CHAPTER 7

General Discussion

In a world where numbers and numerosities are everywhere, this dissertation aimed to expand our understanding and provide novel insights on children's early numerosity processing skills – nonsymbolic and symbolic – and how these can set the foundations of their math achievement. As outlined in Chapter 1, however, in order to address this question, one must first address the mechanisms that underlie nonsymbolic and symbolic numerosity processing and what kind of a role these mechanisms may play in the interplay between numerosity processing and making our fist developmental steps towards mathematics achievement.

THE ROLE OF WM IN NONSYMBOLIC AND SYMBOLIC NUMEROSITY PROCESSING

A bird's eye-view on our findings regarding the relationship of children's WM capacities and nonsymbolic and symbolic numerosity processing leads to three general conclusions:

1. WM processing underlies nonsymbolic (Chapters 2, 3, 4) and symbolic numerosity processing (Chapters 3, 4, 5, 6).
2. However, WM capacity does not always override the unique predictive role of nonsymbolic and symbolic numerosity processing. It depends on the task-format and the age of the children. We found that with the sequential-large task-format, nonsymbolic and symbolic numerosity processing predicts children's later math achievement above and beyond any math-specific or math nonspecific WM capacity (Chapters 4 & 6).
3. Nevertheless, uncovering which WM component(s) underlie(s) children's performance in nonsymbolic and symbolic numerosity processing provides important insight into how children mentally represent and manipulate these numerosities and how they process a given nonsymbolic or symbolic cognitive task at a given developmental stage (Chapters 2,3). Contrary to the popular assumption that symbolic processing may map onto our pre-existing nonsymbolic representations, our results suggest that various "mapping" or "transcoding" processes may take place. Numerosities may be mentally transcoded even from the symbolic to the nonsymbolic code and WM plays a fundamental role in these mental representation and manipulation processes (Chapter 3).

We found that the assumed "innate" ability to estimate abstract (nonsymbolic) quantities in nature, i.e., the Approximate Number System (ANS; for reviews see De Smedt, Noël, Gilmore, & Ansari, 2013; Dehaene, 2011; Feigenson, Libertus, & Halberda, 2013), necessitates WM processing (Chapters 2,3). Five year-olds appear to use their Central Executive (CE) component of WM to solve a nonsymbolic approximate arithmetic task (Chapter 2). When this component was interfered with, their nonsymbolic approximate addition performance completely broke down. Surprisingly, performance in the visual and spatial interference conditions was not affected. Apparently, kindergarteners process large nonsymbolic numerosities as condensed whole quantities. In general, Chapter 2 brought forth new insight into the manner with which kindergarteners conduct nonsymbolic approximate addition but it also invoked important questions for future research. The CE is a powerful but unfortunately poorly defined component of WM (Baddeley, 1996). It controls, monitors and regulates the processes of the more clearly defined visuospatial (VSSP) and phonological loop (PL) components of WM, but its functions are related to other executive functions too, such as inhibition, shifting and updating (Baddeley, 1996). The CE task that we used to interfere with the 5 year-olds' nonsymbolic approximate addition was a task that tapped the interaction of the CE with the PL (Repovs & Baddeley, 2006), i.e., the regulation of phonological information within WM. However, that does not exclude the assumption that perhaps a CE-VSSP task might have an effect too. Additionally, when one considers the steps involved in the nonsymbolic approximate addition task, it is easy to assume that "updating" may play a role: the child mentally represents the first nonsymbolic quantity that hides behind an occluder, then he or she should update this representation with the second nonsymbolic quantity, which also hides behind the occluder and then compare this summed representation with a third quantity. In general, updating plays a primary role in children's math abilities (Friso-van den Bos, van der Ven, Kroesbergen, & van Luit, 2013).

On the other hand, Gilmore et al. (2013) recently showed that another executive function may play a fundamental role in nonsymbolic magnitude comparison processing, that of inhibitory control. Actually, Gilmore et al.'s (2013) findings suggest that inhibition mediates the effect between nonsymbolic magnitude processing and math achievement. Notably though in Gilmore et al.'s nonsymbolic task, the comparison quantities were presented simultaneously. In this respect, our findings in Chapter 6 are in line with Gilmore et al.'s (2013) assumptions: when controlling for the effect of WM capacity – including performance in various CE tasks – performance in the nonsymbolic simultaneous task-format did not predict later math achievement. However, the nonsymbolic sequential task-format did, in kindergarten and grade 1. It is important, therefore, to acquire a better understanding into the specific WM and executive resources employed in different nonsymbolic numerosity processing task-formats. Surprisingly, however, so far research into the role of WM and executive resources in nonsymbolic numerosity processing is very limited (for a review see also Cragg & Gilmore, 2014).

The aforementioned findings relate to large nonsymbolic numerosity processing. Small nonsymbolic numerosity processing appears to employ different WM resources (Chapter 3). In chapter 3, we used a similar nonsymbolic approximate addition task as in chapter 2, only now the numerosities ranged from 1 to 9 instead of 6 to 70. In this case, results indicated that single-digit nonsymbolic *approximate* addition can be mentally represented in the 5 year-olds' readily accessible mental model, i.e., their VSSP (Rasmussen & Bisanz, 2005). We also saw that different cognitive mechanisms are employed for nonsymbolic and symbolic numerosity processing, which is line with recent findings by Friso-van den Bos, Kroesbergen, and van Luit (2014). Specifically, we found that symbolic approximate addition employed more WM resources than its nonsymbolic counterpart. Actually, the prediction pattern for the symbolic approximate task indicated that some kindergarteners might attempt to transcode the symbolic elements to the nonsymbolic code in order to process them in their readily accessible VSSP. In single-digit *exact* addition different mechanisms were employed. For the nonsymbolic exact task, children seemed to employ a reversed transcoding process, i.e., perhaps the children transcoded the nonsymbolic information to the symbolic code. On the other hand, the prediction pattern for the familiar symbolic exact addition format suggested that kindergarteners stored symbolic information in their PL, which is not so readily accessible or usable for novice learners, making it again a more difficult task than its nonsymbolic counterpart. Accordingly, Caviola, Mammarella, Cornoldi, & Lucangeli (2012), have demonstrated that different WM resources relate to primary school children's complex symbolic exact and approximate addition.

All these different findings may be overwhelming. You might find yourself wondering: So, which component of WM plays a role in what type of numerosity processing? One simplified answer would be the following: All of them can potentially play a role. It depends on the stimuli used (nonsymbolic or symbolic), the range of numerosities (small or large) and the response type (approximate or exact). To make things even more complicated research has established that the roles of the different WM components also change with age since, as children grow, they develop more sophisticated WM strategies to process a given cognitive task (e.g. Rasmussen & Bisanz, 2005). Overall WM appears to play an overarching role in children's mental arithmetic and math achievement, this was clearly evident in Chapter 4: WM predicted individual differences not only in nonsymbolic and symbolic approximation, but also in math achievement with a regression weight much higher than the approximation predictors. The extent to which WM will play a role in a given mental arithmetic task depends highly on the task's presentation format (for reviews see DeStefano & LeFevre, 2004; Friso-van den Bos et al., 2013; Raghubar, Barnes, & Hecht, 2010). This dissertation's findings extend the existing literature by also demonstrating the fundamental role that WM plays in nonsymbolic and symbolic approximate mental arithmetic. But not only mental arithmetic; in Chapter 4, we saw that WM capacity also predicted large numerosity magnitude comparison (nonsymbolic and symbolic). Recently, Friso-van den Bos et al., (2014) extended this also to nonsymbolic and symbolic number line tasks and small numerosity magnitude comparison tasks. So, WM appears to underlie all types of nonsymbolic and symbolic numerosity processing skills.

In general, reviewing the literature in early numerical cognition one notices that various different nonsymbolic and symbolic tasks and presentation formats are employed, for example with small or large numerosities, presented simultaneously or sequentially, as well as different number line tasks (e.g., Bartelet, Vaessen, Blomert, & Ansari, 2014; Friso-van den Bos et al., 2014; Gilmore, Attridge, De Smedt, & Inglis, 2014; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; for a review see also De Smedt et al., 2013). Performance in all these different tasks is often viewed under the same theoretical umbrella. So, the question that now arises is: Which nonsymbolic or symbolic numerosity skill overrides the effect of WM? Our findings demonstrate that in order to identify the unique predictive role that performance in a given nonsymbolic or symbolic numerosity-processing task may play in children's math achievement, WM capacity should be controlled for (Chapter 6).

THE ANS AND SYMBOLIC PROCESSING: A UNITARY VIEW ON THE ROLES THEY PLAY

"σὺν Ἀθηνᾷ καὶ χεῖρα κίνει"

"Along with Athena (the goddess of wisdom), you must act too"

(Ancient Greek Saying)

The "nonsymbolic versus symbolic" debate is actually similar to the "nature versus nurture" debate. Admittedly, the assumption that we have an "innate" ability, the ANS, to estimate and manipulate nonsymbolic quantities in nature and that this ability may foster our math achievement comprises a very compelling story (Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004; Feigenson et al., 2013; Starr, Libertus, & Brannon, 2013). On the other hand, the assumption that it does not play any role in the early developmental steps of math achievement (Noël & Rousselle, 2011) and what is of primary importance is how well children learn to compare symbolic numerals, is also quite a compelling theoretical account for different reasons (De Smedt et al., 2013; De Smedt, Verschaffel, & Ghesquière, 2009; Lyons et al., 2014; Sasanguie, Defever, Maertens, & Reynvoet, 2014). But as life usually shows us, the truth is somewhere in the middle and that is precisely what this dissertation demonstrates.

As outlined in Chapter 1, currently there are two prevalent theoretical classes on which skill – nonsymbolic or symbolic – fosters and enhances children's first developmental steps into math achievement. One theoretical class focuses on nonsymbolic numerosity processing, i.e. the ANS: a skill we share with other primates. In humans it is evident from infancy (Izard, Sann, Spelke, & Streri, 2009; Van Herwegen, Ansari, Xu, & Karmiloff-Smith, 2008; Xu & Spelke, 2000), it improves with age (Halberda & Feigenson, 2008) and correlates with or predicts later math achievement (Gilmore, McCarthy, & Spelke, 2010; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011; Starr, Libertus, & Brannon, 2013). According to this theoretical class, the ANS comprises the primary cognitive building block upon which symbolic processing and general math achievement are built and enhanced. It has been suggested that symbolic representations map onto their pre-existing nonsymbolic representations (Lipton & Spelke, 2005; Mundy & Gilmore, 2009). On the other hand, several other studies which concurrently also assessed symbolic processing failed to find an association between ANS measures and symbolic processing or math. According to these theorists, children's ability to estimate the magnitude of digits is the most consistent and important cognitive building block of early math achievement (e.g., Bartelet et al., 2014; De Smedt et al., 2013; Lyons et al., 2014; Sasanguie et al., 2014). Within this context it is suggested that symbols are cognitively represented and manipulated in a fundamentally different manner (e.g., Holloway & Ansari, 2009; Lyons, Ansari, & Beilock, 2012; Sasanguie et al., 2014). In the following section, we outline how this dissertation's studies fill in previous gaps in the literature and bring forth findings that confirm certain aspects of both theoretical classes – and disconfirm others – leading to a unitary view on the issue.

Nonsymbolic and Symbolic Approximation Before School Instruction

Before the start of formal schooling, i.e. at the kindergarten stage, children are already "equipped" with certain numeracy skills: they can estimate and manipulate large nonsymbolic numerosities (Chapter 2, 4). They have also probably received some symbolic numeracy skills within their homes. In the Netherlands, children do not receive formal math instruction before grade 1 (in Dutch: groep 3). In Chapter 4, we examined the role that nonsymbolic and symbolic approximation skills (addition and comparison) concurrently play as building blocks of math achievement controlling for WM capacity. Contrary to previous studies, we conducted a large-scale study, which allowed us to examine for the first time the factorial structure and integrative relations of the skills in question. Furthermore, in this study kindergarteners' WM capacity was accounted for. First, we examined whether approximation is one single construct or two separate ones (nonsymbolic and symbolic). Our results highlighted the distinct but also integrative relation between nonsymbolic and symbolic representations of quantity. The ability of representing nonsymbolic numerosities was distinct to that of representing symbolic numerals (Holloway & Ansari, 2009) but the two abilities did correlate as recently confirmed by further research (Friso-van den Bos et al., 2014; Gilmore et al., 2014). Also, both nonsymbolic (Gilmore et al., 2010) and symbolic approximation (De Smedt & Gilmore, 2011) correlated with math achievement. But what specific role do these skills, which seem to be interrelated in childhood (Gilmore et al., 2014), play in kindergarten math achievement? When structural paths were entered into the models, a clear prediction pattern emerged. Nonsymbolic approximation explained individual differences in symbolic approximation. It did not, however, directly explain individual differences in kindergarten math achievement; symbolic approximation completely mediated its effect. Individual differences in symbolic approximation were explained by nonsymbolic approximation and WM capacity. In turn, symbolic approximation together with WM explained individual differences in math achievement. Noticeably, symbolic approximation predicted math achievement beyond WM capacity and nonsymbolic approximation. So, before formal schooling, both nonsymbolic and symbolic numerosity processing skills play important but different roles in explaining individual differences in math achievement. This study demonstrated how we should move beyond asking ourselves which is the best predictor. All were good predictors (WM, nonsymbolic & symbolic approximation). It was their *specific integrative relations* that led to our model explaining 87.2 % of the variance in kindergarten math achievement. Our findings on the integrative relations between WM, nonsymbolic and symbolic numerosity and the important roles they play in kindergarten math achievement have been recently confirmed within a study conducted by Hornung, Schiltz, Brunner, & Martin (2014).

How numbers are named matters

Let us take a step back now and consider what symbolic approximation, the skill that predicted math achievement beyond the ANS and WM in Chapter 4, practically meant: it reflected the kindergarteners' ability to compare large digits ranging from 6 to 70, as well as their ability to add them in an approximate manner. Intuitively, one may have not expected that a five year-old can add large numbers and correctly assess, for example, that 30 + 12 is larger than 24. This surprising finding was first demonstrated by Gilmore et al. (2007). Gilmore et al. (2007) showed that this form of symbolic arithmetic starts at the age of five without needing previous instruction. In their study, nonsymbolic and symbolic approximation demonstrated the same signature effects (e.g. the ratio effect), strengthening the assumption that children are capable of such arithmetic without needing instruction because they map symbolic representations to their readily accessible nonsymbolic representations. Contrary to Gilmore et al. (2007), however, our kindergarteners did not demonstrate the expected ratio effect in symbolic approximate addition. Actually, our sample performed quite poorly in symbolic approximate addition. This difference across the two studies could have been due to sample or task characteristic differences between our study (Chapter 4) and Gilmore et al. (2007). In Chapter 5, we conducted two experiments, which solidified that the difference was not due to task differences. What seemed to cause the contradicting results was the language difference; Gilmore et al. (2007) assessed English-speaking children, whereas we assessed Dutch-speaking children.

As outlined in Chapter 1, the naming procedure of multi-digit numbers in English is quite different compared to Dutch (as well as German and other, see Comrie, 2005). That is because the latter carries the *inversion property*. In English, two-digit numbers above twenty, such as the number twenty-four, are named in the same order as they are written: first the tens and then the units. In Dutch, however, it is the opposite: first, one names the units and then the tens. So, the number "24" is actually named "four and twenty" (in

Dutch: "vierentwintig"). In English, the phonological representation of an Arabic two-digit number could involve the following two steps: First, the child (silently) vocalizes the tens, which he or she then approximately positions on an assumed mental number line. Second, the child vocalizes the units with which he or she fine-tunes approximately the position on the mental number line. In Dutch, the corresponding process appears to be cognitively more demanding. The child first (silently) vocalizes the units. However, this step does not allow him or her to make an approximate decision on the entire number's position on a mental number line. Instead, this action must be delayed till after the child has vocalized the tens. Meanwhile, the child has to hold the units in his or her working memory, or inspect them again before the fine-tuning can start. So, now imagine again a Dutch kindergartener trying to estimate if 30 + 12 is smaller or larger than 24. If he or she is influenced by the invertion property than he or she may mistake the number 24 with the number 42!

In accordance to the aforementioned reasoning Helmreich et al., (2011) have found German speaking (inversion language) children's number line skills to be less accurate compared to Italian speaking (non-inversion language) children. Helmreich et al.'s (2011) study was conducted with first grade children. However, they did not account for the two groups' general ability to estimate quantities, i.e. their nonsymbolic skills. We, therefore, conducted a study examining the effect the language of numbers can have on children's approximation skills, which play a core role in developing math achievement (De Smedt et al., 2013; Feigenson et al., 2004; Xenidou-Dervou et al., 2013). In Chapter 5, we saw that for Dutch-speaking children the ability to conduct symbolic approximate arithmetic does not onset in kindergarten; it starts in grade 1, when their knowledge of symbols and counting skills have increased and have become automatized. Nonsymbolic and symbolic approximate arithmetic demonstrated different developmental trajectories. Therefore, children do not execute the symbolic arithmetic tasks solely by mapping the symbolic representations to their ANS. When comparing the Dutch kindergarteners' performance with that of an English-speaking sample, which had similar math achievement level and SES background, it was evident that the Dutch-speaking children were at a disadvantage in symbolic approximate arithmetic, not nonsymbolic. They performed worse than the English-speaking sample on the easy ratio of the symbolic approximate addition task: that is the ratio where all trials included a two-digit number that could be inverted. Also, the Dutch-speaking sample demonstrated a WM overload in symbolic approximate addition. It seems that this load can be relieved only after Dutch-speaking children receive formal instruction about numbers and start automatizing them. We also saw that the ability to name correctly two-digit numbers highly correlated with symbolic approximate arithmetic and not nonsymbolic. Clearly, the distinct cognitive system children possess to represent and manipulate symbolic numerosities is affected by language. Note that in our symbolic tasks the numbers were never read aloud by the experimenters. The phonological representation of a digit is activated just by merely seeing a digit. Increasingly more studies have been demonstrating the negative effects the inversion property can have during childhood in several symbolic math skills (Göbel, Moeller, Pixner, Kaufmann, & Nuerk, 2014; Göbel, Shaki, & Fischer, 2011; Klein et al., 2013) and even in adulthood (Nuerk, Weger, & Willmes, 2005).

From kindergarten till grade 2: developmental pathways to math achievement

Chapter 4 introduced a unitary view on how nonsymbolic and symbolic magnitude processing "work together" to enhance kindergarteners' math achievement. But this was a correlational study. What are their developmental roles? Which skills predict later math achievement? Also, why is it that so many different studies report contradictory results (for a review see De Smedt et al., 2013)? So far, the developmental pathways of nonsymbolic and symbolic numerosity processing have been unclear. Notably, previous research had been examining children of different ages, with different nonsymbolic and symbolic magnitude processing task-formats and children's WM capacity was rarely controlled for.

Chapter 6 addressed these important gaps in the literature. We assessed a single large sample of children in kindergarten, grade 1 and grade 2. These children were assessed on two well-known nonsymbolic and symbolic magnitude processing task-formats, which have so far been viewed under the same "theoretical umbrella": one where small numerosties (nonsymbolic or symbolic) are presented simultaneously (e.g., De Smedt et al., 2009; Friso-van den Bos et al., 2014; Holloway & Ansari, 2009; Sasanguie et al., 2013) and one where large numerosities (nonsymbolic or symbolic) are presented sequentially (e.g., Barth et al., 2006; Gilmore et al., 2010; Xenidou-Dervou, De Smedt, van der Schoot, & van Lieshout, 2013; Xenidou-Dervou, van Lieshout, & van der Schoot, 2014). Furthermore, we assessed these children's initial IQ, their WM skills within each grade and collected scores of their general mathematical achievement at the end of grade 2. If symbolic representations map onto nonsymbolic representations then they should demonstrate similar developmental trajectories. However, that was not the case. Nonsymbolic and symbolic magnitude comparison in both task-formats demonstrated different developmental trajectories.

This is in line with the theoretical view that symbols are represented and manipulated in a fundamentally different cognitive construct (Lyons et al., 2012; Xenidou-Dervou et al., 2013). Symbolic processing underwent larger developmental increases as children's knowledge of symbols increased within school instruction. Thus, symbolic magnitude representations may be related to nonsymbolic representations (Xenidou-Dervou et al., 2013) but that does not mean that the first necessarily map only onto the second. At the same time, we also found that the two-task formats demonstrated different developmental trajectories. This suggests that the field of numerical cognition should not be generalizing conclusions acquired from these two fundamentally different task formats within the same theoretical framework. Taken together, this novel finding of a task presentation-format (sequential-large vs simultaneous small) and stimulus (nonsymbolic vs symbolic) developmental interaction is in line with recent findings demonstrating that performance in these different tasks correlates in childhood (Gilmore et al., 2014) but not in adulthood (Gilmore, Attridge, & Inglis, 2011).

Our findings so far indicated that these four magnitude-processing tasks (nonsymbolic and symbolic, sequential-large, simultaneous-small) probably play different developmental roles in children's first steps towards mathematical achievement. Notably, kindergarten accuracy in all these tasks correlated with math achievement three years later (i.e., at the end of grade 2). However, only development in the symbolic sequential-large task correlated with distant math achievement. Nonsymbolic sequential-large performance uniquely predicted distant math achievement in kindergarten and grade 1, beyond symbolic magnitude processing, WM capacity, and initial IQ. Symbolic magnitude processing, on the other hand, uniquely predicted distant math achievement across all three grades, beyond WM capacity, initial IQ and nonsymbolic processing. In general, symbolic magnitude processing was consistently a stronger longitudinal predictor than nonsymbolic (De Smedt et al., 2013). These findings correspond with different findings across the literature highlighting the important role that the ANS plays early on in development (Feigenson et al., 2004, 2013; Inglis, Attridge, Batchelor, & Gilmore, 2011; Libertus et al., 2011; Mazzocco et al., 2011). However, as in previous studies, performance in the nonsymbolic simultaneous magnitude processing task did not uniquely predict distant math achievement at any developmental stage (e.g., Bartelet et al., 2014; Holloway & Ansari, 2009; Lyons et al., 2014; Sasanguie, De Smedt, Defever, & Reynvoet, 2011; Sasanguie et al., 2014). Studies so far reporting a non-significant relation between ANS and symbolic processing with math achievement had concurrently assessed the role of nonsymbolic and symbolic magnitude processing but only with *simultaneous* presentation formats, primarily with small numerosities but also large (Bartelet et al., 2014; Lyons et al., 2014). Our findings suggest that perhaps the simultaneous presentation format does not tap the ANS as effectively as the sequential presentation format. In essence, chapter 6 demonstrated once again that the ANS and symbolic processing “work together” in children’s first developmental steps towards math achievement, i.e., before formal school instruction starts and during the first grade. Later on, symbolic processing completely takes over. It should be noted that small symbolic numerosity processing is more important in kindergarten and large symbolic numerosity processing after school instruction starts.

Thus, Chapter 6 appeared to unite findings across the two prevalent theoretical classes in the nonsymbolic and symbolic numerosity processing literature. Most importantly, however, it demonstrated that this intense research focus that is being placed the last decades on the roles of nonsymbolic and symbolic numerosity processing is not without a very good reason. Our results suggested that they play unique predictive roles above and beyond overarching general processing capacities such as WM and IQ.

To sum up, this dissertation’s results regarding the roles of nonsymbolic and symbolic numerosity processing in young children’s math achievement lead to five main conclusions:

1. The ANS plays a unique and important role for children’s math achievement until the first year of formal schooling. It probably fosters children’s ability to compare the magnitude of digits.
2. Symbolic approximate arithmetic and magnitude processing do not necessarily map only onto children’s readily accessible nonsymbolic representations, i.e., the ANS. Symbolic numerosity processing is a related but notably distinct cognitive system to nonsymbolic processing. It needs instruction for Arabic numbers and demonstrates larger developmental changes than the ANS when formal education starts.
3. Overall, symbolic numerosity processing is a stronger predictor of children’s math achievement than nonsymbolic at least from kindergarten and on.
4. The presentation format of nonsymbolic and symbolic numerosity processing tasks is of primary importance. Tasks with fundamental design characteristic differences should not be used interchangeably.
5. The language of numbers can affect the developmental onset of symbolic numerosity processing.

Let us revisit the question that was raised in Chapter 1 as to whether we are bound to process numerosities just like animals do (Dehaene, 2011). Maybe we do in infancy or before we start receiving any education. Using discrete symbols to represent and manipulate numerosities, though, clearly distinguishes humans from other primates. In humans, nonsymbolic and symbolic magnitude processing capacities have an integrative relation with flexible WM capacities (Chapter 4, Chapter 5). Thus, we can potentially use sophisticated WM strategies, for example to retrieve knowledge of the order of numbers (Lyons & Beilock, 2011), transcode numerosities to a nonsymbolic or symbolic format depending on the task-format (Chapter 3), or at latter developmental stages perhaps by using phonological strategies to "tag" quantities in order to compare them, e.g. by labelling a quantity with words such as "much", "more" or "less".

EDUCATIONAL IMPLICATIONS: ASSESSMENT & INTERVENTIONS

"Πᾶν μέτρον ἄριστον"

"All in good measure" or "Excellence lies in the norm"

(Ancient Greek Saying)

This dissertation's findings bring forth important implications for educational assessment and instruction. It suggests that high WM capacity but also proficiency in nonsymbolic and symbolic numerosity processing skills appear to be enhancing and protective factors of young children's development of math achievement. On the other hand, low WM capacity and low symbolic and nonsymbolic – especially until grade 1 –numerosity processing skills may constitute risk factors. Indeed research with children with developmental dyscalculia is indicating that both WM and numerosity processing (especially symbolic) appear to be contributing factors to the disorder (De Smedt & Gilmore, 2011; De Smedt et al., 2013; Fias, Menon, & Szucs, 2013; Noël & Rousselle, 2011; Piazza et al., 2010).

With respect to nonsymbolic and symbolic numerosity processing, this thesis suggests that assessment and instructional designs should change focus across grades (Chapter 6). Our results indicate there is a sensitive period for the association between the ANS and math achievement. Pre-school education should focus on fostering children's nonsymbolic numerosity processing skills in a sequential-large format and small and large symbolic numerosity processing. In grade 1, again focus should be placed on nonsymbolic numerosity processing and this time primarily large symbolic numerosity processing, i.e., the ability to compare and manipulate large Arabic numerals. In grade 2 educational assessment and instruction should concentrate on large symbolic numerosity skills.

Attempts to intervene and foster early math achievement are steadily increasing (for reviews see De Smedt et al., 2013; Kadosh, Dowker, Heine, Kaufmann, & Kucian, 2013), although so far most include both nonsymbolic and symbolic stimuli, which makes it difficult to distinguish between the effects of nonsymbolic and symbolic numerosity processing separately. Recently, in accordance with the assumptions of this dissertation, Hyde, Khanum, & Spelke (2014) evidenced that a brief practise on a nonsymbolic approximate task enhanced first graders' exact arithmetic skills.

In general, our findings also highlight the importance of setting solid cognitive foundations from early on for developing math achievement skills. We evidenced that kindergarten abilities predict children's general mathematics achievement three years later. This highlights the fact that good pre-school programs can improve children's educational outcomes in the long run (Weiland & Yoshikawa, 2013).

Given the evident primary role that symbolic numerosity processing seems to play as a foundational building block towards math achievement, special reference should be made for educational systems in countries like the Netherlands where the language of numbers is not as transparent as in other languages. Dutch-speaking (as well as German-, Arabic-speaking, and others, see Göbel et al., 2011) children face an extra hurdle in their path towards math achievement due to the cognitively demanding number naming systems. The inversion property can be confusing and difficult to learn and as evidenced in this dissertation, it can lead to WM overload and a delayed onset of large numerosity symbolic processing. Results in chapter 5 suggest that in inversion-speaking languages formal instruction of two-digit Arabic numerals should start earlier since the demanding number naming system can lead to a developmental delay in symbolic processing skills compared to their peers from other countries that use more transparent number naming systems. The seriousness of this issue was also witnessed from my personal contacts with the schools and teachers. It was especially surprising when a Dutch primary school teacher told me that she overheard a pupil telling another while solving a symbolic arithmetic task: "Just do it in English, it's easier". After presenting our results to teachers, a common first reaction was "Do you mean we should change our number naming system?". Allow me to clarify that this is not the message we would like to convey. What is important is that formal instruction on two-digit numbers starts earlier, so that children can have a better chance of catching up with all their international peers in our ever-changing globalized society, where numbers and numeracy affect our daily personal and professional lives.

SUGGESTIONS FOR FUTURE DIRECTIONS

"ἓν οἶδα ὅτι οὐδὲν οἶδα"

"All I know is that I know nothing" or

"The only true wisdom is in knowing you know nothing"

(*Socrates* paraphrased from Plato›s *Apology*)

Further research is needed in order to understand the underlying mechanisms of nonsymbolic and symbolic numerosity processing in different task-formats. How exactly do children use their WM capacities to solve a task with nonsymbolic or symbolic, small or large numerosities presented simultaneously or sequentially? Also, what kind of cognitive strategies do children use to solve the different nonsymbolic and symbolic numerosity-processing tasks? Identifying these abilities' underlying mechanisms and the effective or not so effective strategies that may be employed can inform educational instruction and interventions. It is still surprising how a five year-old child can solve a symbolic approximate addition task with numbers ranging from 6 to 70 in an unfamiliar problem format. We evidenced that symbolic processing does not only map onto children's pre-existing nonsymbolic representations. So, the question is: what else takes place? How do children use their WM to solve symbolic approximate addition problems? Also, further research is needed into understanding cultural differences in the development of symbolic processing and mathematical achievement.

This dissertation demonstrated how nonsymbolic and symbolic numerosity processing skills can play different developmental roles in setting the foundations for children's initial steps towards *general* math achievement. But what about different types of math achievement skills? For example, do nonsymbolic and symbolic numerosity processing skills play different roles in predicting mathematical word problems or pictorial problems? Furthermore, we evidenced how different magnitude processing task-formats can play different developmental roles in children's math achievement. Future research should design intervention studies that take into account the differential effect of the different magnitude processing task-formats.

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