantic-linguistic skills play in the solving of routine and non-routine word problems in early (second) and later (sixth) grades of elementary school.

The second objective of this thesis is to investigate how teachers implement an innovative instructional approach – based on the didactical use of visual-schematic representation – in their own classroom teaching practice. This instructional approach requires teachers to use visual-

schematic representations that visualize the problem structure in a diverse and flexible way as well as to vary the kinds of representations in a way that suits problem characteristics. Moreover, they are expected to model the representation process transparently, correctly and completely, as well as to construct visual representations that correctly and completely depict the relations between all the components relevant to the solution of the problem.

Approach

To achieve the objectives of this thesis we conducted five cross-sectional studies in the field of educational psychology in which both second ($N = 47$) and sixth grade ($N = 128$) elementary school students were examined. In addition, we conducted one study in which we investigated the way in which eight mainstream – sixth grade – teachers implemented a teaching intervention for supporting non-routine word problem solving.

Furthermore, we conducted a feasibility study in which we examined four second-grade students who performed poorly in word problem solving. The feasibility study has been included as an Appendix to the scientific part of the thesis: it has been included in order to give an illustration of a word problem solving instruction which could be suitable in the early grades of elementary school.

Chapter overview

The first three studies conducted for the research presented in this thesis focus on two component skills and two basic abilities of word problem solving. The component skills examined are: 1) the production of visual-schematic representation, and 2) relational processing (i.e., deriving the correct relations between solution-relevant elements of the word problem text base). The two basic abilities examined are: 1) spatial ability, and 2) reading comprehension. These component skills and the underlying basic abilities related to them belong to two different processing domains: the visual-spatial and the semantic-linguistic domain.

In **Chapter 2** a study is reported in which the two component skills and two basic abilities were investigated in one theoretical model (see Figure 3), in order to examine the extent to which they explain unique variance in sixth graders' word problem solving performances.

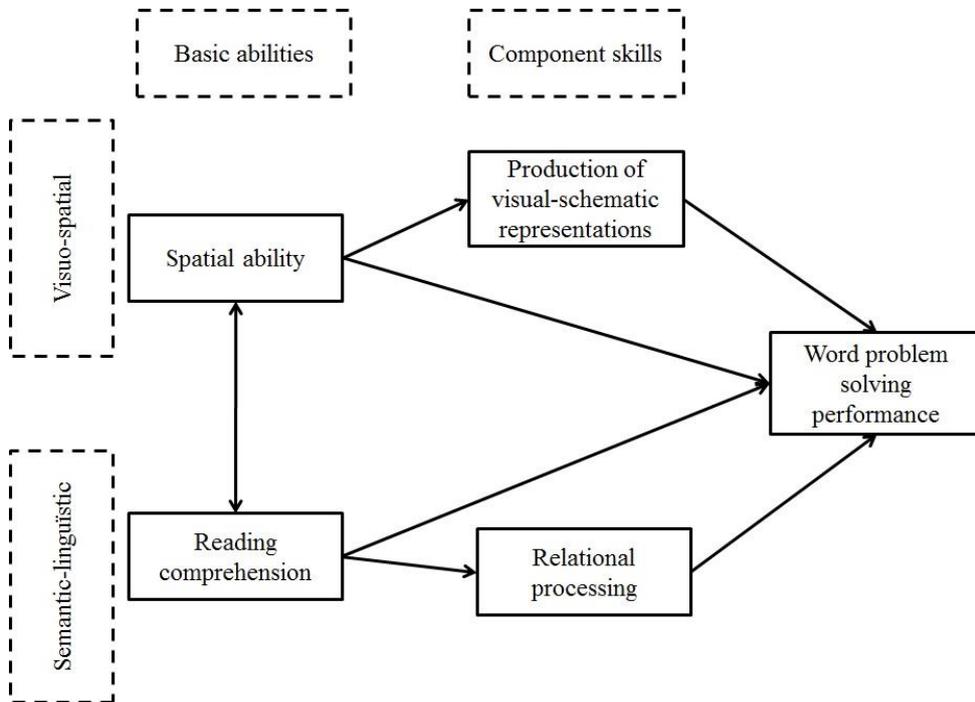


Figure 3. Path model with all hypothesized pathways

The purpose of the study reported in **Chapter 3** was to demonstrate that word problem solving instruction in Realistic Math Education seems to pay too little attention to the teaching of semantic-linguistic skills (i.e., reading comprehension) that allow sixth grade students to handle linguistic complexities in a word problem. The study attempted to show the importance of semantic-linguistic skills for the solution of semantically complex and less complex inconsistent compare word problems, in a group of successful and less successful sixth grade word problem solvers following the Dutch Realistic Math curriculum.

In **Chapter 4** the findings are examined of a study that investigated the importance of different types of visual representation, spatial ability, and reading comprehension on the word

problem solving performance of sixth grade students. In contrast to previous studies, an item-level approach was used in this study rather than a test-level approach.

This change in statistical modeling generated a more thorough and sophisticated understanding of the process and enabled us to examine if and to what extent the production of a specific kind of visual representation affected the chance of successfully solving the word problem of which the visual representation had been made. Moreover, we wanted to examine if we were able to reproduce the findings of test-level analysis with regard to the importance of spatial ability and reading comprehension, by using an item-level analysis. This made it possible to identify any level of analysis discrepancies.

Chapter 5 is focused on the importance of spatial ability, and the role it plays in the relation between (early) constructive play activities and word problem solving performance of sixth grade elementary school students. The aim of the study described in this chapter was to investigate whether spatial ability acted as a mediator in the relation between constructive play and mathematical word problem solving performance in 128 sixth grade elementary school children.

The studies described in chapters 2 to 5 focused on the strategies, solution processes and performances on word problems of students in higher grades of elementary school (i.e., grade 6). Word problems are, however, already offered in the first grades of elementary school. Moreover, scientific research has shown that students from first and second grade of elementary school already experience difficulties solving word problems. Therefore, in **Chapter 6** a study that investigated the word problem solving performances of second grade elementary school students is reported. The findings of this study reveal a plausible reason for second grade students' differing performances on three commonly investigated routine word problem types, namely combine, change and compare problems.

The studies described in chapters 2 to 6 of this thesis are focused on the difficulties experienced by students in solving word problems. Their findings accentuate the importance of skills that help students to identify and represent the word problem text correctly in order to generate a

deep understanding of the problem situation. Evidence-based word problem solving instructional programs that could help develop these skills are, however scarce, limited in scope and often not adapted to the educational practice of mainstream classrooms. In the study described in **Chapter 7** this is addressed by examining teachers' use of and their competence in making visual-schematic representations while executing an innovative word problem solving instruction in their own classrooms. The study was performed in the context of a teaching intervention that involved eight teachers who felt confident about teaching mathematics and using visual representations, and who were motivated to participate in and contribute to research in this area.

Chapter 8 reflects on the findings from the series of studies conducted as part of this research on word problem solving, and examines them in light of the objectives of this thesis. Furthermore, the implications of these findings for educational practice are considered and a list of recommendations for teacher training and teacher professionalization is presented and discussed. Finally, the main findings of this study are discussed in a broader perspective. That is, in the perspective of the current debate on the importance of building bridges between educational research and the educational practice. Central to this debate is the question whether the outcomes of educational research can be directly implemented in the classroom practice of students and teachers.

As already indicated above, besides the six studies discussed as part of the research presented in this thesis, **Appendix I** contains a feasibility study in which a word problem instruction based on the principles of the Solve it! method and schema-based instruction is evaluated. The aim of this feasibility study was to experiment with a word problem solving instruction, examine the extent to which it was (un)successful, and see whether second grade students were able to execute the cognitive steps of this instruction and improve their word problem solving performances on combine, change and compare problems. Although the feasibility study does not have significant scientific value, it does provide an example of a word problem solving instruction that can be given to students who are still in the early grades of elementary school.