Indirect Effects in Cost-Benefit Analysis

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Abstract

The debate about the effectiveness of investments in public infrastructure initiated by Aschauer suggests that there may be substantial discrepancies between the results of conventional cost-benefit analysis and the ultimate effects of such investments on welfare. This paper takes a closer look at this issue by investigating the existence of secondary or indirect effects under conditions of monopolistic competition. We find that such effects will in general exist, and that they are potentially large, but that they can also be negative, depending on the specification of the model. With linear demand curves, indirect effects can be positive, zero or negative, with Dixit-Stiglitz they are always nonnegative and closely related to the taste for diversity, while with the logit model they are always identically zero. Free entry reinforces the positive indirect effects in the Dixit-Stiglitz model, and causes negative indirect effects in the logit model. Given this variety of results, robust empirical measurement of the indirect effects appears to be difficult.

KEYWORDS: project evaluation, indirect (or secondary) effects, monopolistic competition

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1 Introduction

In many countries plans for investments in infrastructure are subject to public debate in which those who are in favour of it tend to stress the importance of good infrastructure not only for those who (hope to) benefit directly from the investment, but for the total economy. According to this argument, there are, at least potentially, substantial secondary or indirect effects of improvements in public infrastructure that should be taken into account.

However, the point that there can also be substantial indirect effects associated with investment in public infrastructure that are neglected in private investment analysis but should be taken into account in social CBA is generally regarded as problematic. Standard texts on benefit-cost analysis, for instance Boardman et al. (2005), discuss the case of valuing costs and benefits in distorted markets and describe how negative effects in primary markets could under some conditions be outweighed by positive impacts in secondary markets in a second-best world. Although the theoretical motivation for including indirect effects in CBA is therefore in principle as clear as that for incorporation of externalities which was forcefully made by Pigou (1920) and others, an important difference is that the significance of these indirect effects is difficult to assess in actual situations and that even their sign can be indeterminate. For this reason, the profession tends to be rather sceptical about the incorporation of indirect or secondary effects in CBA.

Indeed, there are prominent examples of misuse of indirect effect arguments in CBA, for example by the Bureau of Reclamation in economic assessments of large water projects in the U.S. in the first half of the 20th century (see Eckstein, 1958) and even present day analyses do not always fully deal with the complexity of this accounting.1

An elementary, but important issue is the danger of double counting associated with the incorporation of secondary effects in social CBA. Take the example of investments in public infrastructure. They result in changes in the price of one or more goods, such as transport or communication. The direct effect of such a price change is the increase in the surplus under the demand curve of that good. If it is a consumer good, the surplus is the consumer surplus and the welfare effect is clearly identical to the direct effect. If it is an input or an intermediate good, it is not obvious that the direct effect is identical to the ultimate effect that the price change will have on consumer welfare. That ultimate effect will be realized through a (possibly complicated) sequence of indirect effects. The simplest possibility is that the input is used to produce a consumer good, but it is also possible that it will be used to produce another intermediate

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1 See, for instance, Bhatia et al. (eds) (2009). Some of the studies in this book do not adequately account for opportunity costs, and suffer from other serious shortcomings.
good and that consumer goods will only be affected through a sequence of input-output relations. In the course of this process, the profits or the number of firms may change in the industries through which the effect passes, unemployment may decrease and the prices of some consumer goods may go down. Since it is not obvious that the ultimate effect on consumer welfare is equal to the direct effect there are potentially indirect effects.

This discussion makes clear that for the purposes of social CBA one should look for differences between the direct effect (the change in surplus under the demand function of the product whose price changes) and the ultimate effect of this price change on consumer welfare. Only the difference between the ultimate effect and the initial effect, which could be called the additional indirect effect, is relevant for social CBA. It may not always be easy to identify such additional indirect effects and to determine their size.

The relevance of indirect effects for social CBA may be illustrated by the debate on the effects of public investment initiated by Aschauer (1989), who argued that the effects of investments in public infrastructure are empirically very large. His results suggested that conventional cost benefit analysis underestimates the total gains associated with such expenditure and hence the presence of important additional indirect effects. The ensuing debate, reviewed in Gramlich (1994), questioned the original results of Aschauer in many ways. However, one feature that has been highlighted is the stimulating effect that public infrastructure has on private investment (see Munnel, 1992). Nevertheless, one can easily argue that investment in traffic infrastructure will naturally generate some spatial redistribution of economic activities which may appear in the data as investment activity. The associated welfare effects may therefore be properly taken into account if the long run demand curve for transport is used for the determination of the direct effect. If this is indeed the case, it would be wrong to interpret the investment activity as an indirect effect that has to be added to the change in consumer surplus.

This discussion makes clear that it is difficult to assess the importance of indirect effects because a chain of cross-market price effects is involved, while the magnitudes, types and interactions of economic distortions that would affect their size are often not known sufficiently well. Presumably for this reason, many empirical pieces that acknowledge the existence of indirect effects either bring the issue up as an after-thought or discuss a special case only. This paper attempts to contribute to improve the analyses of indirect effects by formalizing their derivation under imperfect competition when inputs or intermediate goods are involved in a general way. By doing this, it will be shown that even for certain common models found in the literature, under certain circumstances, contrary to

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2 See, for instance, Moreno et al. (2002) for an in-depth analysis of the specific effect of investments in public infrastructure on the performance of manufacturing industries.
what many might think, these indirect effects are not insignificant. In fact the magnitudes can be extremely large. Furthermore, there are also circumstances under which the indirect effects can be negative. The paper provides a rigorous way in which these indirect effects can be derived. In the next section we start with a review of the relevant literature. Section 3 contains a brief discussion of the monopoly situation that serves to introduce the methodology used in the next two sections and provides a useful starting point for the analysis of indirect effects under monopolistic competition. Section 4 introduces the class of models studied in the present paper and provides an analysis for situations in which the number of firms is fixed. Section 5 considers the situation of free entry. In section 6 a number of specific examples are discussed. Section 7 concludes with a discussion of the implications for social CBA.

2 A review of the relevant literature

The theory of Cost-Benefit Analysis (CBA) evaluates an investment on the basis of its effects on welfare (see, for instance, Drèze and Stern, 1986). In principle this requires the use of a general equilibrium model to determine the values of the determinants of welfare before and after the realization of the project. This ‘conceptually straightforward approach’ (Diewert, 1983) is often difficult to carry out in practice, thus partial equilibrium analyses are more common.

This is an important reason why applications often concentrate on the direct effects, and then make appropriate adjustments for market failures like external effects. This practice means that apart from market failures the CBA is identical to the assessment a private entrepreneur would make. In all market economies, private investment decisions are taken in this way, and it is well known that the implied focus of attention on the direct effects is correct if the economy operates under perfect competition. This practice implies that investments that result in improvements in the production of intermediate products (and therefore have no direct effects on consumer welfare) can nevertheless be evaluated by the induced change in the area under their demand curve because all decreases in cost will be transformed into equivalent decreases in the price of final products. Analogously, social CBA of public investment that does not (or partly) concern consumers can be restricted to a consideration of the direct effects. For instance, if a new road is constructed, it is sufficient to compare the cost of this investment with the benefits as measured by the consumer surplus under the demand curve for trips on this road, even though there may be measurable effects of its construction in other sectors as well.

This argument has been put forward by Hicks (1946a, b), who argued that the effect of a decrease in the price of an input (for instance because of a subsidy) can be measured equivalently as the change in consumers’ surplus under the
demand curve for the input concerned, or through the changes in the consumers’ surplus of the demand curves of the final products that used this input. Hicks’ analysis was used by Sugden and Williams (1978) who give the example of a decrease in transport cost, which leads to a change in housing demand. In that case the welfare effect can be measured in two equivalent ways: as the change in consumers’ surplus under the demand curve for transport (the direct effect) or as the change in consumers’ surplus under the demand curve for housing (an indirect effect). Incorporating both effects in a CBA would imply (literally) double counting. Blackorby and Donaldson (1999) provide a more formal analysis of Hicks’ equivalence proposition and prove its validity in the context of an economy with perfect competition.

The proposition that the welfare effect of changes in input prices can be measured completely by changes in consumers’ surplus under the demand curve for an input (that is, a commodity that is not directly consumed at all) was considered from a different angle, and formally proved, in a series of articles in the American Economic Review in the 1970s. These analyses use partial equilibrium models of an industry that produces a consumer good and consider the effect of a change in the price of one input. For the purposes of the present paper it is important that also some attention was paid to the monopoly case. This made clear that the welfare effect of a decrease in the price of a monopolist’s input was not completely measured by its direct effect. In the next section we return to that model and use it as a starting point for the analysis in subsequent sections.

These two branches of literature make it abundantly clear that under perfect competition indirect effects should never be incorporated into a social CBA in addition to direct effects. Additional indirect effects can only occur with market imperfections. In other words: additional indirect effects have to be taken into account into social CBA for the same reason as environmental damage and other relevant welfare consequences of market failure.

At the end of the 1970s interest in markets characterized by monopolistic competition increased and this type of models have become very useful standard tools of economic analysis since then. The ‘monopolistic competition revolution’ has transformed the field of international trade and economic geography. The international trade literature that uses models based on monopolistic competition provides interesting suggestions about the presence of indirect effects. For instance, in a review of the literature on regional economic integration Baldwin

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3 It appears that this second branch of literature emerged independently of Hicks’ analysis.
4 This discussion focused on the measurement of the welfare effect of a change in the price of an input of an industry producing a consumers’ good. See Schmalensee (1971), Wisecarver (1974), Anderson (1976), Schmalensee (1976) and Jacobsen (1979).
and Venables (1995) conclude that the welfare effects of regional economic integration ‘may be many times larger if industries are imperfectly, rather than perfectly competitive’ (Baldwin and Venables, 1995, p. 1606). They illustrate this with a Dixit-Stiglitz model of imperfect competition and iceberg transport costs in the context of a model in which two to three countries integrate. One reason is the shift of industries towards the integrating countries; another is the larger scale of production in the integrated economies. However, the authors observe that unambiguous general results on the net effects of economic integration are difficult to obtain. Baldwin et al. (2003) use a similar framework for the decomposition of the welfare effects in their analysis of trade policies. The analysis of these two review papers suggests that imperfect competition can make a substantial difference for the evaluation of public policy. The indirect effects of public policy that are the subject of the present paper arise in much the same way as these non-traditional effects of trade policy, but they are not related to the relocation of firms across national boundaries. It remains to be seen therefore whether similar results will show up. In economic geography the main development was, of course, Krugman’s core periphery model which implies that under monopolistic competition small differences in transport costs can have large implication for the geographical structure of economic geography.

Closer to the topic of the present paper are a few studies that concentrate on the effects of imperfect competition in the context of a non-spatial CBA. Venables and Gasiorek (1998) have provided simulation results that suggest that in an economy with vertical linkages where industries are characterized by monopolistic competition, the benefits of transport improvements may be substantially larger than is suggested by conventional cost-benefit analysis in which attention is restricted to direct effects. On the other hand, Newbery (1998) provided counterexamples that show that in some situations conventional cost-benefit analysis might overestimate the welfare gains from public investment. That analysis shows that additional indirect effects may well be negative. Bröcker (1998) presents a related analysis of the welfare-improving effects of a transport subsidy in a Hotelling-Smithies type of partial spatial equilibrium. The models used in these papers are rather specific and it is therefore unclear what level of generality can be attached to them.

The main conclusion to be drawn from this review is that the question about the importance of additional indirect effects is still open in the context of market imperfections. Additional indirect effects are consequences of market failures that should in principle be taken into account in a social CBA. There are important suggestions from models of imperfect competition in related fields that suggest they can be substantial. Moreover, there are some specific examples of cases in which they are positive and indeed substantial and also one in which it

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6 Their chapter 17 on infrastructure policies is mainly concerned with economic growth.
the additional indirect effect is negative. It is the purpose of the present paper to address the question about the relevance of additional indirect effects for social CBA for a class of models that cover a number of popular specifications of monopolistic competition to see if general conclusions with respect to the sign and magnitude of additional indirect effects can be derived. This will be done by adopting the framework of Schmalensee (1971) and the related literature in the context of imperfect competition. That is: we consider an industry that produces a consumer good and consider the effect of a change in an input price. In particular, we ask the question whether such a change in an input price generates additional indirect effects. The decrease in price may, for instance, have been caused by an investment in public infrastructure such as a road or glass fibre cable network.

The framework adopted here is probably the simplest one in which additional indirect effects can occur and seems therefore a natural starting point. Possible generalizations will be discussed in the concluding section of this paper.

3 Monopoly

In this section we introduce the methodology to be used for the analysis of monopolistic competition by means of a brief reconsideration the monopoly situation. We assume that the monopolist produces a single product,\(^7\) takes the demand function for its output as given and is able to set the price of its output. It does so in order to maximize its profits, which are defined as:

\[
Z = ry - px ,
\]

where \(Z\) denotes profits, \(r\) the price of the firm’s output, which is a consumers’ good, \(y\) its output volume, \(p\) the vector of input prices and \(x\) input volumes. The demand for the firm’s output is a function of its price: \(y = y(r)\).

The maximization problem can be considered as consisting of two parts: minimization of the cost \(K\) at a given output volume and determination of the optimal output volume. Cost minimization leads to a cost function, which we denote as \(K(p, y)\). Substitution of the demand function and the cost function into the definition of profits leads to the firm’s profit function, which has the price \(r\) as the firm’s only remaining decision variable:

\[
Z = ry(r) - K(p, y(r))
\]

The welfare effect of a change in the price of an input is measured as the change in social surplus \(\Delta SS\), i.e. sum of the changes in consumers’ surplus \(\Delta CS\) and in profits \(\Delta Z\):

\(^7\) Jacobsen (1979) has also looked at multiproduct firms.
\[ \Delta SS = \Delta CS + \Delta Z \]

The change in social surplus gives the ultimate effect of the input price on welfare and is therefore equal to what has been called in the introductory section the indirect effect. It has to be compared with the direct effect that will be defined below.

The change in consumers’ surplus occurs because of the change in the (profit maximizing) output price induced by the change in the input price. Let the changing input price be that of the \( i \)-th input. We denote its initial value with a superfix \( 0 \) and its new value with a superfix \( 1 \). Then we can write for the change in consumers’ surplus:

\[ \Delta CS = \int_{\rho_i^0}^{\rho_i^1} y(r) \frac{dr}{dp_i} dp_i \]

The change in profits equals:

\[ \Delta Z = -\int_{\rho_i^0}^{\rho_i^1} \frac{\partial Z}{\partial p_i} dp_i \]

\[ = -\int_{\rho_i^0}^{\rho_i^1} y(r) \frac{dr}{dp_i} dp_i - \int_{\rho_i^0}^{\rho_i^1} (r - mc) \frac{\partial y}{\partial r} \frac{\partial r}{\partial p_i} dp_i + \int_{\rho_i^0}^{\rho_i^1} \frac{\partial K}{\partial p_i} dp_i, \]

where \( mc = \frac{\partial K}{\partial y} \), the marginal cost; the second line makes use of (2) and takes into account that the output price \( r \) depends on \( p_i \), and also that output volume \( y \) is influenced by \( p_i \). The change in output volume influences revenues as well as costs.

The first expression on the right-hand-side in the second line of (5) is equal to minus the change in consumers’ surplus. The change in social surplus is therefore equal to the sum of the second and third term:

\[ \Delta SS = -\int_{\rho_i^0}^{\rho_i^1} (r - mc) \frac{\partial y}{\partial r} \frac{\partial r}{\partial p_i} dz + \int_{\rho_i^0}^{\rho_i^1} \frac{\partial K}{\partial p_i} dp_i \]

The first term on the right-hand-side of (6) (including the minus-sign) is positive when the price exceeds marginal cost, since the demand curve is negatively
sloped and the output price is an increasing function of the input price. The second term is also positive.

The direct effect of the input price change is the induced change in the area under the input demand curve. In measuring this change, we should take into account that input demand might change because of substitution between inputs and because of a change in the demand for the final output. The latter is the result of the change in the price of the output induced by the changing input price. Denoting the input demand curve as \( x_i(p,y(r)) \), and taking into account that the profit maximizing price is a function of the input prices, we define the direct effect of the price change as \( \Delta S \):

\[
\Delta S = \int_{p_i^0}^{p_i^1} x_i(p,y(r(p)))dp_i,
\]

where the second line makes use of the fact that the demand \( x_i(p,y(r)) \) is the partial derivative of the cost function with respect to the price of input \( i \).

Comparison of eqs. 6 and 7 reveals that apart from the direct effect, there is an indirect effect (\( IE \)) associated with the lower input price, that is overlooked by conventional cost-benefit analysis:

\[
IE = \int_{p_i^0}^{p_i^1} (r - mc) \left( \frac{\partial y}{\partial r} \right) \frac{\partial r}{\partial p_i} dp_i.
\]

The expression under the integral sign is the product of the difference between price and marginal cost, (the absolute value of) the slope of the demand curve and the partial derivative of the output price with respect to the price of the \( i \)-th input. The \( IE \) is positive only when the price differs from the marginal cost. Moreover, it is clear from (8) that the presence of a nonzero \( IE \) requires a downward sloping demand curve, which is a necessary condition for monopoly behavior. There is a close relation between the price elasticity of demand and the size of the indirect effect: with zero price elasticity the \( IE \) is also zero, while it increases in the absolute value of the price elasticity.

The formula therefore shows that there is an intimate relation between the presence of an \( IE \) and monopolistic behavior. Indeed, the reason for the beneficial

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\(^8\) This follows from the first order condition for profit maximization if marginal cost is increasing in the price of input \( i \).
IE is that a decrease in the price of an input decreases this market distortion caused by this behavior to some extent and may therefore be viewed as a second best measure to cure this effect. The effect of such a decrease is related to that of a subsidy given to the monopolist and the presence of an IE is therefore related to the well known ‘paradoxical result’ (Tirole, 1988, p. 68) that welfare increases when a monopolist’s output is subsidized. The reason is that the monopolists’ product is underconsumed, and the subsidy corrects for this distortion. The lower input price has a similar effect as a subsidy when it leads to a lower output price. Note, finally, that a nonnegative IE requires that the change in the input price induces a change in the output price. This condition is violated if the input only affects the fixed cost of production.

The various effects are illustrated in Figure 1, which refers to a monopolist with a linear demand curve and constant marginal cost. The figure shows the effect of a decrease in the marginal cost, which has been caused by a lower input price. The change in the marginal cost induces the monopolist to set a lower output price, which causes an increase in consumer surplus, as indicated by the vertically shaded area. The lower price in itself implies lower profits, but this effect is more than compensated by the decrease in the monopolist’s lower cost, given by the direct effect, and the additional indirect effect, which is equal to the mark up times the additional output.

Jacobsen (1979) observed that, apart from its (positive) sign, little can be learned from (8) in general. It is indeed the case that is gives us very little information about the size of the additional indirect effect apart from it positive sign. We can rewrite (8) as:

\[
IE = \int_{p_i^1}^{p_i^0} y(r) \frac{\partial r}{\partial p_i} dp_i, \tag{8'}
\]

This uses the fact that for the profit maximizing price \( r = mc + 1/(\partial y/\partial r) y \).

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which shows that the $IE$ is equal to the change in consumers’ surplus. More pronounced results can be reached in special cases. When the demand function is linear and the cost function is homogeneous of degree 1 in output, the $IE$ is equal to 50% of the direct effect; when the demand function is log-linear and the cost function is homogeneous of degree 1, the $IE$ is equal to $\frac{\epsilon}{(\epsilon - 1)}$ times the direct effect, where $\epsilon$ is absolute value of the (constant) price elasticity of demand. This implies that for $\epsilon$ (larger than but) close to 1, the $IE$ can be a large multiple of the direct effect, whereas for a large $\epsilon$ it will be close to 100% of the direct effect. It appears therefore that the size of the $IE$ depends to a considerable extent on the curvature of the demand curve. The large difference between the conclusions reached for linear and log-linear demand functions suggests that, for measuring the size of the $IE$ correctly, it is of crucial importance to know if and how the price elasticity changes with the price (i.e. the second order derivative of the demand curve). In practice it may not be easy to get the relevant information.

The next section will show how additional indirect effects can occur under monopolistic competition.
4 Monopolistic competition with a given number of firms

In the class of models we consider there is a (possibly) large number of firms $n$ who each produce one variety of a differentiated product. All firms have identical linear cost functions and aggregate demand functions are symmetric in the sense that a permutation of the indices of the firms would not change the demand functions. The demand functions in the class of models we consider can be generated from a consumers’ surplus function $CS(r_1,\ldots,r_n)$ as follows:

$$y_j = -\frac{\partial CS}{\partial r_j}$$

In general, the demand curve for $j$’s product depends not only on the price $r_j$ of firm $j$, but also on that of all other firms that are active in the market. This class covers, among others, the Dixit-Stiglitz model and the logit model of monopolistic competition.

All firms maximize their profits:

$$Z_j = r_jy_j - K(y_j, p)$$

where $K$ denotes the cost function:

$$K(y_j, p) = F(p) + y_jv(p)$$

by setting their price and producing the corresponding market demand. The profit-maximizing price satisfies:

$$r_j = mc - \frac{r_j}{\varepsilon_j}, \quad \varepsilon_j = \frac{\partial y_j}{\partial r_j}$$

Market equilibrium in such an industry is characterized by equal prices and production volumes for all firms. In what follows we confine our attention to such market equilibria. Moreover, in the present section, we will take the number of firms as given.

Our welfare measure is the social surplus, now defined as the sum of the consumer surplus and the profits associated with each product (or firm). The change in this surplus that occurs, for instance, as a consequence of the lower price for input $i$ is:
\[ \Delta SS = \Delta CS + \sum_{j=1}^{n} \Delta Z_j \]  

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The change in consumer surplus that results from an infinitesimal change in the identical equilibrium prices of all firms is:  
\[ dCS = \sum_j (\partial CS/\partial r_j) dr = - \sum_j y_j dr . \]

We can therefore write the total change in consumers’ surplus of a finite change in \( p_i \) as:

\[ \Delta CS = \int_{p_i'}^{p_i} \sum_j y_j \frac{\partial r}{\partial p_i} dp_i . \]  

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The change in profits of firm \( j \) can be written in a similar way as:

\[ \Delta Z_j = - \int_{p_i'}^{p_i} y_j (r) \frac{\partial r}{\partial p_i} dp_i - \int_{p_i'}^{p_i} (r - mc_j) \frac{\partial y_j}{\partial r} \frac{\partial r}{\partial p_i} dp_i + \int_{p_i'}^{p_i} \frac{\partial K_j}{\partial p_i} dp_i \]  

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whereas the direct effect of the change in input price \( i \) is:

\[ \Delta S = \sum_{j=1}^{n} \int_{p_i'}^{p_i} x_{ji}(p, y(r(p))) dp_i , \]  

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\[ = \sum_{j=1}^{n} \int_{p_i'}^{p_i} \frac{\partial K_j}{\partial p_i} dp_i . \]

where \( x_{ji} \) denotes the demand for input \( i \) by firm \( j \).

For later reference, we note that cost function (11) implies that we can rewrite the direct effect (16) as:

\[ \Delta S = \sum_{j=1}^{n} \int_{p_i'}^{p_i} \left( \frac{\partial F}{\partial p_i} + y_j \frac{\partial v}{\partial p_i} \right) dp_i . \]  

17
This shows that the direct effect consists of two parts: one caused by the change in fixed cost, the other by the change in variable cost. The change in fixed cost does not result in a change in the output price\textsuperscript{10} and will only affect profits.

It is now easy to obtain the total $IE$:

$$IE = \sum_{j=1}^{n} \left( - \int_{p_i}^{p_i'} (r - mc_j) \frac{\partial y_j}{\partial r} \frac{\partial r}{\partial p_i} dp_i \right)$$

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It must, however, be noted that in this case we cannot establish equality between the $IE$ and the sum of the consumers’ surpluses. The reason is that the mark-up of each firm is now determined on the basis of the price elasticity of its demand with respect to its own price, i.e. holding the prices of all other varieties constant (cf. equation 12), whereas the partial derivatives $\partial y_j/\partial r$ refer to changes in the prices of all varieties. We can, however, rewrite (18) as:

$$IE = \sum_{j=1}^{n} \left( \int_{p_i}^{p_i'} y_j \frac{\partial y_j}{\partial r} \frac{\partial r}{\partial p_i} dp_i \right)$$

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where $\partial y_j/\partial r_j$ is the partial derivative of the demand function for firm $j$ with respect to its own price. The ratio of the partial derivatives that appears on the right-hand-side is smaller than 1, implying that the $IE$ is smaller than the sum of the changes in the consumers’ surpluses.

As in the monopoly case, we find that the $IE$ is always positive when the mark up is positive, demand curves slope downward and the change in the input price induces a change in the output price. Note, however, that the demand curve for firm $j$ must be downward sloping when the price of firm $j$’s output changes and also when the prices of the outputs of all firms 1,...,n change simultaneously. Economic theory does not imply that the latter condition is satisfied, and this opens up the possibility of a negative $IE$. Clearly, the general result for $IE$’s under monopolistic competition is even less informative than that under monopoly (where at least the sign of the $IE$ was determined) and we have to look for special cases to derive more specific results.

If the number of firms is large, the $IE$ will decrease if the mark-up becomes smaller or (equivalently) if the absolute value of the price elasticity of demand increases. In the next section we will consider the model under conditions of free entry.

\textsuperscript{10} Cf. (12).
5 Monopolistic competition with free entry

The importance of considering the effects of free entry in relation to social CBA of investments in public infrastructure is suggested, for instance, by the work of Holtz-Eakin and Lovely (1996) who found that such investments left the size of the firms unchanged, but altered the number of firms substantially.

To study the presence of IE’s under free entry in the context of the model developed in the previous section, we write consumers’ surplus as a function of the equilibrium price $r$ and the number of firms $n$: $CS = CS(r, n)$ The latter variable will now be treated as continuous. Both $r$ and $n$ can depend on the price of input $i$ via the variable cost $v(p)$ and/or the fixed cost $F(p)$. In general, $n$ and $r$ are general functions of both cost terms: $n = n(v, F), r = r(v, F)$. We can substitute these relationships into $CS$ and in this way write consumer surplus as a function of the two cost terms. After doing so, we can write the change in consumers’ surplus that results from a change in the price of input $i$ as:

$$
\Delta CS = -\int_{p_i^0}^{p_i^1} \left( \frac{\partial CS}{\partial v(p)} \frac{\partial v(p)}{\partial p_i} + \frac{\partial CS}{\partial F(p)} \frac{\partial F(p)}{\partial p_i} \right) dp_i
$$

Under free entry profits will always be equal to 0, and the change in social surplus is therefore equal to the change in consumers’ surplus: $\Delta SS = \Delta CS$.

The direct effect of the change in the price of input $i$ is:

$$
\Delta S = \int_{p_i^0}^{p_i^1} n \frac{\partial K}{\partial p_i} dp_i
$$

$$
= \int_{p_i^0}^{p_i^1} \left( ny_j \frac{\partial v(p)}{\partial p_i} + n \frac{\partial F(p)}{\partial p_i} \right) dp_i,
$$

where it must, again, be taken into account that both $n$ and $y_j$ are functions of the input prices.

The $IE$ is equal to the difference between the change in social surplus and the direct effect. Since the former is equal to the change in consumers’ surplus, we can compute it by subtracting (21) from (20):

$$
IE = \int_{p_i^0}^{p_i^1} \left( -\frac{\partial CS}{\partial v(p)} - ny_j \right) \frac{\partial v(p)}{\partial p_i} dp_i + \int_{p_i^0}^{p_i^1} \left( -\frac{\partial CS}{\partial F(p)} - n \right) \frac{\partial F(p)}{\partial p_i} dp_i
$$
An important aspect of this relationship is that it is not clear that the $IE$ will always be nonnegative. In order to investigate the issue somewhat further, we write the partial derivatives of consumer surplus by first differentiating explicitly to $n$ and $r$, and then further to the two cost components $v$ and $F$. We note from (9) that $\partial CS/\partial r = -ny_j$ and after substituting this result we find the following expression for the $IE$:

$$IE = \int_{p_i}^{p_0} \left( -\frac{\partial CS}{\partial n} \frac{\partial n}{\partial v} + ny_j \left( \frac{\partial r}{\partial v} - 1 \right) \frac{\partial v}{\partial p_i} \right) dp_i + \int_{p_i}^{p_0} \left( -\frac{\partial CS}{\partial n} \frac{\partial n}{\partial F} + n \left( y_j \frac{\partial r}{\partial F} - 1 \right) \frac{\partial F}{\partial p_i} \right) dp_i$$

We expect consumers’ surplus to be increasing in the number of firms and the number of firms to be non-increasing in variable and fixed costs. We can also note that monopolistic price setting behavior usually implies that $\partial r/\partial v(p)$ is at least equal to 1.\textsuperscript{11} This implies that the $IE$ can only be negative when $y_j (\partial r/\partial F) < 1.$ Since we cannot exclude this a priori, we must conclude that a negative $IE$ is a real possibility under monopolistic competition with free entry.

6 Examples

The analysis of the previous two sections has taken us to a position in which it is useful to consider some specific examples. The section starts with considering three well known models of monopolistic competition with a given number of firms, and subsequently considers the free entry case for two of them.

6.1 Linear demand curves

We consider a market with a representative consumer who has a quasi-linear indirect utility function.\textsuperscript{12}

\textsuperscript{11} This is the case when the demand curve is downward sloping ($\partial y/\partial r < 0$) and the second-order derivative of the natural logarithm of demand with respect to price is positive ($\partial^2 \ln(y) / \partial r^2 \geq 0$). The latter condition says that the demand function is log-convex and most (if not all) demand curves used in empirical work satisfy this property.

\textsuperscript{12} This model is used (with a continuum of varieties) in Ottaviano, Tabuchi and Thisse (2002) and in Fujita and Thisse (2002, chapter 8,9).
\[ u = \frac{x - \left( a \sum_{i=1}^{n} r_i + \frac{1}{2} b \sum_{i=1}^{n} r_i^2 + \frac{1}{2} c \left( \sum_{i=1}^{n} \sum_{j=1}^{n} r_i r_j \right) \right)}{d} \]  \hfill (24)

where \( x \) denotes total consumption budget and the parameters \( a, b, c \) and \( d \) can be functions of the prices of consumer goods produced in other industries. For quasilinear utility functions, consumers’ surplus is a valid welfare measure, since Chipman and Moore (1976, 1980) have shown that for such functions the compensating variation is exactly equal to consumers’ surplus.\(^{13}\) Consumers’ surplus \( CS \) in (24) is equal to the term in brackets behind the minus sign in the numerator of the right-hand-side.

The demand for variety \( j \) can be derived by means of Roy’s identity (or, equivalently, from consumers’ surplus) as:

\[ y_j = a + b r_j + c \sum_{i=1}^{n} r_i \]  \hfill (25)

We assume that \( b<0 \), and that \( b+c<0 \) in order to have a downward sloping demand curve for all firms. Moreover, total demand decreases when the market equilibrium price increases only if \( b+nc<0 \).

Cost and profit functions for the firms have been defined above, and we proceed immediately to the computation of the \( IE \) as given in (18). From the profit-maximizing price, the mark-up can be found as:

\[ r_j - \nu = - \frac{y_j}{b + c} \]  \hfill (26)

The partial derivative of the demand function with respect to a change in the market equilibrium price follows from (25) as:

\[ \frac{\partial y_j}{\partial r} = b + nc \]  \hfill (27)

whereas the partial derivative of the equilibrium price with respect to a change in input \( i \) can be determined from the mark-up equation (26) as:\(^{14}\)

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\(^{13}\) As long as we measure all prices relative to those of a numeraire outside consumption good whose price is kept constant.

\(^{14}\) Note that demand \( y_j \) also depends on the price \( p_i \) through its dependence on \( r \).
Adding the pieces together leads to the following expression for the IE:

\[
IE = -\frac{n(b + nc)}{c(n-1)} \left( \int y \frac{\partial v}{\partial p_i} dp_i \right)
\]

where \( y \) denotes the sum of the demands for all varieties.

It was noted above that in the case of a monopoly with a linear demand curve the IE equals 50% of the direct effect. We must now conclude that under monopolistic competition with linear demand curves, such a fixed ratio between direct and indirect effects does not exist. It can be inferred from (29), (14) and (28) that the IE is equal to a fraction \((b + nc)/(b + c)\) of consumers’ surplus. This fraction is positive and smaller than 1 if \(b+nc<0\), zero if \(b+nc=0\) and negative if \(b+nc>0\). Depending on the parameters \(b\) and \(c\), and the number of firms \(n\), the IE can be a negligible fraction or a huge multiple of the direct effect of either sign. The analysis suggests that the number of firms, the steepness of the demand curves and the size of the substitution effects are the determinants of the sign and magnitude of the IE.

6.2 The Dixit-Stiglitz model

The Dixit-Stiglitz (1977) model is one of the oldest and best known models of monopolistic competition. It is used extensively in the new economic growth and new economic geography literature. Dixit and Stiglitz model the demand for a differentiated product by means of a representative consumer. We specify the utility of this consumer as 15:

\[
U = y^\alpha_0 \left( \sum_{j=1}^{n} y_j^\rho \right)^{1/\rho}
\]

where \( \alpha \) and \( \rho \) should be positive, moreover \( \rho \) should be smaller than 1.

Utility is maximized under a budget restriction:

15 Dixit and Stiglitz consider two generalizations of this utility function. The specification given here is usually employed in the subsequent literature.
\[ y_0 + \sum_{j=1}^{n} r_j y_j = X \]  

The demand functions that result from this problem are:

\[ y_j = \frac{r_j^{1/(\rho-1)} X}{\sum_{k} r_k^{\rho/(\rho-1)} (1 + \alpha)} \]  

Since the utility function in (30) is homothetic, consumers’ surplus \(CS\) provides a correct welfare measure as long as income does not change (Chipman and Moore, 1980). Consumers’ surplus is:

\[ CS = -\frac{X}{1 + \alpha} \rho^{-1} \ln \left( \sum_{j=1}^{n} r_j^{\rho/(\rho-1)} \right) \]  

and it is easy to verify that \( y_j = -\frac{\partial CS}{\partial r_j} \).

Dixit and Stiglitz assume that firms take the denominator of the first term of demand function in (32) as given. The price set by such firms can then be derived as:

\[ r_j = \frac{v(p)}{\rho} \]  

Using these equations, it is easy to compute the ingredients of (18) and the result is:

\[ IE = (1 - \rho) \frac{X}{1 + \alpha} \frac{1}{\rho} \int_{p_i}^{p} \frac{\partial v}{\partial p_i} dp_i \]  

When fixed costs do not change, the ratio between the \(IE\) and the direct effect is \((1 - \rho)/\rho\), which can take on any nonnegative value. When \(\rho = 1\) all varieties are

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16 However, it should be noted that in the change in consumers’ surplus is not equal to the compensating variation \(CV\). The two are related as follows: \(CV/X = 1 - \exp(-\Delta CS/X)\) (see Chipman and Moore, 1980, p.945). This implies that different results with respect to the AIE will be reached if the CV is used.
perfect substitutes and in this case the IE vanishes, which is intuitive. When $\rho$ is close to 0, the IE may be a large multiple of the direct effects.

Since $\rho$ is inversely related to the ‘preference for diversity’ of the representative consumer, we conclude that a strong preference for diversity leads to a large additional indirect effect. This result is in line with the one reached for monopoly under a loglinear demand curve. The result is also intuitive, since a strong preference for diversity implies relatively large monopoly power for the individual firms. The IE is equal to a fraction $(1-\rho)$ of the change in consumers’ surplus.

Note that the number of firms, $n$, does not appear in (35). This implies that the size of the IE does not tend to zero when the number of firms gets large. The results of the present subsection are somewhat more positive than that of the previous one in that we can exclude negative IE in the Dixit-Stiglitz model and that the size of the IE can be related to a key parameter of the model in an intuitive way.

6.3 The logit model of monopolistic competition
Anderson, de Palma and Thisse (1992, and earlier papers referred to in that book) used the logit model in order to study an industry with differentiated products. They made use of the representative consumer theory of that model developed in McFadden (1981).

The indirect utility function of the representative consumer is:

$$V = \ln \left( \sum_{j=1}^{n} \exp \left( \frac{X - r_j}{\mu} \right) \right)$$

$$= \frac{X}{\mu} + \ln \left( \sum_{j} \exp \left( \frac{-r_j}{\mu} \right) \right)$$

This indirect utility function is quasi-linear, like the one in the model with linear demand equations. This implies that consumers’ surplus, which is equal to $\mu$ times the second term on the right-hand-side can be used as the appropriate welfare measure. The representative consumer buys one unit of the product per period and the size of the market is equal to $N$. The probability that variety $k$ will be chosen is:

17 This is sometimes referred to as the logsum.
\[ \pi_j = \sum_k e^{-r_k/\mu} e^{-r_j/\mu} \]

The expected demand for the product \( j \) is therefore equal to \( N\pi \). We treat the \( \pi \)'s as (deterministic) market shares. The implied demand function is similar to that of the Dixit-Stiglitz model with the (natural) logarithm of the price used instead of the untransformed price. The comparison between these two models is therefore similar to that between the linear and loglinear demand function for the monopoly case.

The optimal price is:

\[ r_j = v(p) + \frac{\mu}{1 - \pi_j} \]

Computation of the IE is particularly easy for the logit model, since total demand is always equal to \( N \). In a symmetric equilibrium each firm has a market share \( 1/n \), independent of the value of the equilibrium price \( r \). Hence \( \partial y_j / \partial r = 0 \) for each firm and the IE is always equal to 0.

6.4 Dixit-Stiglitz model with free entry

CS for the Dixit-Stiglitz model was given in (33) and now we rewrite it as:

\[ CS = \frac{X}{1 + \alpha} \left( \frac{1 - \rho}{\rho} \ln(n) - \ln(r) \right). \]

The profit maximizing price was given in (34), and the zero profit condition implies that the number of firms \( n \) equals:

\[ n = (1 - \rho) \frac{X}{1 + \alpha} \frac{1}{F(p)} \]

After substitution of these results in (39) we can compute the partial derivatives:

\[ \frac{\partial CS}{\partial v(p)} = -\frac{X}{1 + \alpha} \frac{1}{v(p)} \]
\[
\frac{\partial CS}{\partial F(p)} = -\frac{1 - \rho}{\rho(1 + \alpha)} \frac{X}{F(p)} \frac{1}{1 + \alpha} F(p)
\]

From (22) we now derive the IE as:

\[
AIE = (1 - \rho) \frac{X}{1 + \alpha} \left( \int_{V^{\phi}} \frac{\partial v(p)}{\partial p_i} dp_i + \frac{(1 - \rho)^{\phi_i}}{\rho} \int_{\phi_i} \frac{\partial F(p)}{\partial p_i} dp_i \right)
\]

Comparison with (35) shows that a new term has been added that represents the effect of entry. Entry therefore causes a second additional indirect effect that is also positive. The conclusions reached for the Dixit-Stiglitz model do therefore not change substantially when we allow for entry (or exit) of firms.

6.5 The logit model with free entry
From (36) we rewrite the sum of the consumers’ surpluses for the \(N\) consumers in the logit model in market equilibrium as:

\[
CS = N(\mu \ln(n) - r)
\]

The profit maximizing price was given in (38) and with free entry the number of firms is:

\[
n = \frac{N\mu}{F(p)} + 1.
\]

This shows that the number of firms is independent of the variable cost, that is, \(\partial n / \partial v = 0\). Note, from (38), that \(\partial r / \partial v(p) = 1\). This implies that the first term on the right-hand-side of (22') (or (22)) is identically 0.

To find the sign of the second term, note that it follows from (45) that:

\[
\frac{\partial n}{\partial F(p)} = -\frac{N\mu}{F(p)^2}.
\]

Using this result we find for the IE:

\[
IE = \int_{\phi_i} \left( -\frac{N\mu}{N\mu + F(p)} \right) \frac{\partial F}{\partial p_i} dp_i
\]
The most important aspect of (47) is its sign: the additional indirect effect turns out to be negative in this case. Even though we could, in section 5, not determine the sign of the $IE$ in the case of free entry, it seemed intuitively plausible that it would be positive. The negative sign of the $IE$ in equation (47) shows that in the case of the logit model with free entry the change in consumers’ surplus overestimates the true welfare gain of the project. Note, however, that the bias associated with using the direct effect is small if the preference for diversity (measured by $\mu$), is small, or if the fixed cost is large (or both).

7 Implications for Cost-Benefit Analysis

Investments in public infrastructure often result in decreases in cost for industries that use this infrastructure. If these industries are imperfectly competitive, they may generate $IE$s. If we want to incorporate these effects in a $CBA$, a study of the way the effects of policy measures pass through the economy to the consumers is necessary. The setting we considered in the previous sections is one of the simplest that serves this purpose. A number of conclusions can be drawn.

First, under the class of monopolistic competition models that we have considered, $IE$’s exist in general and they may take on any logically possible value. That is, they may have the same order of magnitude as the conventionally measured direct effects, may be much smaller or may exceed them substantially. Thus there is a real possibility that conventional CBA substantially underestimates or overestimates the true welfare gains of a project resulting in an input price change.

Second, since the sign of the additional indirect effect is indeterminate: it is possible that conventional CBA overestimates the welfare effect of an investment resulting in a change in an input price by ignoring the additional indirect.

Third, the details of the model specification or even the size of the parameters can be of crucial importance for the sign and size of the $IE$’s. This is most dramatically the case for the model with linear demand curves. This conclusion confirms the few results that are available in the literature: Venables and Gasiorek (1998) found substantial positive $IE$s in simulation results, whereas Newbery (1998) showed that the $IE$ may be negative. Since the only difference between the various models we considered here is the specification of the demand functions, we can conclude that their curvature is of crucial importance for the generation of $IE$s.

The relevance of these results for the ‘Aschauer debate’ is that they provide a possible explanation for the wide range of empirical results with respect to the total welfare effects of investments in public infrastructure. These results may not only be due to differences in econometric techniques and data quality, but
may also have to do with (apparently) small differences in the demand functions that are relevant for the cases studied. Also misspecification of the demand functions may have important consequences for the conclusions drawn.

The analysis in the present paper suggests that trustworthy measurement of the IE’s associated with actual investments in public infrastructure may well be practically impossible. A major conclusion is that ‘the devil is in the details’ and since these details may differ between investments that are similar in many respects (for instance because expanding the capacity of road infrastructure affects different industries depending on the location of the investment) generally applicable rules of thumb are unlikely to exist. Moreover, an in-depth investigation of the IE’s associated with a specific project will usually be too costly. Since small differences in model specifications may well lead to substantially different conclusions the ability for such research to be of assistance in settling debates about indirect effects is probably limited. In short, the possibilities for addressing IE’s in social CBA seem to be fairly limited at present.

The presence of IE’s is directly related to distortions in the resource allocation process associated with imperfect competition. Including these IEs in a CBA implies that one takes into account the mitigating (or exaggerating) effect of the project on existing market distortions. In general, it would be preferable to address these distortions by direct means such as the improvement of competition. Even though the presence of imperfect competition in many parts of the economy suggests that such first best policies are out of reach, it is nevertheless important to keep in mind that there may be other ways, which are possibly to be preferred, to mitigate the effects of such market distortions.

References


