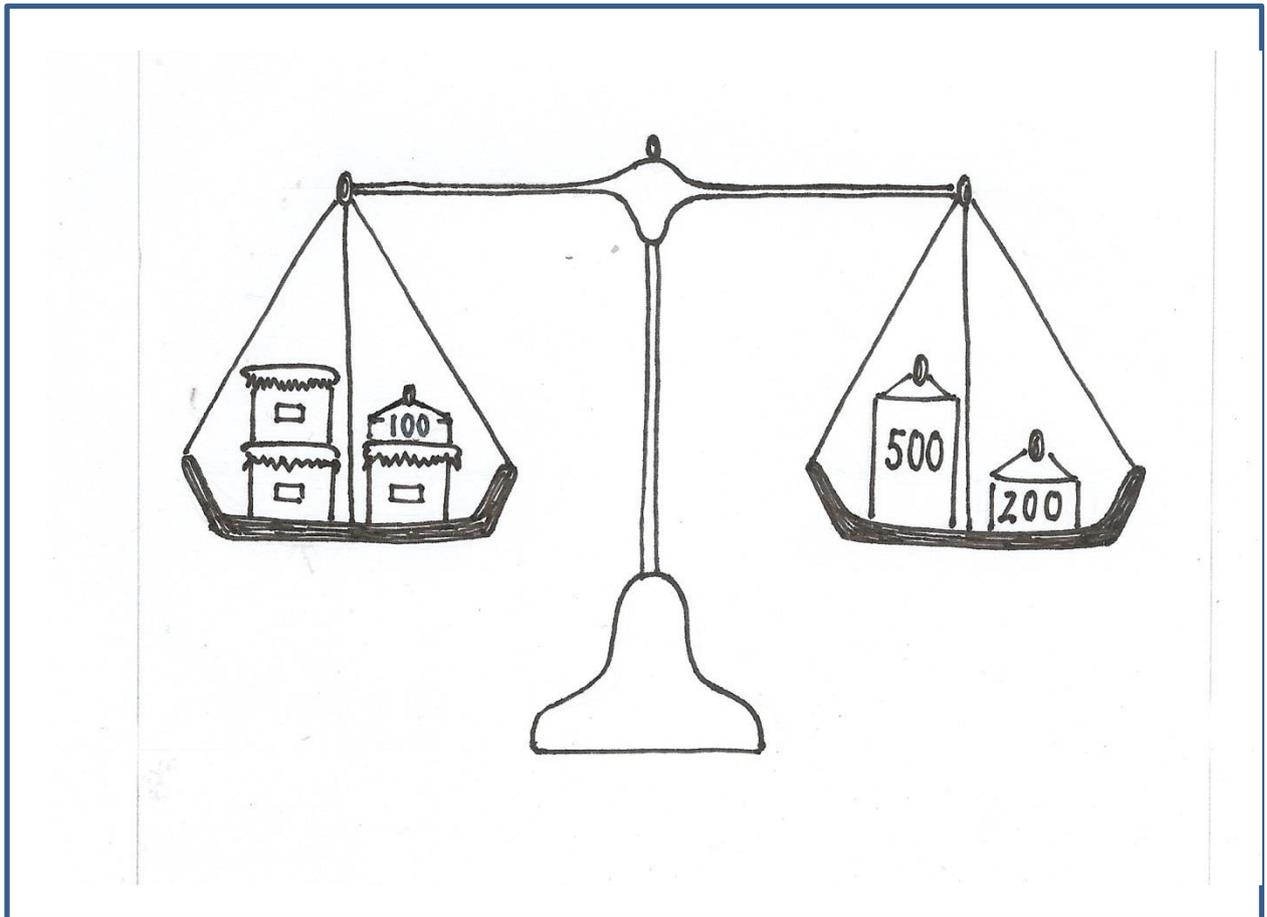


APPENDIX I

Teaching low performing second grade students to solve combine, change and compare mathematical word problems: A feasibility study in four subjects

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(submitted)



On one side of a scale there are three pots of jam and a 100 g weight. On the other side there are a 200 g and a 500 g weight. The scale is balanced. What is the weight of a pot of jam?

Abstract

The solution of combine, change and compare mathematical word problems causes many difficulties in especially young elementary school students. The principles underlying instructional programs like *Solve It!* and *schema-based instruction* could prove helpful in mastering these types of word problems.

This feasibility study examined four second-grade students who were less successful word problem solvers. These students received protocolled instruction during a five-week intervention period. The effectiveness of the word problem solving instruction was reported by comparing students' performances on the combine, change and compare problems before and after the intervention period, as well as by examining whether they executed the solution steps of the instruction correctly.

The results of the pre- and post-test comparison showed that the total word problem solving performance of all four students had improved. However, this improvement was not always visible in all three types of word problems. The study showed that the extent to which the solution steps had been executed correctly was a determining factor for the correct solution of the word problems.

While our findings do not imply that every student will benefit from a word problem instruction like the one we investigated, this feasibility study does provide important insights with regard to varying ways in which a word problem solving instruction can influence the solution strategies and performances of students who perform poorly on mathematical word problems.

Keywords: combine word problems; change word problems; compare word problems; solution strategies

Introduction

[Word problem example]

Mary has 9 marbles. She has 4 marbles more than John. How many marbles does John have?

Tim, a seven-year-old boy who is in the second grade of elementary school, has difficulties with solving word problems like the one that is given in the example above. While solving these word problems, Tim often uses an impulsive, superficial solution strategy. Notably, he only focuses on selecting the presented numbers (9 and 4) and identifying the relational keywords (more than), which subsequently form the basis for his mathematical calculations. Tim's strategy often leads to an incorrect answer to the word problem. In this situation, Tim performed an addition operation where a subtraction operation was required, that is $9 + 4 = 13$ instead of $9 - 4 = 5$. The incorrect answer is not the result of a lack of calculation ability, but a result of a problem with deeply and correctly understanding the word problem text.

Mathematical word problem solving plays a prominent role in the curriculum of contemporary approaches to teaching mathematics (see Barnes, 2005; Elia, Van den Heuvel-Panhuizen, & Kovolou, 2009; Gravemeijer & Doorman, 1999; Van den Heuvel-Panhuizen, 2003). The solution of a word problem generally depends on two major phases: (1) *problem representation*, and (2) *problem solution*. The problem representation phase involves the identification and representation of the problem structure of the word problem. The identification and representation of the problem structure facilitate the correct understanding of the word problem text and help distill the mathematical operation(s) that should be performed. In the problem solution phase, on the other hand, the mathematical operations to be used are identified and the planned computations are executed to solve the problem (Krawec, 2010; Lewis & Mayer, 1987). Hence, errors in word problem solutions frequently occur in the problem representation phase, rather than in the problem solution

phase. Improving students' problem representation skills is therefore of pivotal importance in order to help them master these word problems.

This article describes a feasibility study in which a word problem instruction was used to help students overcome their difficulties with understanding and representing the word problem text. It was set up as a multiple single-case study involving four subjects who participated in a five-week educational intervention course. The effectiveness of the instruction was reported by comparing students' performances before and after the intervention period, as well as by examining whether they executed the solution steps of the instruction correctly.

All of the subjects were second grade students who performed poorly as word problem solvers. Students from the second grade of elementary school were used as subjects in this study because difficulties with solving word problems already arise at an early age (see the example of Tim mentioned above, Cummins, Kintsch, Reusser, & Weimer, 1988). In our view, the current article presents convincing evidence that early intervention in the first grades of elementary school is imperative in order to address the difficulties experienced by young children. Our findings show that dedicated word problem solving instruction by means of a direct instruction method can play a pivotal role in this respect.

In the remainder of this introduction we provide some background information concerning the two instructional programs which were used as the basis of in our study: the *Solve it!* method and *schema-based instruction*.

Solve it! and schema-based instruction

Many researchers in the domain of mathematical word problem solving have documented the fact that students of all ages experience difficulties with solving a variety of types of word problems (e.g., Cummins et al., 1988; Hegarty, Mayer, & Green, 1992; Hegarty, Mayer, & Monk, 1995; Van der Schoot, Bakker-Arkema, Horsley & Van Lieshout, 2009). Yet, there is a scarcity of effective evidence-based instructional programs that address both the identification and representation of the problem

structure (phase 1, see above), as well as the execution of the planned mathematical operations (phase 2, see above).

Instructional programs focusing on the explicit instruction of cognitive strategies to help students identify and represent the problem structure of a word problem seem to be effective (Jitendra et al., 2013; Jitendra & Star, 2012; Jitendra et al., 2009; Krawec, 2012). An example of this type of instruction is the *Solve It!* method (Montague, 2003). This is a heuristic instructional approach that teaches both elementary and middle school students how to: (a) read the problem for understanding; (b) paraphrase the problem by putting it into their own words; (c) visualize the problem (i.e., by constructing an external or internal visual-schematic representation); (d) set up a plan for solving the problem; (e) compute; and (f) verify the solution of the problem (Montague, Warger & Morgan, 2000).

Another commonly investigated example of a word problem solving instruction is *schema-based instruction* (SBI, developed by Jitendra et al.,). Schema-based instruction is generally provided in the early grades of elementary school and uses schema training to help students see the underlying (mathematical) structure of the word problem. Students are taught to recognize the similarities and differences between types of word problems and to identify and represent their problem structures (Jitendra, George, Sood, & Price, 2010). Schema-based instruction is particularly prescriptive in nature, because visual representations of the problem structure of word problems are provided to the children. However, several studies have recently shown that it is more effective to teach students to construct their own visual-schematic representations, instead of providing them with the representations (Van Dijk, Van Oers, & Terwel, 2003; Van Dijk, Van Oers, Terwel, & Van den Eeden, 2003a). Moreover, SBI has been shown to be difficult to master for low achieving students (Jitendra et al., 2002, 2013).

The aim of the present study is to describe and evaluate an instructional approach primarily based on the principles of the *Solve it!* method and of SBI. An important ‘technical’ element of the approach taken by us is that the students should be trained to identify and construct the visual

representations themselves, instead of being provided with visual representations by teachers. Specifically, this approach addresses visual representations of the problem structures of the three types of word problems that are frequently offered in the first grades of elementary school, namely combine, change and compare word problems.

Methods

Participants and instruments

Four second-grade students (three boys *Hugo, Peter, Tim* and a girl *Lisa*) took part in the study. They attended a mainstream suburban elementary school in the Netherlands and were native Dutch speakers. All students were healthy and their intelligence was in the normal range. Table 1 shows students' age and their performances on the nationwide standardized norm-referenced CITO (Institute for Educational Measurement; www.cito.nl) Mathematics and Technical Reading test. According to these norms, level A corresponds to the highest 25% of the norm-referenced population, level B to the above-average 25%, level C to the below-average 25%, and level D and E together to the lowest 25%. The four students were selected on the basis of their performance on a nine-item *word problem pre-test* (Cronbach's $\alpha = .82$). This test included three combine, three change and three compare problems (see Table 2). The word problem items were presented on a different page and administered by the teacher in a session – attended by the four subjects - of approximately 30 minutes. Each word problem was read out loud twice by the teacher to control for differences in decoding skills. After reading the word problem, students had to solve the word problem within three minutes and during this time the teacher did not speak to the student. An examination of the number of word problems completed (see Table 1) showed that all four students experienced significant difficulties solving them. In contrast to their classmates (who solved the most of the nine word problems correctly; $M = 8.53$, $SD = 1.23$), the four research subjects solved less than half of the nine word problems items correctly.

After the intervention period a post-test with nine similar word problems was administered. Although the problem structure of these word problems was identical to the word problems that were included in the pre-test, we adjusted the figures in order to prevent the occurrence of a learning effect. The female teacher who executed all testing and instruction was 28 years old and had three years of experience in teaching second grade students. She had also obtained her master degree in Educational Sciences.

Table 1. *Age, Mathematical and Technical Reading achievement and pre-test score of the four research subjects*

Variable	Student			
	Hugo	Peter	Tim	Lisa
Age (months)	84	90	94	92
Achievement				
Mathematics ^a	B	B	C	B
Technical Reading ^a	E	D	B	C
Pre-test score	4/9	0/9	2/9	4/9

Note: ^a Norm scores on the nationwide standardized CITO (Institute for Educational Measurement) Mathematics & Technical reading test (2012). Between brackets: A= high 25%, B = above average 25%, C= average 25%, D = below average 25%, E = low 25%.

Table 2. *The nine items of the word problem solving pre-test (taken from Cummins et al., 1988)*

<i>Combine word problems</i>
1. Mary has 2 marbles. John has 5 marbles. How many marbles do they have altogether?
2. Mary has 4 marbles. John has some marbles. They have 7 marbles altogether. How many marbles does John have?

3. Mary and John have 8 marbles altogether. Mary has 7 marbles. How many marbles does John have?

Change word problems

1. Mary had 3 marbles. Then John gave her 5 marbles. How many marbles does Mary have now?

2. Mary had 2 marbles. Then John gave her some marbles. Now Mary has 9 marbles. How many marbles did John give to her?

3. Mary had some marbles. Then John gave her 3 marbles. Now Mary has 5 marbles. How many marbles did Mary have in the beginning?

Compare word problems

1. Mary has 5 marbles. John has 8 marbles. How many marbles does John have more than Mary?

2. Mary has 3 marbles. John has 4 marbles more than Mary. How many marbles does John have?

3. Mary has 4 marbles. She has 3 marbles less than John. How many marbles does John have?

Intervention materials

The intervention was a word problem solving instruction primarily based on the Solve It! instruction program (Montague, 2003) and on SBI (Jitendra, DiPipi, & Perron-Jones, 2002, Jitendra et al., 2009; Jitendra & Star, 2012). Specifically, the solution steps that were part of both instructional methods were merged and adjusted in such a way that they were understandable for second grade students and followed the solution process of three different types of word problems. This resulted in five solution steps that taught students to:

Step 1: READ the word problem for understanding: Students were taught to read each sentence of the word problem text critically and not only look for numbers and keywords, like more than, times, just as much, etcetera;

Step 2: VISUALIZE the word problem: Students were taught to identify and externally represent the problem structures of the combine, change and compare word problem type (i.e., the production of a visual-schematic representation);

Step 3: Add a QUESTION MARK to the visual-schematic representation to indicate the variable that has to be calculated;

Step 4: COMPUTE the required operation; and

Step 5: DRAW A CIRCLE around the variable that had to be calculated in order to check if the required solution was reported.

Procedure: The execution of the solution steps of the instruction

Over the course of five weeks, the four students were taught to use these five solution steps to improve their solution strategies and performances. Each of the ten sessions that were offered to the students (in two group sessions of 30 minutes per week) included teacher-mediated instruction that addressed the use of the solution steps. The amount and intensity of the instructional support of the teacher was gradually faded within and across the sessions. The way in which the instruction for each of the three types of word problems was offered is elaborated below.

The solution of combine word problems (week 1 of the instruction)

In the first week the five solution steps of the instructional approach were introduced by teaching the students to solve the type of problem known as a combine word problem (see word problem example 1).

[Word problem example 1]

Mary has 3 marbles. John has 5 marbles. How many marbles do they have altogether?

In a combine word problem a subset or superset must be computed given the information about two other sets. This type of problem involves understanding part-whole relationships and knowing that the whole is equal to the sum of its parts (Cummins et al., 1988; Jitendra, 2002, Jitendra et al., 2002).

The five solution steps of the instruction to solve word problem example 1 are offered to the students in the following way:

Step 1: The combine problem is read aloud twice by one of the four students.

Step 2: Students are taught to visualize the problem structure of the combine problem by making an external visual representation.

The three marbles that Mary has are drawn first (see Figure 1):

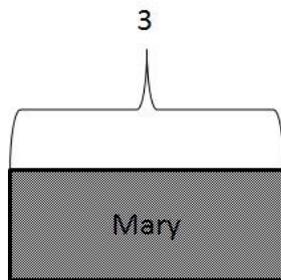


Figure 1. Step 2: VISUALIZE the marbles that Mary has

Next, the five marbles that John has are drawn (see Figure 2):

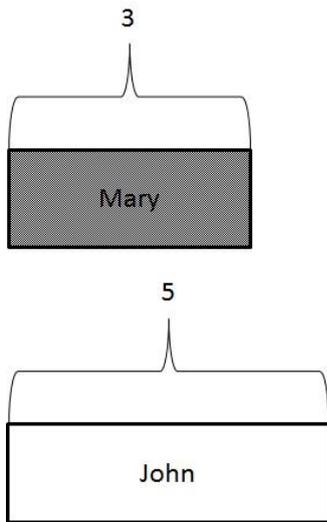


Figure 2. Step 2: VISUALIZE the marbles that John has

Step 3: A question mark is added to the visual-schematic representation to indicate the 'unknown' value that should be calculated (see Figure 3):

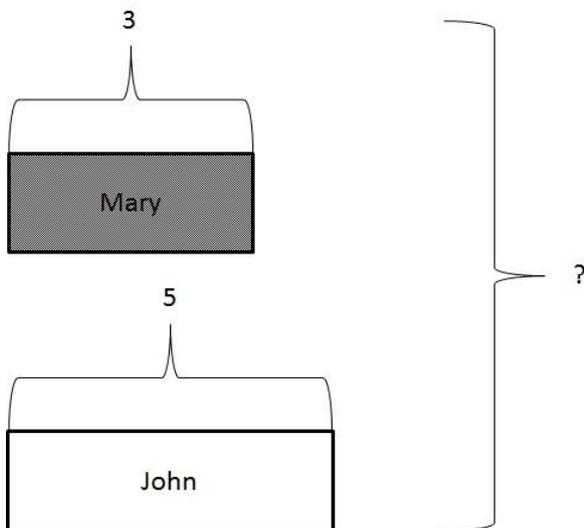


Figure 3. Step 3: Add a QUESTION MARK to the visual-schematic representation to indicate the variable that has to be calculated

Step 4: The required mathematical operation is written down ($3 + 5 =$) and solved ($3 + 5 = 8$).

Step 5: A circle is drawn around the 'unknown' value (i.e., 8), to be sure that the required answer is reported (see Figure 4).

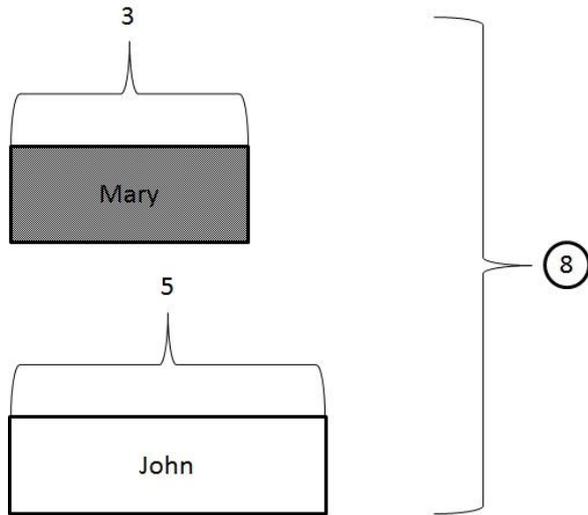


Figure 4. Step 5: DRAW A CIRCLE around the variable that had to be calculated

The solution of change word problems (week 2 of the instruction)

In the second week of the intervention period the four students were trained to solve change word problems (see word problem example 2).

[Word problem example 2]

Mary had 2 marbles. Then John gave her some marbles. Now Mary has 9 marbles. How many marbles did John give to her?

A change problem starts with a beginning set in which the object identity and the amount of the object are defined. Then a change occurs to the beginning set that results in an 'ending set' in which the new amount is defined (Jitendra, 2002). The solution of a change problem is instructed in the following way:

Step 1: The change problem was read aloud twice by the one of the students.

Step 2: Students are taught to visualize the problem structure of the change problem by making an external visual representation.

The two marbles that Mary has are drawn first (see Figure 5):

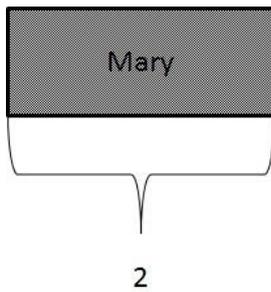


Figure 5. Step 2: VISUALIZE the marbles that Mary has

Next, the marbles that John gave to Mary are drawn (see Figure 6):

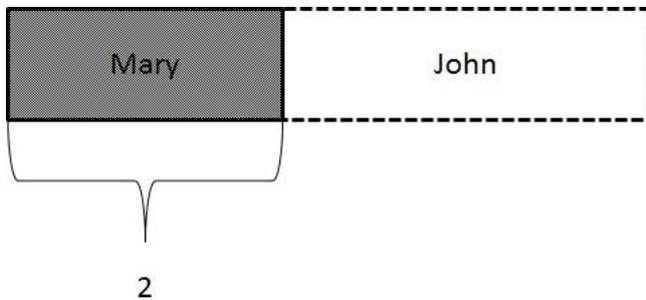


Figure 6. Step 2: VISUALIZE the marbles that John gave to Mary

Finally, the total amount of marbles is added to the visual-schematic representation (see Figure 7).

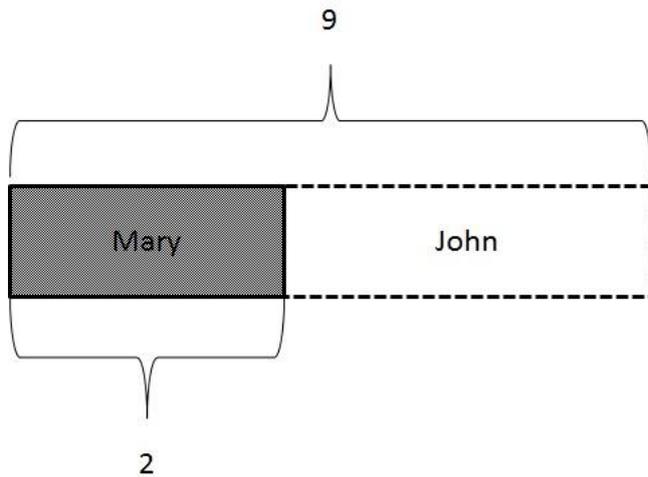


Figure 7. Step 2: VISUALIZE the amount of marbles that Mary and John have

Step 3: A question mark is added to the visual-schematic representation to indicate the 'unknown' value that should be calculated (see Figure 8):

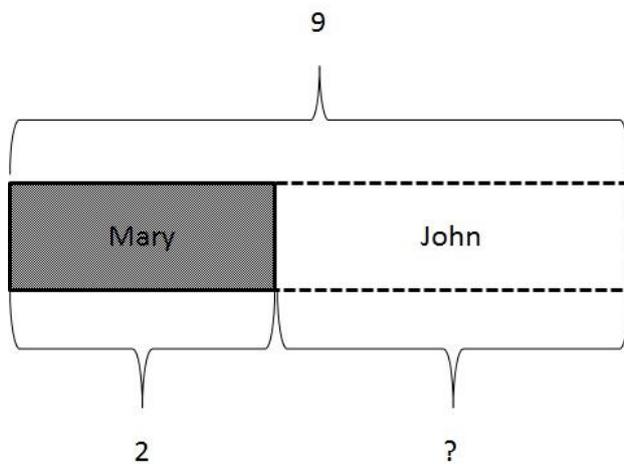


Figure 8. Step 3: Add a QUESTION MARK to the visual-schematic representation to indicate the variable that has to be calculated

Step 4: The mathematical operation is distilled from the visual representation, written down ($2 + ? = 9$) and solved ($2 + 7 = 9$).

Step 5: A circle is drawn around the 'unknown' value (i.e., 7), to be sure that the required answer is reported (see Figure 9).

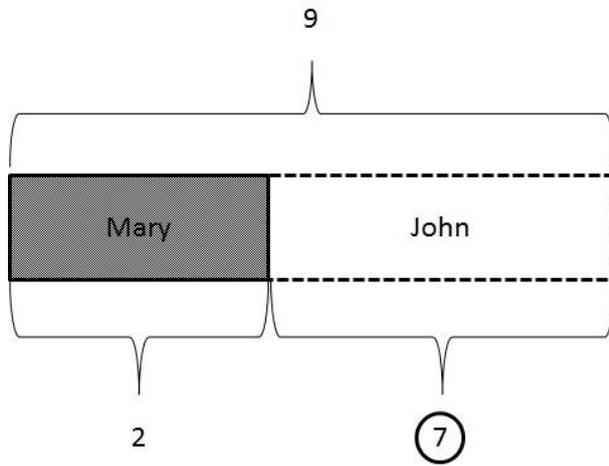


Figure 9. Step 5: DRAW A CIRCLE around the variable that had to be calculated

The solution of compare word problems (week 3 of the instruction)

In the third week of the intervention period the four students were trained to solve compare word problems (see word problem example 3).

[Word problem example 3]

Mary has 5 marbles. John has 8 marbles. How many marbles does John have more than Mary?

In compare problems the cardinality of one set must be computed by comparing the information given about relative sizes of the other set sizes; one set serves as the comparison set and the other as the referent set. The solution of a compare problem is instructed in the following way:

Step 1: The compare problem was read aloud twice by the one of the students.

Step 2: Students are taught to visualize the problem structure of the compare problem by making an external visual representation.

The five marbles that Mary has are drawn first (see Figure 10):

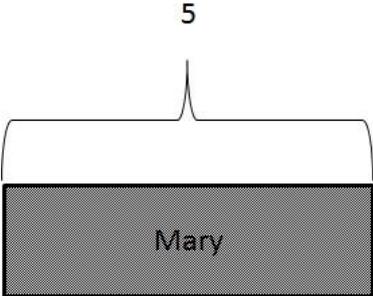


Figure 10. Step 2: VISUALIZE the marbles that Mary has

Next, the 8 marbles that John has are drawn (see Figure 11):

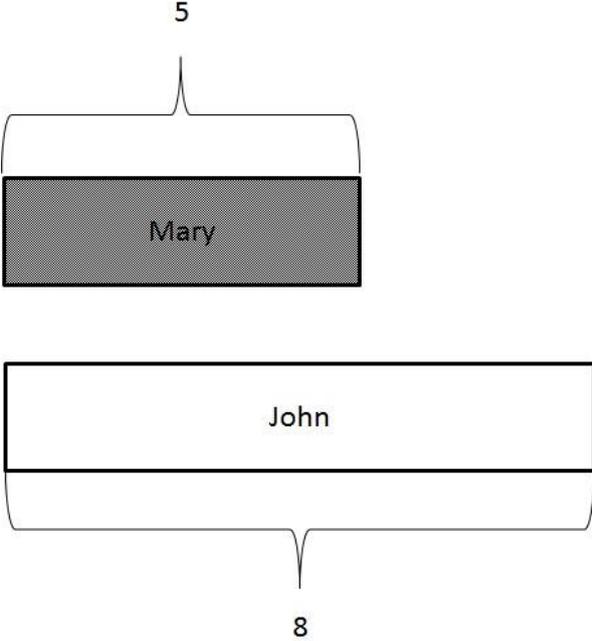


Figure 11. Step 2: VISUALIZE the marbles that John has

Step 3: A question mark is added to the visual-schematic representation to indicate the 'unknown' value that should be calculated (see Figure 12):

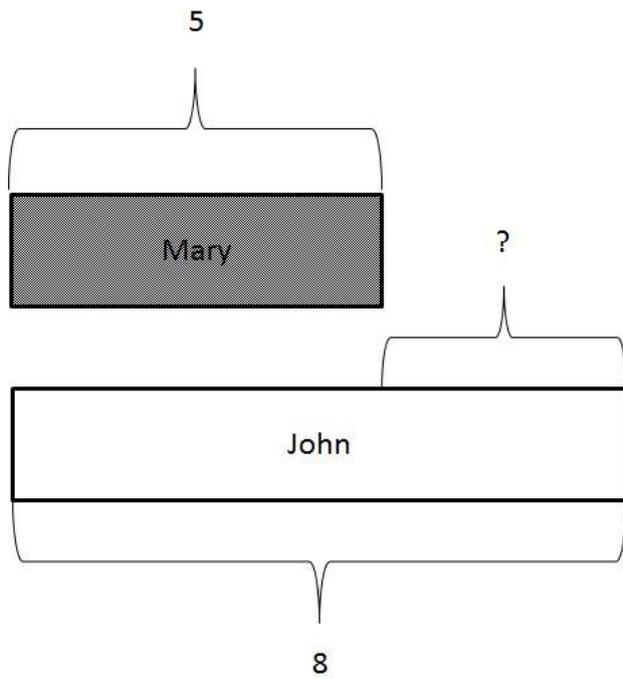


Figure 12. Step 3: Add a QUESTION MARK to the visual-schematic representation to indicate the variable that has to be calculated

Step 4: The required mathematical operation is written down ($5 + ? = 8$) and solved ($5 + 3 = 8$).

Step 5: A circle is drawn around the 'unknown' value (i.e., 3), to be sure that the required answer is reported (see Figure 13).

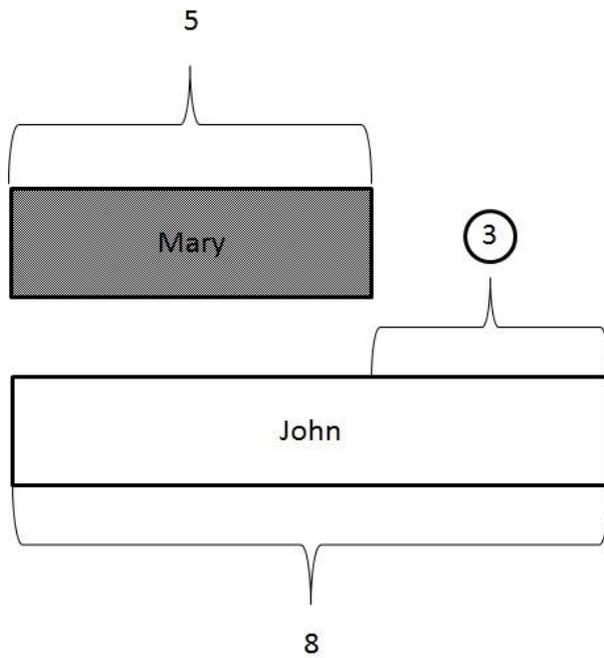


Figure 13. Step 5: DRAW A CIRCLE around the variable that had to be calculated

In the fourth and fifth week the three types of word problems were repeated and randomly presented to the students to check whether they were able to make the transfer between the problem structures of each type of word problem.

Students' word problem solving performances and use of strategy in the pre- and post-test

Table 3 gives an overview with respect to the performances on combine, change and compare problems before and after the intervention period.

Table 3. Performance pre- and post-test for combine, change and compare problems

Subject	Type of word problem	Pre-test score	Post-test score
Hugo	Combine	1	3
	Change	1	2
	Compare	2	1

	Total	4	6
Peter	Combine	0	3
	Change	0	3
	Compare	0	3
	Total	0	9
Tim	Combine	1	3
	Change	1	3
	Compare	0	3
	Total	2	9
Lisa	Combine	1	2
	Change	2	0
	Compare	1	3
	Total	4	5

For each of the four subjects the performances and solution strategies used are reported for each type of word problem separately.

Combine word problems

The following three word-problem items were included in the pre-test and post-test (between brackets the adjusted figures of the post-test):

Combine 1. Mary has 2 (3) marbles. John has 5 (6) marbles. How many marbles do they have altogether?

Combine 2. Mary has 4 (4) marbles. John has some marbles. They have 7 (6) marbles altogether. How many marbles does John have?

Combine 3. Mary and John have 8 (10) marbles altogether. Mary has 7 (9) marbles. How many marbles does John have?

Hugo

Pre-test. The results of the pre-test showed that Combine 1 was solved correctly by Hugo (i.e., $2 + 5 = ?$ [7]). Combine 2 was, however, solved incorrectly (answer Hugo = 7, required answer = 3). The mathematical operation that he reported showed that Hugo had difficulties finding the required answer (Hugo: $4 + 3 = ?$ [7]; required: $4 + ? = 7$ [3]). Also Combine 3 was solved incorrectly (Hugo: $7 + 8 = ?$ [15]; required: $7 + ? = 8$ [1]). Hugo's decision to add the two known figures of Combine 3 reflected a difficulty with comprehending the text of the word problem. In the pre-test Hugo only wrote down the mathematical operations that he performed and reported no other solution strategies.

Post-test. The results of the post-test showed that Combine 1 still did not cause any difficulties (i.e., $3 + 6 = ?$ [9]). Also Combine 2 and Combine 3 were solved correctly after the intervention period (Combine 2: $4 + ? = 6$ [2]; Combine 3: $9 + ? = 10$ [1]). With regard to the solution strategies that were used in the post-test, Hugo correctly visualized the problem structure, added a question mark in the right place in the visual-schematic representation, and correctly drew a circle around the 'unknown' variable in all three word-problem items.

Peter

Pre-test. The results of the pre-test showed that Peter solved Combine 1 incorrectly (Peter: $5 \times 2 = ?$ [10]; required: $2 + 5 = ?$ [7]). Instead of performing an addition operation, Peter used a multiplication operation, reflecting a difficulty with distilling the correct mathematical operation from the word problem text. Combine 2 was also solved incorrectly (answer Peter = 7; required answer = 3). The

mathematical operation that he reported showed that Peter had difficulties finding the required answer (Peter: $4 + 3 = ?$ [7]; required: $4 + ? = 7$ [3]). The same situation applied to Combine 3, where Peter reported the incorrect mathematical operation (Peter: $7 + 1 = ?$ [8]; required: $7 + ? = 8$ [1]). In the pre-test Peter only wrote down the mathematical operations that he performed, and reported no other solution strategies.

Post-test. The results of the post-test showed that Combine 1 was solved correctly (i.e., $3 + 6 = ?$ [9]). Also Combine 2 and Combine 3 were solved correctly after the intervention period (Combine 2: $4 + ? = 6$ [2]; Combine 3: $9 + ? = 10$ [1]). With regard to the solution strategies that were used in the post-test, Peter correctly visualized the problem structure, added a question mark in the right place in the visual-schematic representation, and correctly drew a circle around the 'unknown' variable in Combine 1 and 2. In Combine 3 Peter only used one specific solution step: he correctly drew a circle around the unknown variable.

Tim

Pre-test. The results of the pre-test showed that Combine 1 did not cause any difficulties (i.e., $2 + 5 = ?$ [7]). Combine 2 and Combine 3 were, however, solved incorrectly by Tim (answer Tim Combine 2 = 10; required answer = 3; answer Tim Combine 3 = 8; required answer = 1). These errors were a result of the fact that Tim had difficulties understanding the text of the word problem. This was reflected by the mathematical operations that he reported (Combine 2: $3 + 7 = ?$ [10]; required: $4 + ? = 7$ [3]; Combine 3: $4 + 4 = ?$ [8]; required $7 + ? = 8$ [1]). In the pre-test Tim only wrote down the mathematical operations that he performed and reported no other solution strategies.

Post-test. The results of the post-test showed that Combine 1 still did not cause any difficulties (i.e., $3 + 6 = ?$ [9]). Also Combine 2 and Combine 3 were solved correctly after the intervention period (Combine 2: $4 + ? = 6$ [2]; Combine 3: $9 + ? = 10$ [1]). With respect to the solution strategy that was used in the post-test, Tim only used the step in which a circle had to be drawn around the 'unknown' variable. He executed this step in all three Combine word problems correctly.

Lisa

Pre-test. The results of the pre-test showed that Combine 1 did not cause any difficulties (i.e., $2 + 5 = ?$ [7]). Combine 2 was, however, solved incorrectly (answer Lisa = 10; required answer = 3). Lisa made two errors with respect to this specific word problem: (1) she reported the incorrect mathematical operation (Lisa: $4 + 7 = ?$; required: $4 + ? = 7$); and (2) she made a calculation error (Lisa: $4 + 7 = 10$; required: $4 + 7 = 11$). Also Combine 3 was solved incorrectly (Lisa: $7 + 8 = ?$ [15]; required: $7 + ? = 8$ [1]). Lisa's decision to add the two known figures in Combine 3 reflected a difficulty with comprehending the text of a word problem. In the pre-test Lisa only wrote down the mathematical operations that she performed, and reported no other solution strategies.

Post-test. The results of the post-test showed that Combine 1 still did not cause any difficulties (i.e., $3 + 6 = ?$ [9]). Looking at the solution strategies that were used, the results showed that Lisa correctly visualized the problem structure, added a question mark in the right place in the visual-schematic representation, and correctly drew a circle around the unknown variable. Although these three steps were also correctly executed in Combine 2, Lisa's answer on this word problem was incorrect (Lisa: $4 + 2 = ?$ [6]; required: $4 + ? = 6$ [2]). This incorrect answer was a result of her difficulty with finding the required answer. Combine 3 was solved correctly by Lisa (i.e., $9 + ? = 10$ [1]). In this word problem Lisa executed only the last solution step.

Change word problems

The following three word-problem items are included in the pre-test and post-test (between brackets the adjusted figures of the post-test):

Change 1. Mary had 3 (2) marbles. Then John gave her 5 (4) marbles. How many marbles does Mary have now?

Change 2. Mary had 2 (3) marbles. Then John gave her some marbles. Now Mary has 9 (7) marbles.

How many marbles did John give to her?

Change 3. Mary had some marbles. Then John gave her 3 (4) marbles. Now Mary has 5 (9) marbles.

How many marbles did Mary have in the beginning?

Hugo

Pre-test. The results of the pre-test showed that Change 1 was solved correctly by Hugo (i.e., $3 + 5 = ?$ [8]). Change 2 was, however, solved incorrectly (Hugo: $2 + 9 = ?$ [11]; required: $2 + ? = 9$ [7]). Hugo's decision to add the two known figures in Change 2 reflected a difficulty with comprehending the text of a word problem. Also Change 3 was solved incorrectly (Hugo: $2 + 5 = ?$ [7]; required: $3 + ? = 5$ [2]). Hugo's solution of this word problem showed that he had difficulties comprehending the word problem text and distilling the correct mathematical operation. In the pre-test Hugo only wrote down the mathematical operations that he performed and reported no other solution strategies.

Post-test. The results of the post-test showed that Change 1 still did not cause any difficulties (i.e., $2 + 4 = ?$ [6]). Also Change 2 was solved correctly after the intervention period (Hugo: $3 + ? = 7$ [4]).

With regard to the solution strategies that were used in Change 1 and Change 2, Hugo correctly visualized the problem structure, added a question mark in the right place in the visual-schematic representation, and correctly drew a circle around the 'unknown' variable in these two word problem items. Change 3 was, however, solved incorrectly (no answer was given). Looking at the execution of the solution steps, Hugo seemed to have difficulties visualizing the problem structure and adding a question mark in the right place in the visual-schematic representation (see Figure 14).

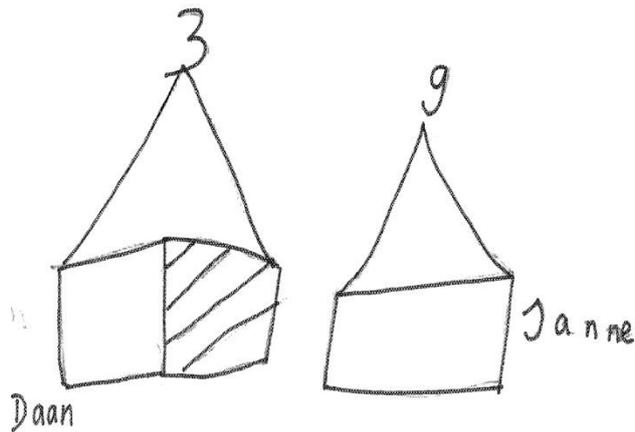


Figure 14. Hugo's incorrect visualization of Change 3 (Note: in the Dutch translation of the word problem test *Mary* and *John* were replaced by *Sanne* and *Daan*)

Peter

Pre-test. The results of the pre-test showed that Peter solved Change 1 incorrectly (Peter: $5 \times 3 = ?$ [15]; required $3 + 5 = ?$ [8]). Instead of performing an addition operation, Peter used a multiplication operation, reflecting a difficulty with distilling the correct mathematical operation from the word problem text. Change 2 was also solved incorrectly (answer Peter = 9; required answer = 7). The mathematical operation that was reported showed that Hugo had difficulties finding the required answer (Peter: $2 + 7 = ?$ [9]; required: $2 + ? = 9$ [3]). The same applies to Change 3 where Peter reported the incorrect mathematical operation (Peter: $2 + 3 = ?$ [5]; required: $3 + ? = 5$ [2]). In the pre-test Peter only wrote down the mathematical operations that he performed and reported no other solution strategies.

Post-test. The results of the post-test showed that Change 1 was solved correctly (i.e., $2 + 4 = ?$ [6]). Also Change 2 and Change 3 were solved correctly after the intervention period (Change 2: $3 + ? = 7$ [4]; Change 3: $4 + ? = 9$ [5]). With regard to the solution strategies that were used in the post-test, Peter correctly visualized the problem structure, added a question mark in the right place in the

visual-schematic representation, and correctly drew a circle around the 'unknown' variable in all Change word problems.

Tim

Pre-test. The results of the pre-test showed that Change 1 did not cause any difficulties (i.e., $5 + 3 = ?$ [8]). Change 2 and Change 3 were, however, solved incorrectly (answer Tim Change 2 = 9; required answer = 7; answer Tim Change 3 = 5; required answer = 2). The mathematical operations that were reported showed that Tim had difficulties finding the required answer (Change 2: $2 + 7 = ?$ [9]; required: $2 + ? = 9$ [7]; Change 3: $3 + 2 = ?$ [5]; required: $3 + ? = 5$ [2]). In the pre-test Tim only wrote down the mathematical operations that he performed, and reported no other solution strategies.

Post-test. The results of the post-test showed that Change 1 still did not cause any difficulties (i.e., $3 + 4 = ?$ [6]). Also Change 2 and Change 3 were solved correctly after the intervention period (Change 2: $3 + ? = 7$ [4]; Change 3: $4 + ? = 9$ [5]). With regard to the solution strategies that were used in the post-test, Tim only used the step in which a circle should be drawn around the 'unknown' variable. This step was executed correctly in all three Change word problems.

Lisa

Pre-test. The results of the pre-test showed that Change 1 did not cause any difficulties (i.e., $3 + 5 = ?$ [8]). Change 2 was, however, solved incorrectly (answer Lisa = 11; required answer = 7). Lisa reported the incorrect mathematical operation (Lisa: $2 + 9 = ?$; required: $2 + ? = 9$). Her decision to add the two known figures in Change 2 reflected a difficulty with comprehending the text of a word problem.

Change 3 was solved correctly (i.e., $3 + ? = 5$ [2]). In the pre-test Lisa only wrote down the mathematical operations that she performed and reported no other solution strategies.

Post-test. The results of the post-test showed that Change 1 was not answered. Looking at the execution of the solution steps, Lisa seemed to have difficulties visualizing the problem structure (see Figure 15).

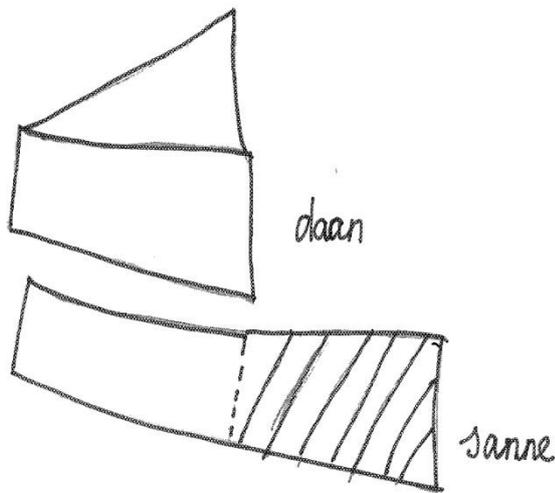


Figure 15. Lisa's incorrect visualization of Change 1 (Note: in the Dutch translation of the word problem test *Mary* and *John* were replaced by *Sanne* and *Daan*)

Also Change 2 was solved incorrectly by Lisa (Lisa: $4 + ? = 7$ [3]; required: $3 + ? = 7$ [4]). In this word problem only the last solution step was used, but incorrectly executed. Lisa seemed to have problems finding the 'unknown' variable. The same applies to Change 3 (Lisa: $5 + ? = 9$ [4]; required: $4 + ? = 9$ [5]).

Compare word problems

The following three word-problem items are included in the pre-test and post-test (between brackets the adjusted figures of the post-test):

Compare 1. Mary has 5 (4) marbles. John has 8 (9) marbles. How many marbles does John have more than Mary?

Compare 2. Mary has 3 (2) marbles. John has 4 (5) marbles more than Mary. How many marbles does John have?

Compare 3. Mary has 4 (5) marbles. She has 3 (2) marbles less than John. How many marbles does John have?

Hugo

Pre-test. The results of the pre-test showed that Hugo answered Compare 1 incorrectly (answer Hugo = 13; required answer = 3) His decision to add the two known figures, indicated that Hugo had difficulties comprehending the word problem text (Hugo: $5 + 8 = ?$ [13]; required: $5 + ? = 8$ [3]). Compare 2 was, however, solved correctly (i.e., $3 + 4 = ?$ [7]). Also Compare 3 was correctly answered, despite of complex problem structure of this word problem (i.e., $4 + 3 = ?$ [3]). Although this word problem had almost the same problem structure as Compare 1, no comprehension or calculation errors were made by Hugo while solving Compare 3.

Post-test. The results of the post-test showed that Compare 1 was answered correctly (i.e., $4 + ? = 9$ [5]). With regard to the solution strategies that were used in Compare 1, Hugo correctly visualized the problem structure, added a question mark in the right place in the visual-schematic representation, and correctly drew a circle around the 'unknown' variable. However, Compare 2 was solved incorrectly in the post-test (Hugo: $5 + ? = 7$ [2]; required: $2 + 5 = ?$ [7]). Hugo seemed to have difficulties finding the 'unknown' variable. This is also reflected in the incorrect visual-schematic representation that he made (see Figure 16).

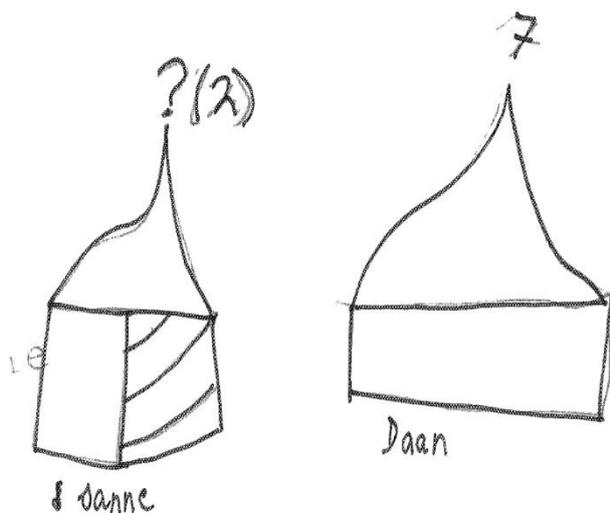


Figure 16. Hugo's incorrect visualization of Compare 2 (Note: in the Dutch translation of the word problem test *Mary* and *John* were replaced by *Sanne* and *Daan*)

Also Compare 3 was solved incorrectly (Hugo: $5 - 2 = ?$ [3]; required: $5 + 2 = ?$ [7]). Hugo apparently was distracted by the relational keyword 'less than', and performed a subtraction operation instead of the required addition operation. The step in which the problem structure was visualized was executed correctly. However, Hugo drew a circle around the wrong variable.

Peter

Pre-test. The results of the pre-test showed that Peter solved Compare 1 incorrectly (Peter: $5 - 3 = ?$ [2]; required: $5 + ? = 8$ [3]). Peter had difficulties distilling the correct mathematical operation from the word problem text. Also Compare 2 was solved incorrectly (Peter: $2 \times 2 = ?$ [4]). It seemed that Peter randomly performed a mathematical operation for this word problem. Compare 3 was solved incorrectly because Peter performed a subtraction operation (Peter: $4 - 3 = ?$ [1]), instead of an addition operation (required: $4 + 3 = ?$ [7]). Peter was probably distracted by the relational keyword 'less than'.

Post-test. The results of the post-test showed that Compare 1 was solved correctly (i.e., $4 + ? = 9$ [5]). Also Compare 2 and Compare 3 were solved correctly after the intervention period (Compare 2: $2 + 5 = ?$ [7]; Compare 3: $5 + 2 = ?$ [7]). With regard to the solution strategies that were used in the post-test, Peter correctly visualized the problem structure, added a question mark in the right place in the visual-schematic representation, and correctly drew a circle around the 'unknown' variable in all Compare word problems.

Tim

Pre-test. The results of the pre-test showed that Tim gave no answer to Compare 1, because he apparently did not know the mathematical operation that had to be performed. The answer on Compare 2 (answer Tim = 8; required answer = 7) suggested that Tim recognized the structure of this word problem, but made a calculation error (Tim: $3 + 4 = ?$ [8]; required: $3 + 4 = ?$ [7]). The incorrect answer on Compare 3, however, did not reflect a calculation error, but rather a comprehension error (Tim: $4 - 3 = ?$ [1]; required $4 + 3 = ?$ [7]). Tim apparently was distracted by the relational keyword 'less than', and performed a subtraction operation instead of the required addition operation. In Compare 2 and Compare 3 Tim only wrote down the mathematical operations that he performed and reported no other solution strategies.

Post-test. The results of the post-test showed that Compare 1 was answered correctly (i.e., $4 + ? = 9$ [5]). Also Compare 2 and Compare 3 were solved correctly after the intervention period (Compare 2: $2 + 5 = ?$ [7]; Compare 3: $5 + 2 = ?$ [7]). With regard to the solution strategies that were used in the post-test, Tim only used the step in which a circle should be drawn around the 'unknown' variable. This step was executed correctly in all three Compare word problems.

Lisa

Pre-test. The results of the pre-test showed that Lisa answered Compare 1 incorrectly (answer Lisa = 13; required answer = 3) Her decision to add the two known figures, indicated that Lisa did not

comprehend the word problem text (Lisa: $5 + 8 = ?$ [13]; required: $5 + ? = 8$ [3]). Compare 2 was, however, solved correctly (i.e., $3 + 4 = ?$ [7]). Compare 3 was solved incorrectly; this was not caused by a comprehension error, but by a calculation error (Lisa: $4 + 3 = ?$ [5]; required: $4 + 3 = ?$ [7]).

Post-test. The results of the post-test showed that Compare 1 was solved correctly (i.e., $4 + ? = 9$ [5]). With regard to the solution steps that were used in Compare 1, Lisa correctly visualized the problem structure, added a question mark in the right place in the visual-schematic representation, and correctly drew a circle around the 'unknown' variable. Also Compare 2 was solved correctly (i.e., $2 + 5 = ?$ [7]). Remarkably, Lisa did not use any solution steps while solving this word problem. The same applies to Compare 3 (i.e., $5 + 2 = ?$ [7]).

Conclusions

The results of the pre-test showed that Peter and Tim experienced difficulties solving all three types of word problems. As Hugo solved two out of three compare problems correctly, his difficulties lay mainly in solving combine and change problems. Lisa, on the other hand, had relatively less difficulty with solving change problems compared to combine and compare problems (see Table 3). The types of errors that were made in the pre-test can be distinguished as: (1) calculation errors, and (2) comprehension errors. The four research subjects made relatively less calculation errors. Incorrect answers to the word problems were mainly the result of comprehension errors. In several situations, the students reported the incorrect mathematical operation, had difficulties finding the required answer, or just performed an addition operation with the known figures in the word problem without carefully understanding and identifying the problem structure. While solving Compare 3 for example, students got distracted by the relational term 'less than', which falsely referred to a subtraction operation. With respect to the solution strategies used during the pre-test, all four students only wrote down the mathematical operations that they performed and reported no other solution strategies.

After the intervention period, Peter and Tim made no errors on the combine, change and compare problems. Although Hugo showed a better total performance in the post-test, he performed poorer on compare problems compared to the pre-test. The same applied to Lisa's performance on change problems. Therefore, we can only tentatively conclude that the difference between pre-test and post-test results can be ascribed to the word problem solving instruction. When we looked at the correct use of the solution steps of this instruction, we saw that Peter mastered all steps and executed them correctly. Apparently, this had a positive influence on his word problem solving performance. Tim, on the other hand, performed only one solution step (i.e., draw a circle on the required answer) in all the word problems during the post-test. The correct execution of this solution step seemed to help him solve all the word problems in the post-test correctly. That it is important to execute the solution steps correctly is also reflected by the performance of Hugo on the change and compare problems. Although Hugo executed all the solution steps, in some situations he had difficulties constructing the correct visual-schematic representation and drawing a circle on the 'unknown' variable. As a result he answered these word problems incorrectly. The findings with regard to the solution strategies used by Lisa showed that she did not master the solution steps for all types of word problems. In two types of word problems (Combine 1 and Compare 1), Lisa executed all the steps correctly. However, in the other type of word problems not all solution steps were executed, or some of the solution steps were executed incorrectly.

It should be noted that our findings do not imply that every student will benefit from a word problem solving instruction like the one that we investigated. Teachers should take the individual differences between students' educational needs into account in all circumstances. Moreover, the absence of a control group deters us from drawing strong conclusions concerning the effectiveness of the word problem solving instruction. Nevertheless, this feasibility study provides insights with regard to the varying ways in which a word problem solving instruction can influence the solution strategies and performances of students who perform poorly. It provides the basis for a more formal

study in the future, conducted using a controlled design. The present study gives us clues about which methodological elements should be taken into account.

Final remarks

This article describes a feasibility study in which the performances of four second-grade students were investigated as individual cases. Therefore, its findings regarding the effectiveness of the word problem solving instruction used should be interpreted with caution. Nevertheless, the word problem solving instruction described in this study shows promise as a tool for teaching students to solve word problems. Future research is required to provide more insights concerning the effectiveness of the instruction on students' performances and use of strategy in solving word problems. This feasibility study marks an important starting point in the search for instructional programs that could be implemented in the educational practice of contemporary math approaches where word problem solving plays a prominent role.