

CHAPTER

1

General Introduction

“Πάντα κατ’ αριθμὸν γίνονται”

“Everything is done on the basis of numbers”

(Pythagoras, 580-490 BC, Ancient Greek Philosopher)

Numbers are in our books and newspapers, our flights and tickets, our payments and taxes. Numbers keep our minds busy, whether thinking of budgets or salaries, time or distance, weight or height. Numbers quantify everything, from love (“They’ve been together for *fifty* years! That’s what I call true love and devotion”) to disasters. A number can also take many forms: for example, there is the *nonsymbolic* representation of five lemons on a tree, the *symbolic* representation of the number “5” in its Arabic form, or its verbal representation “five”. So, numerosities and numbers are everywhere. In the end, all these representations have a common factor: the “fiveness” of the numerosity. Since numeracy and math are everywhere they play an invaluable role in our cognitive development and life success (e.g., Dougherty, 2003; Reyna & Brainerd, 2007; Reyna, Nelson, Han, & Dieckmann, 2009), especially in our technologically driven economies.

Economic and social success are both enhanced by investing in early education and are increasingly included within the policies of fostering human capital (Heckman, 2000, 2011). Therefore, research the past decades has placed a lot of attention on the early building blocks of our sense of numbers and numerosity and the development of mathematical achievement. Identifying which early abilities explain our individual differences in developing mathematical skills can inform educational assessment, practise and design. The studies included in this dissertation aim at bringing forth significant theoretical and methodological contributions to the field of numerical cognitive development by enriching our understanding of children’s early numeracy skills and how they are related to the development of their math achievement. These contributions can inform early assessment and educational practise on how solid foundations can be set for developing proficient math achievement. The following general introductory section outlines the existing theoretical background and the gaps in the literature, which our studies addressed.

NONSYMBOLIC NUMEROSITY PROCESSING

For many years it was believed that numeracy is a uniquely human characteristic, although admittedly the humoristic hints were already out there: *“If you think dogs can’t count, try putting*

three dog biscuits in your pocket and then giving Fido only two of them.” (www.greatfunnyquotes.com). In this case, Fido probably smelled the third biscuit. Nevertheless, as nonsymbolic numerosities exist everywhere in nature, non-human primates, such as monkeys, birds and even fish, have been demonstrated to have an ability to estimate and manipulate abstract quantities (Agrillo, Piffer, & Bisazza, 2011; Cantlon, 2012; Flombaum, Junge, & Hauser, 2005; Roberts, 2010).

Similar to other primates, humans appear to be born with an innate ability to estimate quantities (Izard, Sann, Spelke, & Streri, 2009; McCrink & Wynn, 2007). This ability improves with development (Halberda & Feigenson, 2008) and it is not affected by cross-cultural differences (Pica, Lemer, Izard, & Dehaene, 2004). Pre-school children, before having acquired any math instruction, can effectively compare numerosities and conduct calculations with them, e.g., add or subtract them (Barth et al., 2006; Gilmore, McCarthy, & Spelke, 2010). They can do so even when these quantities are presented in different formats and modalities (Barth, Beckmann, & Spelke, 2008; McNeil, Fuhs, Keultjes, & Gibson, 2011). All these abilities are attributed to an evolutionary ancient, ontogenetic and phylogenetic cognitive system known as the Approximate Number System (ANS; for reviews see De Smedt, Noël, Gilmore, & Ansari, 2013; Dehaene, 2011; Feigenson, Libertus, & Halberda, 2013)

The ANS has been introduced as the pre-linguistic cognitive system where nonsymbolic numerical magnitude is represented and processed (Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004). It is typically assessed with tasks that require participants to simply compare quantities – in the form of “a vs. b”, “which is more?” – or conduct calculations with them in an approximate manner – in the form of “a + b vs. c”, “which is more?” (Gilmore, Attridge, De Smedt, & Inglis, 2014). Performance in these tasks is characterized by the distance- or the ratio- effect. According to the distance effect, the smaller the relative distance between two numerosities is, the harder it is to compare them (e.g., Holloway & Ansari, 2009). According to the ratio-effect, the more the ratio between two quantities approaches 1, the harder it is to compare them (Gilmore et al., 2010). To conceptualize this, imagine seeing a cloud of forty dots; it would be much easier to decide that it is more numerous compared to a cloud of seventy dots (4/7; easy ratio) than a cloud of fifty dots (4/5; difficult ratio). This is assumed to occur irrespective of the physical features of the nonsymbolic stimuli. If you compare a quantity of seven apples to a quantity of four apples, your decision on which is more will not necessarily be based on the quantities they represent. The quantity of seven apples naturally also covers a larger area, so your

decision will most probably be based on the visual aspects of the stimuli. ANS tasks strive to control for such continuous quantity dimensions as much as possible by using dot stimuli where total dot surface area, density and dot size are controlled for (e.g., Dehaene, Izard, & Piazza, 2005; Barth et al., 2005; Gilmore et al., 2010; but see also Gebuis & Reynvoet, 2012). This way responses in ANS tasks are assumed to be driven primarily by the quantities that the nonsymbolic stimuli represent.

It should be clarified that the ANS deals with numerosities of at least above four. The enumeration process of smaller sets reflects a different mechanism, known as subitizing (Revkin, Piazza, Izard, Cohen, & Dehaene, 2008). That is our ability to enumerate rapidly and accurately small sets up to three or four items. Subitizing is also assumed to be a universal ability since it has been evidenced even in animals and preverbal populations (for a review see Desoete, Ceulemans, Roeyers, & Huylebroeck, 2009).

To what extent does the ANS, this presumably “innate” ability, affect children’s numerical and mathematical skills? One class of theorists assumes that the ANS comprises the primary foundational building block that fosters and enhances our ability to learn symbolic numerals and develop mathematics achievement (Butterworth, 2005; Feigenson et al., 2004). In this respect, it is assumed that symbolic numerosity representations directly map onto our pre-existing nonsymbolic representations (Lipton & Spelke, 2005; Mundy & Gilmore, 2009; Piazza & Izard, 2009). It has been shown that infancy and pre-school ANS acuity correlates with or predicts later mathematics achievement (Gilmore et al., 2010; Libertus, Feigenson, & Halberda, 2011; Starr, Libertus, & Brannon, 2013; for a review see Feigenson et al., 2013). Others, however, have failed to find an association between children’s nonsymbolic numerosity processing and their symbolic skills or later math achievement (for a review see De Smedt et al., 2013), which has generated a debate in the field (De Smedt et al., 2013; Noël & Rousselle, 2011). Instead, it has been suggested that symbolic numerical processing is the best and most consistent predictor of children’s mathematics achievement (De Smedt et al., 2013).

SYMBOLIC NUMEROSITY PROCESSING

Of course, contrary to other primates, humans have developed symbolic systems and number words to represent quantities, which we use on a daily basis and they allow us to conduct complex computations, such as $(567 / 45) - (89 + 43)$. In sharp contrast to the fuzziness or indiscreteness of nonsymbolic numerosities, the symbolic system entails

discrete elements that can be formally manipulated. Symbolic numerical processing is typically assessed with the same tasks as those used for nonsymbolic numerosity processing with the difference that now the nonsymbolic stimuli are replaced with their corresponding digits. From three years of age, children start demonstrating an ability to discriminate small and large digits (e.g. 1 from 8); an ability which sharpens with development (Murray & Mayer, 1988). Intuitively, one may expect that since digits are discrete elements, we mentally process them much like computers do. However, as expressed in the words of Dehaene (2011; pp. 62):

“Although numerical symbols have provided us with a unique door to the otherwise inaccessible realms of rigorous arithmetic; they have not severed our roots with the approximate animal representation of quantities. Quite to the contrary, each time we are confronted with an Arabic numeral, our brain cannot but treat it as an analogical quantity and represent it mentally with decreasing precision, pretty much as a rat or a chimpanzee”

Dehaene (2011) derives this assumption from the fact that the ability to compare digits is prone to similar effects as nonsymbolic numerosity processing. Moyer and Landauer showed already in 1967 that digits with large distance are easier to compare than digits with smaller relative distance, i.e., the distance effect (De Smedt, Verschaffel, & Ghesquière, 2009; Holloway & Ansari, 2009). Also, for equal distances, we tend to find it easier to compare small numbers (e.g., 1 vs 2) than larger numbers (e.g., 8 vs 9), known as the size effect (Moyer & Landauer, 1967). Furthermore, similar to nonsymbolic approximation, as the ratio of two digits approaches one, the harder it is to compare them, i.e. the ratio effect (Gilmore, McCarthy, & Spelke, 2007). All these effects have inspired the assumption that we “map numbers onto space” (Dehaene, Izard, Spelke, & Pica, 2008). According to this metaphor, we think of quantities, nonsymbolic or symbolic, on the basis of a left to right oriented – at least in Western societies – mental number line (Dehaene, Bossini, & Giraux, 1993). On this mental line, the bigger the representational overlap of two numerosities, the harder it is to compare them.

A distinctive example of how nonsymbolic and symbolic numerosity processing demonstrate similar effects is a study published by Gilmore and colleagues in 2007. In this study, 5 year-old children were shown to be able to not only compare large symbolic numerosities but they could also add them in an approximate manner. It is widely known that symbolic exact addition, in the familiar written or verbal form of “ $a + b = c$ ” that

entails two-digit numbers, is difficult for children to learn and it is a skill that can take years to master within formal school instruction (e.g., Hamann & Ashcraft, 1985). Thus, it was surprising when Gilmore et al. (2007) demonstrated that 5 year-olds were able to perform well above chance level and demonstrated the characteristic ratio effect in an arithmetic task that included digits ranging from 5 to 58 without having previously received formal math instruction. This arithmetic task required an approximate response, i.e. “ $a + b$ vs. c ”, “which is more?”. Naturally the question that rose was how could children solve this arithmetic problem from such a young age? An explanation was derived from the fact that both nonsymbolic and symbolic approximation skills demonstrated the same signature effects (Gilmore et al., 2007). Nonsymbolic approximation skills seem to be characterized by the following three effects: a) the ratio-effect, b) performance in nonsymbolic approximate addition is as accurate as comparison, and c) nonsymbolic approximate subtraction is harder than comparison (Barth et al., 2006). Gilmore et al. (2007) demonstrated the same three signature effects in symbolic approximation. Thus, it is suggested that symbolic representations map onto their readily accessible ANS representations (Lipton & Spelke, 2005; Mundy & Gilmore, 2009), which also implies that we are bound to process symbols exactly as we process nonsymbolic numerosities.

But contrary to nonsymbolic numerosities, symbolic ones carry with them also the aspect of language (Dehaene, 2011). There are large differences across cultures in the way numbers are named and some number naming systems have been shown to be cognitively more demanding than others (Göbel, Shaki, & Fischer, 2011). For example, the naming procedure of multi-digit numbers in Dutch (as well as German and other, see Comrie, 2005) is quite different compared to English, primarily because the former entails the so-called “inversion property”. In English, two-digit numbers above twenty, such as the number forty-eight, are named in the same order as they are written: first the tens and then the units. In Dutch, however, it is the opposite: first, one names the units and then the tens. So, the number “48” is actually named “eight and forty” (in Dutch: “achtenveertig”). Increasingly more studies are suggesting that this inconsistency between spoken and written numbers can have negative effects on school-aged children’s symbolic processing (e.g., Helmreich et al., 2011). Nonsymbolic approximation skills are not affected by cross-cultural differences (Pica et al., 2004); but can this be the case for symbolic approximation too? To the best of our knowledge, the effect that the language of numbers can have in the development of a core system of numerical cognition such as children’s symbolic approximation skills, controlling for their nonsymbolic approximation skills has not been previously addressed.

In contrast to the aforementioned theoretical framework, another class of theorists supports that symbols are represented and manipulated in a fundamentally different cognitive construct (Lyons, Ansari, & Beilock, 2012; Noël & Rousselle, 2011). That is because several studies have failed to find an association between nonsymbolic and symbolic magnitude processing (Holloway & Ansari, 2009; Sasanguie, De Smedt, Defever, & Reynvoet, 2011; Sasanguie, Defever, Maertens, & Reynvoet, 2014). Also, some have failed to find an association between nonsymbolic processing and mathematical achievement (De Smedt & Gilmore, 2011; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013). In these studies, symbolic magnitude processing was found to be associated with children's math achievement beyond ANS measures. Thus, within this context, symbolic magnitude processing – and not nonsymbolic – is viewed as the best predictor of children's math achievement (see De Smedt et al., 2013). On the basis of these assumptions, nonsymbolic and symbolic numerosity processing should comprise two distinct cognitive constructs that demonstrate different developmental trajectories. Existing studies in the field, however, examined small samples, which did not permit the exploration of the structural organization of nonsymbolic and symbolic processing as latent constructs. Also, previous longitudinal studies that concurrently examined nonsymbolic and symbolic magnitude processing had primarily cross-sectional designs, which did not permit the comparison of these skills' developmental trajectories. In addition, the issue of children's individual growth in nonsymbolic and symbolic magnitude processing and how it is related to their future math achievement has not been addressed so far.

In general, reviewing the literature outlined so far, an outstanding question arises: what are the sources of these contradictory findings and assumptions? One source of contradiction may stem from the fact that different studies examined children of different ages (De Smedt et al., 2013; Noël & Rousselle, 2011). Most probably the role of nonsymbolic and symbolic magnitude processing changes across different developmental stages. Therefore, developmental studies are rendered necessary. Also, different studies have used different task-formats for assessing nonsymbolic and symbolic magnitude processing skills. These task-formats can differ on both design characteristics as well as numerosity ranges (Gilmore et al., 2014). Performance in these task-formats has been shown to correlate in childhood (Gilmore et al., 2014) but not in adulthood (Gilmore, Attridge, & Inglis, 2011), suggesting that they may have different developmental trajectories. Notably, to clarify the role that a given skill can play in our cognitive development, one must first uncover its underlying cognitive processes. However, despite the prominent

roles that are being attributed to either nonsymbolic (Feigenson et al., 2013) or symbolic numerosity processing (De Smedt et al., 2013), very little is known about their underlying cognitive mechanisms.

THE ROLE OF WORKING MEMORY

Working Memory (WM) refers to the domain-general cognitive architectural system of limited capacity, where elements are retained and processed in an online manner when engaging in cognitive tasks (e.g., Baddeley, 2002, 2006, 2012). According to Baddeley's model, WM entails: the Phonological Loop (PL), where phonological information is retained; the Visuospatial Sketchpad (VSSP), where visual and spatial elements are retained; and the Central Executive (CE), which regulates, controls and monitors the processes of the VSSP and the PL and bridges our short-term memory processes with our long-term memory (Baddeley, 2002, 2006, 2012). In computer terms, the VSSP and the PL could be viewed as two distinct (very) limited capacity RAM memories and the CE as a metaphorical CPU (central executive unit of the computer). By default, arithmetic tasks involve the processes of retaining and manipulating numerical information and connecting them to previously acquired knowledge. Therefore, the overarching role of WM in mental arithmetic and mathematics achievement in general, comes as no surprise (Bull, Espy, & Wiebe, 2008; De Smedt, Janssen, et al., 2009; DeStefano & LeFevre, 2004; Noël, 2009; Passolunghi, Vercelloni, & Schadee, 2007; Raghubar, Barnes, & Hecht, 2010; Swanson & Kim, 2007). The question that arises is: Do WM recourses underpin nonsymbolic and symbolic approximation skills? Subsequently, do the aforementioned early nonsymbolic and symbolic processing skills explain individual differences in children's math achievement beyond WM capacity?

Consider for example the steps involved in a well-known nonsymbolic approximate arithmetic task, performance on which, in kindergarten, has been shown to predict later math achievement (Gilmore et al., 2010): a) on the left side of the screen the child sees an amount of blue dots dropping into a grey box, b) then another set of blue dots drops into this box, c) at this point all blue dots are hidden behind the grey box, d) then a set of red dots appears on the right side of the screen. The child is asked to identify which was more numerous: the sum of the blue dots or the red dots. So, the child must retain in his or her memory the first set of blue dots, then update this representation with the second blue set, and subsequently compare this summed representation with

the comparison red quantity. From these steps it becomes obvious that WM must play a part in this process. However, the role of WM in approximate arithmetic had not been previously addressed. In 2005, Rasmussen and Bisanz examined the WM components involved in nonsymbolic and symbolic *exact* arithmetic, i.e. in the form of “ $a + b = c$ ”. They found that kindergarteners’ nonsymbolic exact addition was predicted by their VSSP capacity and proposed that, at this age, children use their readily accessible mental model to represent the stimuli in a one-by-one manner. Does this extend to nonsymbolic approximate arithmetic? How do the cognitive mechanisms employed in nonsymbolic and symbolic, approximate and exact mental arithmetic differ? In general, the role of the different WM components in cognitive processing depends on the age of the participants and the task characteristics (McKenzie, Bull, & Gray, 2003; Rasmussen & Bisanz, 2005; Simmons, Willis, & Adams, 2012; Trbovich & LeFevre, 2003, for a review see DeStefano & LeFevre, 2004). Examining the role of the WM components in approximation skills can uncover the manner in which the elements of the different tasks are stored, manipulated and mentally represented.

DISSERTATION OVERVIEW

The aforementioned theoretical background generates important questions, which highlight the necessity for further research into the early foundations of children’s mathematical achievement. This dissertation reflects the work of the “Understand and Predict” MathChild research group, which is part of the interlinked MathChild project (see <http://vu.mathchild.nl/>). The series of studies presented in this thesis address two general aims:

1. To increase our understanding and provide novel insights into the cognitive underpinnings of children’s nonsymbolic and symbolic numerosity processing (Chapters 2,3,5).
2. To specify the roles that early nonsymbolic (ANS) and symbolic numerosity processing skills play in the development of children’s mathematical achievement (Chapters 4 and 6).

The following section describes the specific research aims of the following chapters.

The articles included may slightly differ from the published or submitted articles in peer-reviewed journals.

Chapter 2 investigates the specific role that WM components play in nonsymbolic approximation. We conducted a dual-task study with pre-schoolers with active phonological, visual, spatial, and central executive interference during the completion of a nonsymbolic approximate addition task. Performance breakdown in one (or more) of the interference conditions can reveal the WM component(s) that are necessary when executing nonsymbolic approximate arithmetic. This study's findings provide insight into the underlying cognitive processes involved in mentally processing and manipulating large nonsymbolic quantities.

Chapter 3 aims to increase our understanding of how kindergarteners solve different single-digit addition problem formats. In this study, we administered single-digit addition problems that differed solely on the basis of two dimensions: response type (approximate or exact), and stimulus type (nonsymbolic or symbolic). We examined how performance differs across these dimensions, and which cognitive mechanism (mental model, transcoding, or phonological storage) underlies performance in each problem format with respect to working memory (WM) resources and mental number line representations. The results of this experiment increase our understanding of the mechanisms underpinning small-numerosity nonsymbolic and symbolic approximate and exact addition.

Chapter 4 explores the interrelationship between nonsymbolic and symbolic approximation, WM and math achievement at the kindergarten age, i.e. before the start of formal schooling. We conducted a large-scale correlational study that allowed us to examine the factorial structure and interrelation of these cognitive skills. With this design, we were able to address the issue of the structure of approximation and which specific role nonsymbolic and symbolic approximation skills (addition and comparison) play in explaining individual differences in math achievement at the kindergarten age.

Chapter 5 introduces novel insights into the development of symbolic approximate arithmetic. In Experiment 1, we compared the developmental trajectories of nonsymbolic and symbolic approximate arithmetic. In our large sample, symbolic approximate arithmetic seemed to onset in grade 1 not in kindergarten contrary to the English-speaking sample in Gilmore et al. (2007). In Experiment 2, we conducted a cross-cultural study, which reveals how beyond nonsymbolic representations the language of numbers can play an important role in the developmental onset of symbolic numerosity processing.

Chapter 6 investigates in a large-scale study the developmental pathways of two different nonsymbolic and symbolic magnitude comparison task-formats and their longitudinal relationship with math achievement. So far in the literature, magnitude

comparison tasks with different design characteristics (simultaneous presentation of the numerosities or in sequential steps) and numerosity ranges (small or large) have been used interchangeably. We compared their developmental trajectories and explored how children's individual growth in each magnitude comparison task relates to their future math achievement. Furthermore, we examined the unique predictive role of each skill beyond WM skills, and how their predictive power dynamically changes across grades: kindergarten, grade 1 and grade 2. This study's findings could explain the contradictory findings in the literature regarding the role of the ANS and symbolic magnitude processing during children's transition from kindergarten to formal schooling.

In **chapter 7**, this dissertation's results are discussed alongside the current literature. Furthermore, our findings' implications for numerical cognition, math achievement cognitive development and implications regarding educational assessment and practise are discussed.

Chapter 8 provides a brief summary of each chapter.

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