

CHAPTER

8

Summary

Chapter 1 outlined the existing theoretical background on the foundations of math achievement and unresolved questions that the individual studies included in this dissertation aimed to address. It presented the main concepts, definitions and theoretical assumptions entailed within the framework of children's nonsymbolic, symbolic numerosity processing skills and their working memory capacities.

Chapter 2 generated novel methodological as well as cognitive and applied educational psychology implications. Firstly, it uncovered important insights into the underlying mechanisms in 5 year-olds' nonsymbolic approximate addition. Secondly, it demonstrated that dual-task studies with active working memory (WM) interference are feasible even with children as young as pre-school. We showed that nonsymbolic approximate arithmetic necessitates WM capacity. Specifically, as expected it necessitated the central executive (CE) component of WM. Five year olds' performance in nonsymbolic approximate addition consistently broke down when the CE component of their WM was interfered with. We had also assumed that the visual or spatial subcomponents of WM would play an active role. That is because previous research has suggested that at this age children may use a mental model for representing nonsymbolic quantities in their WM in a one-by-one manner (Rasmussen & Bisanz, 2005). However, neither visual nor spatial interference interrupted the children's performance in the dot-task. Rasmussen and Bisanz's (2005) nonsymbolic task had small numerosities (1-9) and asked for an exact response. In contrast, our study's nonsymbolic task entailed large numerosities (6 to 70) and asked for an approximate response. Our findings suggest that large nonsymbolic numerosities at this age are mentally represented and processed as condensed whole arrays, perhaps in a pattern recognition manner. We also found a small phonological loop interference effect, which suggested that attention in the form of action control, i.e., sustaining attention, might play an active role too in nonsymbolic approximate arithmetic.

Chapter 3 examined how kindergarteners solve different single-digit addition problem formats, i.e. with numerosities ranging from 1 to 9. We administered problems that differed solely on the basis of two dimensions: response type (approximate or exact), and stimulus type (nonsymbolic, i.e. dots, or symbolic, i.e. Arabic numbers). We found that: a) nonsymbolic tasks are easier than symbolic ones for 5 year-olds, no matter the response-type of the problem format, namely if they are asked to give an exact response (e.g., " $a + b = c$ ") or an approximate response (e.g., " $a + b$ " vs " c ", "which is more?"); b) different cognitive mechanisms underlie performance in the different problem-formats.

As expected, the visuospatial sketchpad was the primary predictor of nonsymbolic

addition. This verified the assumption that kindergarteners use their readily accessible mental model to represent relatively small nonsymbolic quantities (Rasmussen & Bisanz, 2005). Symbolic problem-formats, however, were harder because: 1) they either required the storage and manipulation of quantitative symbols phonologically, which are not readily accessible for novice learners or 2) taxed more WM resources compared to their nonsymbolic counterparts. In symbolic addition, WM and mental number line results showed that when an approximate response was needed, children transcoded the information to the nonsymbolic code. On the other hand, when an exact response was needed, they seemed to phonologically store numerical information in the symbolic code. Lastly, we found that more accurate symbolic mental number line representations were related to better performance in exact addition problem formats, not the approximate ones.

Chapter 4 explored key questions regarding the factorial structure of approximation skills and their interrelation with WM and mathematics achievement at the kindergarten age, i.e. before the start of formal schooling. We assessed the approximation skills (addition and comparison), WM capacity and mathematics achievement of a large sample ($N = 444$) of kindergarteners. This study's design allowed us to use structural equation modelling (SEM) techniques in order to identify these skills' interrelationships and their unique predictive roles. The best fitting measurement model verified that nonsymbolic and symbolic approximation comprise two related but notably distinct abilities; both correlated with kindergarten math achievement beyond WM capacity. Once structural paths were entered into the model, the unique predictive role that each skill plays was demonstrated. WM capacity played an overarching role; it predicted performance in both nonsymbolic and symbolic approximation as well as math achievement. Nonsymbolic approximation had an indirect effect on math achievement; its role was completely mediated by symbolic approximation. On the other hand, symbolic approximation uniquely predicted math achievement above and beyond WM capacity and nonsymbolic approximation. Our final model explained a very high percentage (87.2%) of kindergarteners' individual differences in learning counting and exact addition. This study's findings bring forth an integrative view on how nonsymbolic, symbolic approximation and WM set the foundations that foster math achievement before the start of formal schooling.

Chapter 5 introduced novel insights into the developmental onset and trajectory of symbolic arithmetic. With this study, we addressed the surprising lack of a ratio effect in the kindergarteners' symbolic approximate addition in Chapter 4. It seemed as if this

task was too difficult for our 5 year-olds. Gilmore, McCarthy, & Spelke (2007), however, had previously demonstrated that symbolic approximate arithmetic onsets at the age of 5, before the start of formal schooling. Gilmore et al.'s (2007) results suggested that kindergarteners are able to conduct symbolic approximate arithmetic, which entailed even large numbers, because symbolic representations map onto pre-existing nonsymbolic representations. The difference between the two studies could have been due to task characteristics, e.g., perhaps our task did not tap the desired ability. If this were the case than the ratio effect would not be evident in grade 1 either. In Experiment 1, however, we found that the expected ratio effect in our sample's symbolic approximate addition was significant in grade 1. Thus, in our sample symbolic approximate addition appeared to onset in grade 1, not earlier. We also evidenced that nonsymbolic and symbolic approximate arithmetic demonstrated different developmental trajectories. This is a novel finding, which indicates that symbolic arithmetic representations do not necessarily map only onto their nonsymbolic representations.

Still the difference between our findings and Gilmore et al. (2007) could be due to task differences. More importantly, however, a key difference between the two studies was the fact that we assessed Dutch-speaking children, whereas Gilmore et al. (2007) assessed English-speaking children. In Dutch, naming two-digit numbers – which exist throughout the trials of our symbolic approximate task, especially in the easy ratio – is cognitively more demanding. They entail the “inversion property”, where the written and the spoken format of the number are not consistent. Thus, in Dutch the number 48 is named eight and forty (*achtenveertig*), whereas in English, it would simply be “forty-eight”. To test our assumptions, in Experiment 2, we administered our tasks to an English-speaking sample. Since formal education starts earlier in the UK than in the NL, we assessed a younger sample in the UK. The NL and UK samples did not differ in their exact addition and counting skills or their SES background. As expected, we found that the UK sample performed significantly better than the NL sample in symbolic approximate addition, not nonsymbolic. This difference was localized on the trials of the easy ratio, which all included a two-digit number that could be inversed in Dutch. Furthermore, we found that contrary to the UK sample, the NL sample demonstrated a WM overload in symbolic approximate arithmetic. Also, even though nonsymbolic and symbolic approximation correlated, only symbolic arithmetic correlated with the ability to name two-digit numbers, not nonsymbolic. Lastly, we found that English-speaking pre-schoolers can name better and faster two-digits numbers than their Dutch-speaking peers.

Accumulatively, these results highlight the negative effect the “language of numbers” can have on the developmental onset of a core system of numerical cognition, i.e., that of symbolic approximate arithmetic. Our results suggest that symbolic approximate arithmetic *does* need instruction; it needs instruction for numbers.

Chapter 6 addressed the contradictions brought forth within the nonsymbolic and symbolic numerosity processing literature and proposed a unitary view, clarifying these skills’ developmental pathways and their unique predictive roles from kindergarten up to grade 2. We assessed a large sample in two well-known nonsymbolic and symbolic comparison task-formats: in the one task-format small numerosities (1 to 9) are presented simultaneously on the screen, in the other, large numerosities (6 to 70) are presented sequentially. We found that these task-formats demonstrated different developmental trajectories. This finding constitutes a warning against their interchangeable use within the literature. Also, in both task-formats, nonsymbolic and symbolic numerosity processing demonstrated different developmental trajectories. Consistent with our findings in nonsymbolic and symbolic approximate *arithmetic* (Chapter 5), we saw that children’s symbolic *magnitude processing* does not necessarily map only onto their readily accessible nonsymbolic representations. But how does development in the different nonsymbolic and symbolic comparisons skills relate to children’s future math achievement? Latent growth modelling revealed that kindergarten performance in all four tasks (nonsymbolic and symbolic, simultaneous-small and sequential-large) correlated with later math achievement. However, only developmental growth in the symbolic sequential-large task correlated with later math achievement. Regression analyses, where the children’s WM capacity in the given year and their initial IQ were controlled for, showed that both nonsymbolic and symbolic numerosity processing played a unique predictive role in kindergarten and grade 1. However, in grade 2 symbolic processing completely took over. In general, symbolic magnitude processing was a stronger predictor of later math achievement across all grades. Also, the sequential-large task appeared to be a better predictor than the simultaneous-small task-format, especially after grade 1.

Chapter 7 entailed a general discussion on our findings, connected them to previous and recent research and gave an overview of this dissertation’s main messages. Primarily, this chapter outlined the integrative relations of WM, nonsymbolic and symbolic numerosity processing and the role each predictor plays in setting the foundations for children’s math achievement. Also, in this chapter, the educational implications of our research are presented as well as suggestions for future research.