

SUMMARY

Two objectives were examined in the research presented in this thesis. The first objective was to examine the extent to which students use different types of visual representations, and the role that spatial and semantic-linguistic skills play in the solving of routine and non-routine word problems in early (second) and later (sixth) grades of elementary school. This objective was the focus of the studies described in chapters 2 to 6. The second objective was to investigate how teachers implemented an innovative instructional approach – based on the didactical use of visual representations – in their own classroom teaching practice. This was the focus of the study described in chapter 7.

Students' difficulties with solving mathematical word problems are widely recognized by both researchers and teachers. The difficulties experienced by many students often do not rise from their inability to execute computations, but from difficulties with understanding the problem text. Two component processes, namely the production of visual-schematic representations and relational processing (i.e., deriving the correct relations between solution-relevant elements of the word problem text) and their underlying basic abilities play an important role in the successful understanding of word problems. Previous studies generally examined these component processes separately from each other by distinguishing a visual-spatial (the production of visual-schematic representations and spatial ability) and semantic-linguistic (relational processing and reading comprehension) processing domain. The study described in **Chapter 2** is the first study to examine the two component processes and basic abilities in one hypothesized model. On the basis of the findings of this study we can conclude that these component processes and basic abilities explain 49% of the variance of students' word problem solving performance. Moreover, both processing domains proved important and explained unique variance; 21% of the relation between spatial ability and word problem solving performance was explained by the production of visual-schematic

representations, 34% of the relation between reading comprehension and word problem solving performance was explained by relational processing. On the basis of the path analyses, it appeared that the component processes are parallel processes which aid the successful comprehension of word problems in association with each other.

In spite of the proven importance of both visual representation strategies and semantic-linguistic skills, contemporary realistic math approaches tend to pay limited attention to the training of semantic-linguistic skills during word problem solving instruction. To test this assumption, we designed a study (see **Chapter 3**) in which we not only manipulated the extent to which a sophisticated visual representation strategy was required (by distinguishing consistent and inconsistent word problems), but also varied the semantic complexity of the word problems by using highly semantic complex marked, and low semantic complex unmarked relation terms. Research showed that semantic-linguistic skills (i.e., reading comprehension) are necessary to overcome the semantic complexity of marked relational terms. In this study we classified students as successful and unsuccessful on the basis of their performance on an independent RME-specific mathematics test. The most important finding of this study, namely that successful word problem solvers in the RME curriculum had substantial difficulties in solving marked inconsistent word problems, substantiated the assumption that RME apparently pays little attention to the training of semantic-linguistic skills. It is important to start developing such skills early in elementary school, as word problems get semantically more complex as students progress in their educational career, for example when making the transition from elementary to secondary education.

The study reported in **Chapter 4** examined the importance of different types of visual representations, spatial ability and reading comprehension in word problem solving from an *item-level approach* rather than from the test-level approach that was often used in previous studies. This change in statistical modeling provided a more thorough and sophisticated understanding of the role

of representation, spatial and reading comprehension skills in word problem solving. This item-level approach showed that the distinction between pictorial and visual-schematic representations made by previous studies is too narrow. To be more specific, we demonstrated that only the production of accurate visual-schematic representations was more frequently associated with a correct than with an incorrect answer to a word problem. Accurate, in contrast to inaccurate, visual-schematic representations contain a complete and coherent image of the problem situation, including the correct relations between the word problem's key variables. Accurate visual-schematic representations increased the chance of solving a word problem correctly by almost six times. In contrast, the production of inaccurate visual-schematic representations and pictorial representation decreased the chance of word problem solving success, respectively 2.94 and 2.78 times. As pictorial representations merely concern images of the visual appearance of objects or persons described in the word problem, they probably took the problem solvers' attention away from constructing a coherent model of the word problem, including the appropriate relations between the solution-relevant elements contained in it. Although inaccurate visual-schematic representations do include these relations, they are either incorrectly drawn or missing. As a consequence, this type of representation may have put problem solvers on the wrong track when solving a word problem.

Furthermore, besides contributing to a better understanding of the effects of the type of visual representations on the chance of word problem solving success, in this chapter we tried to reproduce, at the item-level, the findings of previous studies using a test-level approach concerning the importance of spatial ability and reading comprehension in word problem solving. In line with these earlier findings, the current study showed that spatial ability was a significant and relevant basic ability which increased the chance of solving a word problem successfully. However, our findings showed that the extent to which reading comprehension skills increase the chance of problem solving success is very limited. The results of the logistic regression analyses showed that although reading comprehension was a significant predictor in the model (due to the large number of items involved), the relevancy of its contribution was negligible (i.e., reading comprehension

increased the chance of problem solving success only 1.02 times). Our item-level finding that reading comprehension was not a relevant factor contradicted the test-level findings from this study ($r = .45$), as well as previous studies demonstrating that reading comprehension and word problem solving performance were related. In other words, a relation between reading comprehension and word problem solving found at the test level does not imply that reading comprehension positively affects the chance of problem solving success at the item level.

In **Chapter 5** the specific relation between constructive play, spatial ability and word problem solving performance was examined. In previous studies, the relation between constructive play and spatial ability, and between spatial ability and mathematical word problem solving performance was reported. The relation between constructive play and mathematical word problem solving had, however, not been established yet. The findings of our study showed that spatial ability acted as a partial mediator in the relation between these two variables. This implied that children who had frequently engaged in constructive play in their past had better spatial skills and, as a result, showed a higher performance on mathematical word problems. The variables of this study (i.e., constructive play, spatial ability and sex) explained 38.16% of the variance in students' word problem solving performance. Furthermore, 31.58% of the relation between constructive play and mathematical word problem solving performance was explained by spatial ability.

While the first chapters of this thesis focus on the visual strategies, solution processes and performances on non-routine word problems of students in higher grades of elementary school (i.e., grade 6), the study described in **Chapter 6** was focused on second grade students' performances on routine word problems. The findings of this study showed that second grade students made more errors on compare word problems than on combine and change word problems, which was in line with research performed more than 25 years ago. Rather than the existence of a consistency effect (second grade students performed equally on inconsistent and consistent compare problems), a more general difficulty in processing relational terms (like 'more than' and 'less than') seemed to be

a more plausible explanation of the finding. Second graders apparently did not (yet) have the knowledge to comprehend and process the linguistic input of compare problems and recall the appropriate problem structure. So, these students might have difficulties understanding the fact that the quantitative difference between the same sets could be expressed in parallel ways with both the terms more and fewer.

The difficulties that students experience with the comprehension and solution of word problems can be traced back to the role of the teacher in the word problem solving process. The findings of **Chapter 7** showed that most teachers were able to construct mathematical representations (e.g., a proportion table) and that they offered these representations to their students mainly in the solution phase of the word problem solving process. Like their students, several teachers experienced difficulties using accurate visual-schematic representations. Some teachers often used pictorial and inaccurate visual-schematic representations during the comprehension process of word problem solving. The findings of the study in Chapter 4, however, showed that these visual representation types did not increase the chance of solving a word problem correctly. The findings of our study in Chapter 7 revealed that when teachers did use accurate visual-schematic representations, in most cases a *bar model* was used. Other forms of visual-schematic representations like *number lines*, *pie charts* or *own constructions* were used only to a limited extent. In general, the level of diversity of forms of visual representations used by teachers was low. Moreover, the visual representations that teachers used did not always suit the problem characteristics and/or meet the individual needs of the students.

On the basis of the findings of studies described in this thesis, the following recommendations for teacher professionalization and teacher training could be made:

- ✓ School teachers should be competent at constructing accurate visual-schematic representations as an aid in word problem solving, and the development of this competence should have a prominent place in the math curriculum of regular classrooms.

(Student)teachers should have knowledge about the purpose of accurate visual-schematic representations and be trained in the construction and proper use of these types of visual representations.

- ✓ Teacher professionalization and training should focus on the correct use of different forms of visual-schematic representations while solving a word problem. (Student) teachers should learn how to construct *number lines, pie charts and own construction* and other appropriate forms of visual-schematic representations.
- ✓ There should be a particular emphasis on the construction process of these types of visual representation. Teachers should be able to use the construction process in a transparent, correct, and complete manner. (Student) teachers should learn to make their reasoning transparent by explaining which elements of the problem should be represented, and how the representation can be used to solve the problem. This reasoning process should also be correct as well as complete.
- ✓ Finally, teacher training should pay particular attention to teaching how to identify the characteristics of word problems. (Student) teachers should know the distinction between routine and non-routine word problems, and the role that accurate visual-schematic representations play in these word problem types. Namely, in routine word problems (like combine, change and compare problems) the use of only one type of visual representation can suffice, because the problem structure of each of these types of word problems is identical. However, the problem structure of non-routine word problems varies, which makes it inappropriate to offer only one kind of accurate visual-schematic representation. Hence, (student) teachers' should learn to use visual-schematic representations in a way that is both diverse (i.e., demonstrating a varied use of visual representations) and flexible (i.e., offering different visual representations to solve one word problem). Moreover, these visual representations should be functional and suit the specific characteristics of the word problem (e.g., the use of a pie chart while solving word problems involving percentages).

The recommendations listed above provide interesting aspects for further research about the importance of visual representations in word problem solving. Based on the findings of the research presented in this thesis the focus of future studies should initially be on teachers' own competence and didactical use of visual representations during word problem solving instruction. Once teachers have more knowledge about the importance of visualization in the word problem solving process, they can use this knowledge to help their students successfully overcome the difficulties that they are experiencing.

Finally, the feasibility study reported in **Appendix I** examined four second-grade students who were less successful word problem solvers. These students received protocolled instruction during a five-week intervention period. The effectiveness of the word problem solving instruction was reported by comparing students' performances on the combine, change and compare problems before and after the intervention period, as well as by examining whether they executed the solution steps of the instruction correctly. The results of the pre- and post-test comparison showed that the total word problem solving performance of all four students had improved. However, this improvement was not always visible in all three types of word problems. The study showed that the extent to which the solution steps had been executed correctly was a determining factor for the correct solution of the word problems. While our findings do not imply that every student will benefit from a word problem instruction like the one we investigated, this feasibility study does provide important insights with regard to varying ways in which a word problem solving instruction can influence the solution strategies and performances of students who perform poorly on mathematical word problems.