



Tri/Bi-maximal lepton mixing and leptogenesis

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Abstract

In models with flavour symmetries added to the gauge group of the Standard Model the CP-violating asymmetry necessary for leptogenesis may be related with low-energy parameters. A particular case of interest is when the flavour symmetry produces exact Tri/Bi-maximal lepton mixing leading to a vanishing CP-violating asymmetry. In this paper we present a model-independent discussion that confirms this always occurs for unflavoured leptogenesis in type I see-saw scenarios, noting however that Tri/Bi-maximal mixing does not imply a vanishing asymmetry in general scenarios where there is interplay between type I and other see-saws. We also consider a specific model where the exact Tri/Bi-maximal mixing is lifted by corrections that can be parametrised by a small number of degrees of freedom and analyse in detail the existing link between low and high-energy parameters — focusing on how the deviations from Tri/Bi-maximal are connected to the parameters governing leptogenesis.

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1. Introduction

Results from neutrino oscillation experiments [1] have firmly established that neutrinos have tiny but non-zero masses. From a theoretical perspective the smallness of neutrino masses can be well understood within the see-saw mechanism [2], in which the Standard Model (SM) is extended by adding new heavy states. Light neutrino masses are generated through effective operators which are typically suppressed by the masses of the states giving rise to the see-saw. In type I see-saw the extra states are right-handed (RH) neutrinos with large Majorana masses. Apart from providing an explanation for the origin of neutrino masses, the mechanism contains all the necessary ingredients for a dynamical generation of a cosmic lepton asymmetry through the decays of the heavy singlet neutrinos (leptogenesis): (a) Lepton number violation arising from the Majorana mass terms of the new fermionic states; (b) CP-violating sources from complex Yukawa couplings; (c) Departure from thermal equilibrium in the hot primeval plasma at the time the singlet neutrinos start decaying. This lepton asymmetry is then reprocessed into a baryon asymmetry through $B + L$ violating anomalous electroweak processes [3] thus yielding an explanation to the origin of the baryon asymmetry of the Universe [4] i.e. baryogenesis through leptogenesis (for a recent review see [5]).

The structure of mixing in the leptonic sector suggested by experimental data is in sharp contrast with the small mixing that characterises the quark sector. Observations indicate that solar neutrino oscillation is described by a large but non-maximal mixing angle, atmospheric neutrino oscillation is described by maximal or nearly-maximal angle, and reactor data puts a small upper bound on the third angle [6–8]. This mixing pattern is well described by the so-called Tri/Bi-maximal (TB) scheme [9] which corresponds to a unitary matrix of the form

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}, \quad (1)$$

and to the following mixing angles:

$$\sin^2 \theta_{13}^{TB} = 0, \quad \sin^2 \theta_{23}^{TB} = 1/2, \quad \sin^2 \theta_{12}^{TB} = 1/3. \quad (2)$$

This particular mixing structure can be interpreted as a signal of an underlying symmetry¹ and has motivated a great deal of studies aiming to determine the possible flavour symmetry responsible for such a pattern. A large amount of discrete and continuous symmetries have been considered [11–43] and among them discrete non-Abelian ones have been found to be particularly interesting as they can more naturally lead to the required pattern. In the realization of explicit models, a general feature is the breaking of the flavour symmetry: this is a well-known result of a no-go theorem [16,44] that applies in the vast majority of relevant cases; it could be evaded, for example using light Higgs fields charged under the flavour symmetry, but inconsistencies related to flavour-changing neutral current or lepton-flavour violating processes could appear. On the contrary, allowing only heavy Higgs fields charged under the flavour symmetry, it is possible to avoid these dangerous effects [45].

Global fits [7] to the data provides a subtle hint of a deviation from the TB scheme and therefore it is desirable if the flavour symmetry predicts TB at leading order (LO) and allows perturbations at higher orders. It is possible to constrain the amount of these corrections by comparing the TB value of the mixing angles to their experimental measurements: the solar angle is

¹ For a different approach see [10].

known with the lowest relative error and as a result it fixes the upper bound of the deviations at about 0.05. Avoiding any parameter tuning or particular relations among the deviations, we expect that the other LO mixing angles are perturbed by quantities of the same order of magnitude: in particular the corrected θ_{13} is expected to be non-vanishing, but very small.²

In order to explain the baryon asymmetry of the Universe by leptogenesis, CP violation in the leptonic sector is needed. In principle it can be argued that leptogenesis is supported by any observation of CP violation in the leptonic sector, e.g. in neutrino oscillation experiments. However, since generically the baryon asymmetry is insensitive to the low-energy CP-violating phases [46,47] a definitive conclusion cannot be established from such an observation. In contrast, in models based on flavour symmetries that predict the TB mixing pattern, the parameter space is further constrained and as a result one could expect, quite generically, some link between low-energy observables and leptogenesis. As pointed out in [33], in the context of an A_4 flavour symmetry model with type I see-saw the CP-violating asymmetry (ϵ_{N_α}) vanishes in the limit of exact TB mixing, with leptogenesis becoming viable only when deviations from this pattern are taken into account. The explicit structure of the corrections responsible for these deviations are model-dependent and therefore whether a connection between ϵ_{N_α} and low-energy parameters can be established will depend on the particular realization.

In this paper we extend upon the work in [33]. In particular, we study the viability of leptogenesis in the context of models based on an arbitrary flavour symmetry leading to the TB lepton mixing pattern through the see-saw mechanism. When there is only type I see-saw and independently of the nature of the underlying symmetry, we conclude that $\epsilon_{N_\alpha} = 0$ in the limit of exact TB mixing or any other exact mixing schemes where the mixing matrix consists purely of numbers — such as Bi-maximal mixing [48], golden-ratio mixing [49] and some (but not all) cases of Tri-maximal mixing [50,51]. Under these conditions, only deviations from the flavour symmetry imposed pattern yield $\epsilon_{N_\alpha} \neq 0$. It is important to note that this result is not in general valid in the presence of other types of see-saw (e.g. with the interplay of type I and type II).

Following from the model-independent proof we consider particular cases. We check our result by considering several models discussed in the literature. Finally, we also take a specific simple A_4 flavour model [39], where low-energy observables arising from TB deviations can be linked to the CP-violating asymmetry in a straightforward manner and analyse it in more detail.

Our discussion will be entirely devoted to “unflavoured” leptogenesis scenarios: in the framework of flavour symmetry models predicting TB mixing the heavy singlet neutrinos typically have masses above 10^{13} GeV and for $T \gtrsim 10^{12}$ GeV lepton flavours are indistinguishable [52,53].

This paper is organised as follows: in Section 2 we fix our notation and briefly comment upon some generic aspects of leptogenesis. For completeness of our results, in Section 3 we present a brief analysis of randomly generated TB mixing and its implications for the CP-violating asymmetry. We turn to the main subject of this paper in Section 4, showing that an exact mixing scheme enforced by a flavour symmetry in scenarios with just type I see-saw leads to a vanishing CP-violating asymmetry. Leptogenesis becomes potentially viable only when higher-order flavour symmetry corrections lift the pattern — or otherwise if other types of see-saw (e.g. type II) are also present. In Section 5 we confirm our model-independent results in particular realizations, and in Section 6 we analyse in detail a specific model in which low-energy param-

² For an alternative proposal see [43].

ters and the CP-violating asymmetry are directly related in a simple way. Finally in Section 7 we conclude by summarizing our results.

2. The basic framework

In this section we will establish both the notation and a choice of a convenient basis. Let us consider the leptonic part of the SM Lagrangian extended with three fermionic heavy singlets N_α ³

$$-\mathcal{L} = (Y)_{ij} \bar{L}_i H \ell_j^c + (\lambda)_{i\alpha} \bar{L}_i \tilde{H} N_\alpha + \frac{1}{2} (M_R)_{\alpha\beta} (N_\alpha^c)^T N_\beta + \text{h.c.} \quad (3)$$

Here L_i are the lepton $SU(2)$ doublets, ℓ_i^c are the complex conjugate charged lepton $SU(2)$ singlets and H ($\tilde{H} = i\sigma_2 H^*$) is the Higgs $SU(2)$ doublet. Latin indices i, j, \dots label lepton flavour, whereas Greek indices α, β, \dots denote RH species. Y , λ and M_R are 3×3 matrices in flavour space.

At energy scales well below the RH neutrino masses, light neutrino masses are generated via effective operators. The effective Majorana neutrino mass matrix is

$$m_\nu = -m_D M_R^{-1} m_D^T, \quad (4)$$

where $m_D = \lambda v / \sqrt{2}$ ($v \simeq 246$ GeV). We then consider the unitary matrices U_ℓ , U_{ℓ^c} and U_ν , which diagonalise the charged lepton and neutrino mass matrices:

$$\hat{m}_\ell = U_\ell^\dagger Y U_{\ell^c} \frac{v}{\sqrt{2}}, \quad \hat{m}_\nu = U_\nu^T m_\nu U_\nu, \quad (5)$$

where the “ $\hat{}$ ” refers to a diagonal matrix. The lepton mixing matrix is defined by U_ℓ and U_ν :

$$U = (U_\ell)^\dagger U_\nu. \quad (6)$$

From now on we will assume that in the basis in which the charged lepton mass matrix is diagonal, m_ν is exactly diagonalised by the TB mixing matrix U_{TB} and therefore

$$\hat{m}_\nu = D U_{TB}^T m_\nu U_{TB} D, \quad (7)$$

where D accounts for the low-energy Majorana phases

$$D = \text{diag}(e^{i\varphi_1}, e^{i\varphi_2}, 1). \quad (8)$$

In general m_D as well as M_R ($M_R = M_R^T$) are complex matrices which can be diagonalised as follows

$$\begin{aligned} \hat{m}_D &= U_L^\dagger m_D U_R, \\ \hat{M}_R &= V_R^T M_R V_R, \end{aligned} \quad (9)$$

with U_L, U_R, V_R 3×3 unitary matrices, characterised in general by 3 rotation angles and 6 phases.

According to Eq. (9) the effective neutrino mass matrix in (4) can be written as

$$m_\nu = -U_L \hat{m}_D (U_R^\dagger V_R) \hat{M}_R^{-1} (V_R^T U_R^*) \hat{m}_D U_L^T. \quad (10)$$

³ The subsequent analysis is done for three RH neutrinos, but it can be generalised to an arbitrary number with the conclusions being independent of it.

The requirement of having exact TB diagonalisation can be written either in terms of constraints over the light neutrino mass matrix entries, namely

$$\begin{aligned} m_{\nu_{12}} &= m_{\nu_{13}}, \\ m_{\nu_{22}} &= m_{\nu_{33}}, \\ m_{\nu_{11}} &= m_{\nu_{22}} + m_{\nu_{23}} - m_{\nu_{12}}, \end{aligned} \quad (11)$$

or, according to Eqs. (7) and (10), requiring that

$$\hat{m}_\nu = -D(U_{TB}^T U_L) \hat{m}_D (U_R^\dagger V_R) \hat{M}_R^{-1} (V_R^T U_R^*) \hat{m}_D (U_L^T U_{TB}) D \quad (12)$$

is diagonal and real. It is useful to introduce the notation of the Dirac neutrino mass matrix in the basis in which the RH neutrino mass matrix \hat{M}_R is real and diagonal:

$$m_D^R \equiv m_D V_R. \quad (13)$$

2.1. General remarks on leptogenesis

As mentioned in the introduction, singlet neutrinos in flavour symmetry models typically have masses above 10^{13} GeV. Thus, within these frameworks leptogenesis proceeds at temperatures at which lepton flavour effects can be completely neglected. In the standard thermal leptogenesis scenario singlet neutrinos N_α are produced by scattering processes after inflation. Subsequent out-of-equilibrium decays of these heavy states generate a CP-violating asymmetry given by [5,54]

$$\epsilon_{N_\alpha} = \frac{1}{4v^2\pi(m_D^{R\dagger} m_D^R)_{\alpha\alpha}} \sum_{\beta \neq \alpha} \text{Im}[(m_D^{R\dagger} m_D^R)_{\beta\alpha}]^2 f(z_\beta), \quad (14)$$

where $z_\beta = M_\beta^2/M_\alpha^2$ and the loop function can be expressed as

$$f(z_\beta) = \sqrt{z_\beta} \left[\frac{2-z_\beta}{1-z_\beta} - (1+z_\beta) \log\left(\frac{1+z_\beta}{z_\beta}\right) \right]. \quad (15)$$

Depending on the singlet neutrino mass spectrum the loop function can be further simplified. In the hierarchical limit ($M_\alpha \ll M_\beta$) this function becomes

$$f(z_\beta) \rightarrow -\frac{3}{2\sqrt{z_\beta}}, \quad (16)$$

whereas in the case of an almost degenerate heavy neutrino spectrum ($z_\beta = 1 + \delta_\beta$, $\delta_\beta \ll 1$) it can be rewritten as

$$f(1 + \delta_\beta) \simeq -\frac{1}{\delta_\beta}. \quad (17)$$

In any case, as can be seen from Eq. (14), whether the CP-violating asymmetry vanishes will be determined by the Yukawa coupling combination $m_D^{R\dagger} m_D^R$.

3. CP asymmetry and exact TB mixing without any underlying flavour symmetry

While the TB mixing pattern can be well understood as a consequence of an underlying flavour symmetry, in principle it might be that it arises from a random set of parameters (though quite unlikely). For completeness, in this section we consider this possibility and study the consequences

on the CP-violating asymmetry. Neutrino mixing angles are fixed to satisfy the TB mixing pattern and in addition to the measured mass squared differences we have a set of eight constraints on the parameter space: the TB mixing condition enforces the relations in Eq. (11), yielding six constraints (from the real and imaginary parts of the mass matrix entries); the atmospheric and solar mass scales provide the remaining two.

To determine the effect of such constraints on ϵ_{N_α} it is practical to use a parametrisation of m_D that ensures that the TB mixing and the correct neutrino masses are obtained. In the basis in which the RH neutrino mass matrix is diagonal and real it is convenient to introduce the orthogonal complex matrix R defined by the so-called Casas–Ibarra parametrisation [55], namely

$$R^* = (\hat{m}_\nu)^{-1/2} U^T m_D^R (\hat{M}_R)^{-1/2}. \tag{18}$$

All low-energy observables are contained in the leptonic mixing matrix U and in the diagonal and real light neutrino mass matrix \hat{m}_ν . The matrix R turns out to be very useful in expressing the CP-violating asymmetry parameter. Considering for simplicity the case of hierarchical RH neutrinos ($M_1 \ll M_2 \ll M_3$ — thus validating the approximation in Eq. (16)), Eq. (14) can be rewritten as

$$\epsilon_{N_\alpha} = -\frac{3M_\alpha}{8\pi v^2} \frac{\Im[\sum_j m_j^2 R_{j\alpha}^2]}{\sum_j m_j |R_{j\alpha}|^2}, \tag{19}$$

where $m_j \equiv (\hat{m}_\nu)_{jj}$. Once the RH neutrino mass spectrum and low-energy observables are fixed, random values of m_D^R correspond to random values of R . It is shown by Eq. (19) that leptogenesis is completely insensitive to low-energy lepton mixing and CP-violating phases [46]⁴ and therefore the viability of leptogenesis is not at all related with any accidental mixing pattern considered. The CP-violating asymmetry is determined by the values of the entries of R which are arbitrary in the absence of any flavour symmetry, and consequently $\epsilon_{N_\alpha} \neq 0$ in general and its absolute value depends upon the heavy fermionic singlet masses, the light neutrino masses and R .

To illustrate this point we consider the case in which only N_1 decays are relevant for the generation of a lepton asymmetry. We assume normal hierarchy for the light neutrino spectrum and a simple $R = R_{13}(\rho_{13})$ with $\rho_{13} = \omega + i\sigma$ (i.e. R is a ρ_{13} rotation matrix). Under these assumptions the CP-violating asymmetry in Eq. (19) becomes

$$\epsilon_{N_1} = -\frac{3M_1 \sqrt{\Delta m_{\text{atm}}^2}}{2\pi v^2} \frac{\cos \omega \sinh \sigma}{\sqrt{\cosh 2\sigma - \cos 2\omega}}. \tag{20}$$

From Fig. 1 it can be seen that barring the cases $\omega = \pi/2$ and/or $\sigma = 0$ the CP-violating asymmetry does not vanish and its values are well within the range required for successful leptogenesis, regardless of the mixing pattern.

4. Implications of flavour symmetries on the CP asymmetry

We consider now the case in which an underlying flavour symmetry enforces an exact mixing pattern. It will be evident throughout the proof that it holds for any mixing pattern where the mixing matrix consists purely of numbers, but we will assume TB mixing for definiteness.

⁴ This statement is in general also true in flavoured leptogenesis [47].

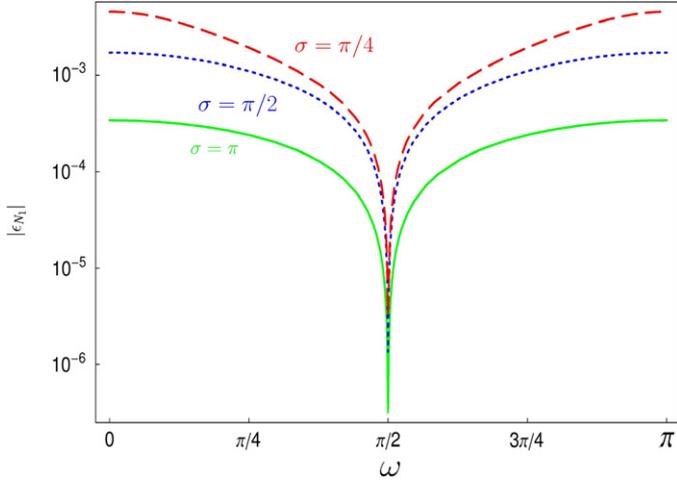


Fig. 1. CP-violating asymmetry as a function of the angle ω for different values of σ . M_1 is fixed to 10^{13} GeV and Δm_{atm}^2 to 2.39×10^{-3} eV² [8] (see the text for further details).

Within the case considered the transformation properties of L_i and N_α under the flavour symmetry group (G_f) determine the structure of m_D and M_R (which are no longer arbitrary). Indeed, these matrices can be regarded as form-diagonalisable matrices [44], i.e. the parameters which determine their eigenvalues are completely independent from the parameters that define their diagonalising matrices. Accordingly, vanishing off-diagonal elements of \hat{m}_ν in Eq. (12) can arise only if

$$U_{TB}^T U_L = P_L O_{D_i} \quad \text{and} \quad U_R^\dagger V_R = O_{D_i}^\dagger P_R O_{R_m}, \quad (21)$$

where $P_{L,R} = \text{diag}(e^{i\alpha_1^{R,L}}, e^{i\alpha_2^{R,L}}, e^{i\alpha_3^{R,L}})$ whereas O_{D_i} and O_{R_m} are respectively unitary and orthogonal matrices that arbitrarily rotate the i and m degenerate eigenvalues of m_D and M_R such that if m_D (M_R) has no degenerate eigenvalues $O_{D_i} = \mathbb{1}$ ($O_{R_m} = \mathbb{1}$). Note that the requirement of having canonical kinetic terms in addition to preserving the m -fold degeneracy of the RH neutrino mass matrix enforce O_{R_m} to be real. Although O_{D_i} and O_{R_m} do not have any effect in Eq. (12) they do affect the structure of $U_{L,R}$ and V_R and correspondingly of m_D (see Eq. (9)). V_R can be defined in such a way that \hat{M}_R is real, and the phases contained in \hat{m}_D are now denoted by γ_i and must obey: $\varphi_i + \alpha_i^R + \alpha_i^L + \gamma_i = 2k\pi$ and $\alpha_3^R + \alpha_3^L + \gamma_3 = 2n\pi$. It is easy to understand the conditions given in Eq. (21) by the use of a *reductio ad absurdum*. Let us consider for simplicity the case without any degeneracy in the eigenvalues of \hat{m}_D and \hat{M}_R : $O_{D_i} = \mathbb{1}$ and $O_{R_m} = \mathbb{1}$. If the products $U_{TB}^T U_L$ and $U_R^\dagger V_R$ are not diagonal, but simply unitary matrices with non-vanishing off-diagonal entries, then the right-hand side of Eq. (12) is in general a matrix whose entries are linear combinations of the mass eigenvalues of \hat{m}_D and of \hat{M}_R . In order to have \hat{m}_ν diagonal, the off-diagonal entries must vanish and this is possible only if the respective linear combinations cancel out. However, there are no *a priori* reasons to have such cancellations, since it corresponds to have well-defined relationships between the eigenvalues of \hat{m}_D and of \hat{M}_R , which is, in other words, a fine-tuning. Avoiding this possibility, the only solution is to consider Eq. (21).

It is useful to classify the number of degenerate eigenvalues of m_D and M_R . There are nine cases in total: 3 for m_D ($i = 1, 2$ or 3-fold degeneracy) and 3 for M_R ($m = 1, 2$ or 3-fold degeneracy). In the following we will identify each case by (i, m) . The cases (3, 3), (2, 3) and

(3, 2) are not consistent with experimental data on neutrino mass splittings, so we are left with six viable cases:

- (a) (1, 1): m_D and M_R have no degenerate eigenvalues;
- (b) (2, 1): m_D with 2 degenerate eigenvalues;
- (c) (1, 2): M_R with 2 degenerate eigenvalues;
- (d) (2, 2): m_D and M_R with 2 degenerate eigenvalues;
- (e) (3, 1): m_D with 3 degenerate eigenvalues;
- (f) (1, 3): M_R with 3 degenerate eigenvalues.

We proceed to show that all the viable cases obey a common expression. In the basis in which the RH neutrinos are diagonal we use m_D^R (see Eq. (13)) and write $\hat{m}_D = \delta_i \text{diag}(v_1, v_2, v_3)$, where we have schematically indicated with δ_i the fact that i values of $\text{diag}(v_1, v_2, v_3)$ are equal. In other words for $\delta_i = \delta_3$ we have $\text{diag}(v_1, v_1, v_1)$ and for $\delta_i = \delta_2$ we have $\text{diag}(v_1, v_2, v_1)$ or one of its possible permutations. We thus have

$$m_D^R = U_{TB} P_L O_{D_i} \delta_i \text{diag}(v_1, v_2, v_3) O_{D_i}^\dagger P_R O_{R_m}. \quad (22)$$

It is clear that in the subspace of the i degenerate eigenvalues the rotation O_{D_i} acts as $O_{D_i} \delta_i \text{diag}(v_1, v_2, v_3) O_{D_i}^\dagger \rightarrow \delta_i \text{diag}(v_1, v_2, v_3)$. Therefore we simplify the expression of m_D^R :

$$m_D^R = U_{TB} P_L \delta_i \text{diag}(v_1, v_2, v_3) P_R O_{R_m}. \quad (23)$$

The next step consists in the redefinition of the v_i by absorbing P_L, P_R . In this way the degeneracy of the i eigenvalues is broken and we finally get

$$\begin{aligned} m_D^R &= U_{TB} \text{diag}(v_1, v_2, v_3) O_{R_m} \\ &= \begin{pmatrix} \sqrt{\frac{2}{3}} v_1 & \frac{v_2}{\sqrt{3}} & 0 \\ -\frac{v_1}{\sqrt{6}} & \frac{v_2}{\sqrt{3}} & -\frac{v_3}{\sqrt{2}} \\ -\frac{v_1}{\sqrt{6}} & \frac{v_2}{\sqrt{3}} & \frac{v_3}{\sqrt{2}} \end{pmatrix} O_{R_m}. \end{aligned} \quad (24)$$

According to our formalism, the RH neutrino mass matrix is trivially given by

$$\hat{M}_R = \delta_m \text{diag}(M_1, M_2, M_3), \quad (25)$$

where δ_m indicates that m eigenvalues of $\text{diag}(M_1, M_2, M_3)$ are degenerate.

We now rewrite Eq. (23) according to the following parametrisation

$$m_D^R = U_{TB} P \hat{v} O_{R_m}, \quad (26)$$

with $\hat{v} = \text{diag}(|v_1|, |v_2|, |v_3|)$ and all the phases absorbed in the diagonal unitary matrix P . In this basis and using the parametrisation given in Eq. (26) for m_D^R , the type I see-saw formula of Eq. (12) is written as

$$\begin{aligned} \hat{m}_\nu &= -D U_{TB}^T (U_{TB} P \hat{v} O_{R_m}) \hat{M}_R^{-1} (O_{R_m}^T \hat{v} P U_{TB}^T) U_{TB} D \\ &= (D P e^{i\pi/2}) \hat{v} \hat{M}_R^{-1} \hat{v} (e^{i\pi/2} P D) = (\hat{v} \hat{M}_R^{-1/2} R^\dagger) (R^* \hat{M}_R^{-1/2} \hat{v}), \end{aligned} \quad (27)$$

where $D = P^* e^{-i\pi/2}$ is a consequence of our definition of \hat{m}_ν in Eq. (12), and where we have introduced the arbitrary orthogonal complex matrix R in the last part of Eq. (27). O_{R_m} acts

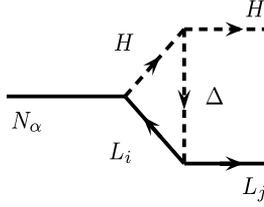


Fig. 2. Vertex correction involving a triplet scalar Δ .

only in the subspace of the degenerate right-handed neutrinos and in this subspace we have by definition $O_{R_m} O_{R_m}^T = \mathbb{1}$. From Eq. (27) we have that

$$\hat{m}_\nu^{-1/2} \hat{\nu} \hat{M}_R^{-1/2} R^\dagger = \mathbb{1}, \quad (28)$$

and remembering that $R^\dagger R^* = R^T R = \mathbb{1}$ we arrive at our parametrisation for R^*

$$R^* = \hat{m}_\nu^{-1/2} \hat{\nu} \hat{M}_R^{-1/2}. \quad (29)$$

By comparing Eq. (29) with the Casas–Ibarra parametrisation given in Eq. (18) we deduce that in the case of exact TB mixing the matrix R is real and according to Eq. (19) the CP-violating asymmetry vanishes.

Note that so far we did not refer to any specific model realisation and we have assumed just exact TB diagonalisation of m_ν within the context of type I see-saw. We not only confirm the result in [33] (in which a model with the A_4 flavour symmetry has exact TB mixing leading to a vanishing CP-violating asymmetry), but also extend it to any possible flavour symmetry responsible for the exact TB scheme.⁵

It is also straightforward to check by replacing U_{TB} with the appropriate mixing matrix that the matrix R still turns out to be real for other exact mixing schemes as long as their mixing matrix also consists purely of numbers (e.g. the corresponding matrix for the Bi-maximal mixing scheme). Note also that although we have only considered three RH neutrinos our result is absolutely generalisable to models with either two RH neutrinos or more than three such as [56].

The proof does not hold however in the presence of additional degrees of freedom, e.g. in models involving type I and type II see-saw. Other contributions to the CP-violating asymmetry will in general not vanish in the limit of exact TB mixing, rendering our result invalid for situations which do not have only type I see-saw. In scenarios with type II see-saw the details concerning the generation of the lepton asymmetry will depend upon the hierarchies between the triplet (Δ) and the lightest RH neutrino masses [57,58]. Even in the case $M_\Delta > M_{N_\alpha}$ (N_α being the lightest RH neutrino) the CP asymmetry will receive an extra contribution from the loop diagram shown in Fig. 2. This contribution will not necessarily vanish, although it is constrained by the TB mixing pattern [60].

An important consequence of our proof is that if the TB mixing pattern is due to any underlying flavour symmetry in a type I see-saw scenario, the viability of leptogenesis depends upon possible departures from the exact pattern. In the context of models based on discrete flavour symmetries that predict TB mixing at LO this is achieved through next to LO (NLO) corrections. Since the size of the deviations from TB mixing are not arbitrary, in principle one might expect

⁵ This result is basis independent and thus remains true even assuming a non-diagonal charged lepton mass matrix.

the CP-violating asymmetry to be constrained by low-energy observables such as θ_{13} and/or the CP-violating phases.

In order to see if this is the case let us consider the most generic situation, in which NLO corrections affect m_ℓ , m_D and M_R . We can perform a linear expansion in the corrections that appear at NLO. First, we note that m_ℓ is no longer diagonal and thus we have to move to the basis in which the charged lepton mass matrix is diagonal:

$$U_\ell^\dagger m_\ell m_\ell^\dagger U_\ell = (m_\ell m_\ell^\dagger)_{diag}, \quad (30)$$

where $U_\ell = \mathbb{1} + U_\ell^{(1)}$, with $U_\ell^{(1)}$ the matrix of the NLO shifts. Eq. (9) is modified as follows

$$\begin{aligned} (\mathbb{1} + U_L^{(1)\dagger}) U_L^\dagger (m_D + m_D^{(1)}) U_R (\mathbb{1} + U_R^{(1)}) &\simeq \hat{m}_D + U_L^\dagger m_D^{(1)} U_R + U_L^{(1)\dagger} \hat{m}_D + \hat{m}_D U_R^{(1)} \\ &\equiv \hat{m}'_D = \hat{m}_D + \hat{m}_D^{(1)}, \\ (\mathbb{1} + V_R^{(1)T}) V_R^T (M_R + M_R^{(1)}) V_R (\mathbb{1} + V_R^{(1)}) &\simeq \hat{M}_R + V_R^T M_R^{(1)} V_R + V_R^{(1)T} \hat{M}_R + \hat{M}_R V_R^{(1)} \\ &= \hat{M}'_R = \hat{M}_R + \hat{M}_R^{(1)}. \end{aligned} \quad (31)$$

Here the unitary matrices are parametrised as the LO terms shifted by the NLO ones. The superscript “ (1) ” refers to the NLO corrections and “ $\hat{}$ ” to the complete mass matrices up to NLO. The corresponding shifts on the light neutrino masses due to the NLO corrections can be estimated according to

$$\mathcal{O}(\hat{m}'_v - \hat{m}_v) \sim \mathcal{O}(\hat{m}_D \hat{m}_D^{(1)} / \hat{M}_R) \sim \mathcal{O}(\hat{m}_D^2 \hat{M}_R^{(1)} / \hat{M}_R^2). \quad (32)$$

Similarly, we can parametrise the shift from the exact TB pattern in the neutrino mixing matrix:

$$U_\nu = U_{TB} (\mathbb{1} + U_{TB}^{(1)}) D, \quad (33)$$

where $U_{TB}^{(1)}$ arises by the interplay between all the corrections. When we constrain the entries of $U_{TB}^{(1)}$ by neutrino experimental data, we obtain constraints on $U_\ell^{(1)}$, $U_L^{(1)}$, $U_R^{(1)}$, $V_R^{(1)}$. Experimental data on neutrino mass splittings constrains $m_D^{(1)}$ and $M_R^{(1)}$.

We write now Eq. (24) in the new basis in which the RH neutrinos and the charged leptons are diagonal:

$$\begin{aligned} m_D^{R'} &= (\mathbb{1} + U_\ell^{(1)\dagger}) U_L (\mathbb{1} + U_L^{(1)}) (\hat{m}_D + \hat{m}_D^{(1)}) (\mathbb{1} + U_R^{(1)\dagger}) U_R^\dagger V_R (\mathbb{1} + V_R^{(1)}) \\ &= m_D^R + U_\ell^{(1)\dagger} m_D^R + U_L U_L^{(1)} \hat{m}_D U_R^\dagger V_R + U_L \hat{m}'_D U_R^\dagger V_R \\ &\quad + U_L \hat{m}_D U_R^{(1)\dagger} U_R^\dagger V_R + m_D^R V_R^{(1)}. \end{aligned} \quad (34)$$

Thus after including NLO corrections the quantity relevant for leptogenesis becomes

$$\begin{aligned} m_D^{R'\dagger} m_D^{R'} &= m_D^{R\dagger} m_D^R + [m_D^{R\dagger} (U_\ell^{(1)\dagger} m_D^R + U_L U_L^{(1)} \hat{m}_D U_R^\dagger V_R + U_L \hat{m}'_D U_R^\dagger V_R \\ &\quad + U_L \hat{m}_D U_R^{(1)\dagger} U_R^\dagger V_R + m_D^R V_R^{(1)}) + \text{h.c.}]. \end{aligned} \quad (35)$$

Some comments are in order concerning this expression. The combination $m_D^{R'\dagger} m_D^R$ is shifted by NLO corrections, and in general it is no longer real — leading to $\epsilon_{N_\alpha} \neq 0$ and enabling viable leptogenesis. The combination of NLO corrections that defines the shift is not directly related with any low-energy observable. Consequently, while we conclude that general model-independent NLO corrections guarantee a non-vanishing CP-violating asymmetry, correlations among low-energy observables in the leptonic sector and ϵ_{N_α} cannot be established unless the nature of the corrections is well known i.e. once the flavour model realisation has been specified.

5. Model building realisations of the different possibilities

In the previous section we have presented a model-independent proof: exact TB mixing produced by any flavour symmetry in a type I see-saw scenario corresponds to vanishing CP-asymmetry. In this section we gather the different models studied in literature which fall under the validity of the proof, and verify that they correspond to one of the six viable cases of Section 4. We have also present a toy model exemplifying the (2, 2) case (i.e. both matrices have two degenerate eigenvalues) which has not been studied yet. We show that all models lead to a vanishing CP-asymmetry and thus this analysis serves as an ample set of examples of the validity our model-independent proof.

Before describing the flavour models proposed in the literature, it is useful to explain the generic approach considered in flavour symmetry model building. The main goal of these models is to explain the fermion mass hierarchies and mixing angles. To do so, an horizontal flavour group G_f is added to the gauge group of the SM and the SM fields transform in a non-trivial way under G_f . Extra fields (flavons) are added to the particle spectrum: the flavons are invariant under $SU(3) \times SU(2) \times U(1)$, but not under G_f ; they can acquire a non-vanishing vacuum expectation value (VEV) which spontaneously breaks the flavour symmetry in a well determined breaking chain. It is through the specific realisation of the breaking chain that one can achieve the goal of explaining fermion data: for example, the lepton mixing matrix becomes the TB structure when G_f is broken down to two distinct and specific subgroups, G_ℓ in the charged lepton sector and G_ν in the neutrino one, with the type of these subgroups defining the flavour structure of the mass matrices for the leptons (which is model-dependent).

In the following analysis we specify only which G_f was used, and the resulting neutrino mass matrices. We leave all other details to the original papers.

(a) $(\mathbf{i}, \mathbf{m}) = (\mathbf{1}, \mathbf{1})$

There are only a few examples of this case in literature. This case is particularly attractive within the context of a Grand Unified Theory (GUT). In some cases the models do not have exact TB only because they account simultaneously for the quark sector [12], with the Cabibbo angle generating LO deviations from exact leptonic TB — therefore they are not as interesting for our current purpose, and in [59] leptogenesis within the sequential dominance framework was considered in detail (note that there is no inconsistency with our model-independent proof). Here we consider instead two other cases explicitly.

1. In [28] the authors present a model in the context of the $SO(10)$ GUT with the addition of the flavour group $G_f = SU(3) \times U(1)$. The breaking of G_f down to the discrete non-Abelian group A_4 provides the TB pattern for the lepton mixing matrix. The neutrino mass matrices have the flavour structure:

$$m_D \propto \begin{pmatrix} A & B & 0 \\ B & \omega A & 0 \\ 0 & 0 & \omega^2 A \end{pmatrix} \quad \text{and} \quad M_R \propto \begin{pmatrix} A' & B' & 0 \\ B' & \omega A' & 0 \\ 0 & 0 & \omega^2 A' \end{pmatrix}, \quad (36)$$

where $\omega = e^{\frac{2i\pi}{3}}$. It is straightforward to show how the correct mixing pattern is recovered by the diagonalisation of the charged lepton mass matrix and we refer to the original paper for the details. For leptogenesis what is relevant are the imaginary parts of the off-diagonal entries of the product $m_D^{R\dagger} m_D^R$, and in this case it is a diagonal matrix.

2. Another pattern has been presented in [34] in the context of an $SO(10)$ GUT model with A_4 as the additional flavour group. The mass matrices have the following structure

$$m_D \propto \begin{pmatrix} A & 0 & B \\ 0 & C & 0 \\ B & 0 & A \end{pmatrix} \quad \text{and} \quad M_R \propto \begin{pmatrix} A' & 0 & B' \\ 0 & C' & 0 \\ B' & 0 & A' \end{pmatrix}. \quad (37)$$

After considering the charged leptons the TB mixing scheme is obtained. Computing $m_D^{R\dagger} m_D^R$, we find that the off-diagonal entries are real.

(b) $(i, m) = (2, 1)$

There are several papers in which the Dirac neutrino mass matrix has only two independent mass eigenvalues: we can divide the discussion in terms of the flavour patterns used for the mass matrices.

1. The first pattern is present in [19,24,29,33,39,41,43]. In the basis of diagonal charged leptons, the neutrino mass matrices have the structure:

$$m_D \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad M_R \propto \begin{pmatrix} A' + 2B' & -B' & -B' \\ -B' & 2B' & A' - B' \\ -B' & A' - B' & 2B' \end{pmatrix}, \quad (38)$$

where M_R is exactly diagonalisable by the TB mixing. The product $m_D^{R\dagger} m_D^R$ is proportional to the identity.

Two different discrete groups have been used: A_4 in [19,29,33,39,41,43] and T' in [24].

2. The other pattern has been presented in [37] where the authors have used the S_4 discrete symmetry and it differs from the previous one in the explicit form of the Majorana mass matrix:

$$m_D \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \text{and} \quad M_R \propto \begin{pmatrix} 2A' & B' - A' & B' - A' \\ B' - A' & 2A' + B' & -A' \\ B' - A' & -A' & 2A' + B' \end{pmatrix}. \quad (39)$$

This pattern corresponds to a completely different neutrino oscillation phenomenology, but the contribution to leptogenesis is still vanishing in the limit of exact TB mixing.

(c) $(i, m) = (1, 2)$

There is only one pattern within this case [38]. The discrete group A_4 is used to construct a Majorana mass matrix with two degenerate eigenvalues and a Dirac mass matrix of the TB-type.⁶ The mass matrices are given by:

$$m_D \propto \begin{pmatrix} 2A + B & -A & -A \\ -A & 2A & B - A \\ -A & B - A & 2A \end{pmatrix} \quad \text{and} \quad M_R \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (40)$$

The product $m_D^\dagger m_D$ is diagonalised by the TB mixing matrix and it is easy to verify that also the light neutrino mass matrix has this property. $m_D^{R\dagger} m_D^R$ does not present any imaginary off-diagonal factor.

⁶ We underline the absence of a relevant contribution to the Dirac mass matrix, the antisymmetric contraction of the two triplets in a singlet [30]. In order to recover the TB pattern it is possible to either assume a fine-tuning on the parameters or alternatively to adapt the model to use another discrete group such as S_4 , in which case this problem is naturally solved by its properties.

(d) $(i, m) = (2, 2)$

There are no models of this kind in the literature. The difficulty consists in the possibility that the degenerate eigenvalues of the Dirac and Majorana matrices conspire to give a degenerate light neutrino spectrum. A fully developed model is beyond the scope of this paper, but we present here an example. Although it requires some *ad hoc* conditions it is sufficient to illustrate a possible setting in which both non-degenerate light neutrino spectrum and TB mixing are achieved.

The flavour group consists of $SO(3)$ (or a subgroup with an irreducible triplet representation). The additional scalar content is a set of four flavon triplets, ϕ_{123} , ϕ_{23} , ϕ_2 and ϕ_3 which get non-vanishing VEVs. At this level we fix only the VEVs of the first two flavons in such a way that $\langle \phi_{123} \rangle \propto (1, 1, 1)$ and $\langle \phi_{23} \rangle \propto (0, 1, -1)$ (these VEVs must be orthogonal). The structure is reminiscent of the models in [12].

The left and RH neutrinos transform as triplets under $SO(3)$. We assume that any additional symmetry allows the Dirac terms

$$(\phi_{123i} \nu_i)(\phi_{2\alpha} N_\alpha), \quad (\phi_{23i} \nu_i)(\phi_{3\alpha} N_\alpha) \quad (41)$$

and the Majorana terms

$$N_\alpha N_\alpha, \quad (\phi_{3\alpha} N_\alpha)(\phi_{3\beta} N_\beta). \quad (42)$$

The term $N_\alpha N_\alpha$ by itself would lead to degenerate masses in the Majorana matrix. The degeneracy is lifted only for one of the states by the VEV $\langle \phi_3 \rangle \propto (0, 0, 1)$ (two eigenvalues remain degenerate). Thus the RH neutrino mass matrix has structure:

$$M_R \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & x \end{pmatrix}, \quad (43)$$

where x parametrises that the entry receives contribution due to $\langle \phi_3 \rangle$. In the Dirac sector one of the eigenvalues is zero. For a non-trivial choice of parameters we end up with exactly two non-zero degenerate eigenstates. With $\langle \phi_2 \rangle \propto (0, 1, 0)$ and through the type I see-saw, the term $(\phi_{123i} \nu_i)(\phi_{2\alpha} N_\alpha)$ will give rise to the solar eigenstate and the term $(\phi_{23i} \nu_i)(\phi_{3\alpha} N_\alpha)$ will give rise to the atmospheric eigenstate. In this case the Dirac mass matrix is:

$$m_D \propto \begin{pmatrix} 0 & t & 0 \\ 0 & t & b \\ 0 & t & -b \end{pmatrix}, \quad (44)$$

where t and b parametrise the contributions of $(\phi_{123\nu})(\phi_2 N)$ and $(\phi_{23\nu})(\phi_3 N)$ respectively. The effective neutrino mass matrix is diagonalised by TB mixing, as this model fits within the framework described in [12]. There is sufficient freedom to fit the squared mass differences (as required by phenomenology), although only strongly hierarchical cases are possible due to the vanishing eigenvalue of m_D . The Dirac matrix has two degenerate masses by requiring $3t^2 = 2b^2$ (completely *ad hoc*, as it requires the conspiracy of the VEVs of the flavons — we can express it as a very specific requirement on the magnitude of $\langle \phi_2 \rangle$). It is straightforward to see that $m_D^{R\dagger} m_D^R$ is a diagonal matrix, leading to vanishing leptogenesis.

(e) $(i, m) = (3, 1)$

This case is the most studied in literature and there are some interesting flavour patterns.

1. The first pattern has been presented in [16,23,26,32] and the flavour group which has been used is A_4 . The mass matrices appear as

$$m_D \propto \mathbb{1} \quad \text{and} \quad M_R \propto \begin{pmatrix} A & 0 & 0 \\ 0 & A & B \\ 0 & B & A \end{pmatrix}. \quad (45)$$

The charged leptons need to be rotated in diagonal form, and the main result is that the lepton mixing matrix is exactly the TB scheme.

2. The second pattern [17,20,22] is similar to the previous one and it still originates in an A_4 context. The mass matrices are the following:

$$m_D \propto \mathbb{1} \quad \text{and} \quad M_R \propto \begin{pmatrix} A & 0 & B \\ 0 & A & 0 \\ B & 0 & A \end{pmatrix}. \quad (46)$$

In the basis of diagonal charged leptons, we obtain the TB pattern for the lepton mixing matrix.

3. The third pattern [18] is also similar to the first one. Once again it is based on the A_4 discrete symmetry. The mass matrices are given by

$$m_D \propto \mathbb{1} \quad \text{and} \quad M_R \propto \begin{pmatrix} C & 0 & 0 \\ 0 & A & B \\ 0 & B & A \end{pmatrix}. \quad (47)$$

Like in the previous cases, when going to the basis of diagonal charged leptons it is easy to see that the lepton mixing matrix is the TB pattern.

For all three patterns, it is trivial to see that $m_D^{R\dagger} m_D^R$ is proportional to the identity matrix.

(f) $(i, m) = (1, 3)$

This case has been studied in two distinct patterns.

1. In [25,30] a flavour model based on the A_4 group has been provided. The model is extremely similar to the previous case, of [16,23,26,32], where the structures of the Dirac and the Majorana mass matrices are exchanged:

$$m_D \propto \begin{pmatrix} A & 0 & 0 \\ 0 & A & B \\ 0 & B & A \end{pmatrix} \quad \text{and} \quad M_R \propto \mathbb{1}. \quad (48)$$

In the basis of diagonal charged leptons the light neutrino mass matrix is diagonalised by the TB scheme and the product $m_D^\dagger m_D$ is real.

2. The second pattern has been presented in [40] and it is similar to that of [17,20,22], discussed in the previous case, exchanging the structure of the Dirac and the Majorana mass matrices:

$$m_D \propto \begin{pmatrix} A & 0 & B \\ 0 & A & 0 \\ B & 0 & A \end{pmatrix} \quad \text{and} \quad M_R \propto \mathbb{1}. \quad (49)$$

This result has been developed in the context of the A_4 flavour symmetry.⁷ The authors themselves have concluded that m_D does not give rise to leptogenesis.

⁷ We underline the presence of the same difficulty previously discussed in (1, 2) about [38], which can be naturally solved by using S_4 instead.

Table 1

Matter and scalar content of the model and their transformation properties under G_f [39].

	L	e^c	μ^c	τ^c	N^c	H^u	H^d	φ_T	ξ'	φ_S	ξ	ζ
A_4	3	1	1	1	3	1	1	3	1'	3	1	1
Z_3	1	1	1	1	ω	1	1	1	1	ω	ω	ω^2
Z_4	1	-1	-i	1	1	1	-i	i	i	1	1	1

To conclude, each pattern in each case agrees with our model-independent result. Exact flavour symmetry imposed TB in type I see-saw leads to vanishing CP-asymmetry (the off-diagonal entries of $m_D^{R\dagger} m_D^R$ are either trivially zero or real).

6. Model dependent perturbations

We concluded Section 4 with the observation that by assuming general perturbations to the TB matrix obtained with an underlying flavour symmetry there are no correlations between low and high-energy scale CP violation parameters. This result was derived from Eq. (34) where it can be seen that the number of free parameters governing the perturbations is quite large and thus no correlation can be expected. In the context of specific flavour models it is possible that the TB scheme is perturbed by a small number of corrections, and in this interesting case correlations between low-energy scale observables and the CP-violating asymmetry may be established.

In this section we consider a supersymmetric model based on the $G_f = A_4 \times Z_3 \times Z_4$ discrete flavour symmetry [39], which for our purposes is attractive due to its elegance and predictivity. The relevant NLO corrections appear only in the Dirac mass and can be parametrised in terms of only 3 complex parameters. Neutrino masses are induced only through type I see-saw so the results from Section 4 hold — in fact we have considered it explicitly in Section 5, as one of the models with the first pattern of class (2, 1).

The three factors in G_f play different roles. The spontaneous breaking of A_4 is directly responsible for the TB mixing. The $Z_3 \times Z_4$ factor avoids large mixing effects between the flavons that give masses to the charged leptons and those giving masses to neutrinos, and it is also responsible for the hierarchy among charged fermion masses. The flavour symmetry breaking sector of the model includes the scalar superfields φ_T , ξ' , φ_S , ξ and ζ . The transformation properties of the lepton superfields L , e^c , μ^c , τ^c , of the electroweak scalar doublets H^u and H^d and of the flavon superfields are reproduced in Table 1 for ease of reference.

We present the Yukawa superpotential of the model as an expansions in $1/\Lambda$, where Λ is the cut-off of the theory: the LO terms are given by

$$\begin{aligned}
\mathcal{W}_\ell = & \frac{1}{\Lambda} y_\tau (L\varphi_T)\tau^c H^d \\
& + \frac{1}{\Lambda^2} y_\mu^{(1)} (L\varphi_T)'' \xi' \mu^c H^d + \frac{1}{\Lambda^2} y_\mu^{(2)} (L\varphi_T\varphi_T)\mu^c H^d \\
& + \frac{1}{\Lambda^3} y_e^{(1)} (L\varphi_T)' (\xi')^2 e^c H^d + \frac{1}{\Lambda^3} y_e^{(2)} (L\varphi_T\varphi_T)'' \xi' e^c H^d \\
& + \frac{1}{\Lambda^3} y_e^{(3)} (L\varphi_T\varphi_T\varphi_T)e^c H^d, \tag{50}
\end{aligned}$$

$$\mathcal{W}_\nu = -\frac{1}{\Lambda} y (LN^c)\zeta H^u + x_a (N^c N^c)\xi + x_b (N^c N^c\varphi_S), \tag{51}$$

where (\dots) , $(\dots)'$ and $(\dots)''$ stand for the contraction in the representations 1, 1' and 1'' of A_4 , respectively.

The flavon superfields acquire the following VEVs:

$$\langle \varphi_T \rangle = \begin{pmatrix} 0 \\ v_T \\ 0 \end{pmatrix}, \quad \langle \xi' \rangle = u', \quad \langle \varphi_S \rangle = \begin{pmatrix} v_S \\ v_S \\ v_S \end{pmatrix}, \quad \langle \xi \rangle = u, \quad \langle \zeta \rangle = w, \quad (52)$$

where v_T , u' , v_S , u and w are the small symmetry breaking parameters of the theory. This pattern of VEVs guarantees that the lepton mixing is approximately TB. It is possible to align these VEVs in a natural way, as the result of the minimisation of the scalar potential [39]: we underline that the symmetry content prevents any deviations from this pattern at NLO and allows the order of magnitude relations between parameters $v_T \sim u'$ and $v_S \sim u \sim w$, assuming at most a mild hierarchy among the two sets.

The charged lepton mass matrix can be approximately written as

$$m_\ell = \begin{pmatrix} \sim \frac{v_T^3}{\Lambda^3} & 0 & 0 \\ 0 & \sim \frac{v_T^2}{\Lambda^2} & 0 \\ 0 & 0 & \sim \frac{v_T}{\Lambda} \end{pmatrix} v^d, \quad (53)$$

where $v^d = \langle H^d \rangle$. A lower bound on the parameters v_T/Λ can be fixed by the requirement that the τ Yukawa coupling y_τ does not become too large, and we can estimate it as

$$\frac{v_T}{\Lambda} = \frac{\tan \beta}{y_\tau} \frac{\sqrt{2} m_\tau}{v} \approx 0.01 \frac{\tan \beta}{y_\tau}, \quad (54)$$

where $v \approx 246$ GeV and $\tan \beta = \langle H^u \rangle / \langle H^d \rangle$. Taking $m_\tau = (1776.84 \pm 0.17)$ MeV and requesting $|y_\tau| < 3$, we find a lower limit on v_T/Λ of 0.007 for $\tan \beta = 2$, the smallest value we consider.

The neutrino mass matrix gets contributions from the type I see-saw according to Eq. (4). We have:

$$m_D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \frac{y w v^u}{\Lambda}, \quad M_R = \begin{pmatrix} b + 2d & -d & -d \\ -d & 2d & b - d \\ -d & b - d & 2d \end{pmatrix} u, \quad (55)$$

with $v^u = \langle H^u \rangle$, $b \equiv 2x_a$ and $d \equiv 2x_b v_S/u$. The mass matrices M_R and m_D are μ - τ symmetric and satisfy the conditions in Eq. (11). Accordingly, M_R and m_ν are diagonalised by the TB mixing matrix, giving as eigenvalues $M_1 = |b + 3d|$, $M_2 = |b|$, $M_3 = |b - 3d|$ and $m_i = (y w v^u)^2 / (\Lambda^2 M_i)$. We already mentioned in Section 5 that the same mass matrices are present in [16,19,21,24]. The phenomenology has already been studied in [37], so we summarise here the main results and refer to [37] for the details. The model can explain both Normal and Inverse Hierarchy (NH and IH) and features lower bounds on the mass of the lightest neutrino: in particular for the NH the lightest neutrino mass has a well defined and narrow range of values between 4.4 meV and 7.3 meV.

In order to estimate the parameter ϵ_{N_α} , we write the Dirac mass matrix in the basis of diagonal and real RH neutrinos:

$$m_D^R = m_D U_{TB} D', \quad (56)$$

where $D' = \text{diag}(e^{i\phi_1/2}, e^{i\phi_2/2}, e^{i\phi_3/2})$ and ϕ_α are the phases of $b + 3d$, b , $b - 3d$ respectively (the eigenvalues of M_R). As was mentioned in Section 5 the product $m_D^{R\dagger} m_D^R$ is a diagonal matrix and therefore $\epsilon_{N_\alpha} = 0$, in perfect agreement with our model-independent proof in Section 4.

A non-vanishing CP-violating asymmetry can be obtained at NLO when the TB mixing is perturbed. In this model the additional discrete Abelian symmetries $Z_3 \times Z_4$ only admit NLO corrections to the Dirac terms. We do not consider terms whose contributions can be reabsorbed in a redefinition of the LO parameters, focusing only on terms that lead to deviations in the mixing angles:

$$-\mathcal{W}_\nu^{NLO} = \frac{1}{\Lambda} y_1 (LN^c)' (\varphi_S \varphi_S)'' H^\mu + \frac{1}{\Lambda} y_2 (LN^c)'' (\varphi_S \varphi_S)' H^\mu + \frac{1}{\Lambda} y_3 ((LN^c)_A \varphi_S) \xi H^\mu, \quad (57)$$

where $(\dots)_A$ refers to the asymmetric contraction of the triplet representation. The deviations to m_D can be written as

$$m_D^{(1)} = \begin{pmatrix} 0 & y_1 + y_3 & y_2 - y_3 \\ y_1 - y_3 & y_2 & y_3 \\ y_2 + y_3 & -y_3 & y_1 \end{pmatrix} v^\mu \frac{v_S^2}{\Lambda^2}, \quad (58)$$

where y_3 accounts for the ratio u/v_S . Note that this correction is of Tri-maximal type [50,51]. As the LO starts out as TB, and TB is also a (special) case of Tri-maximal, the perturbed model fits within that mixing scheme. It is important to clarify that the general Tri-maximal scheme is explicitly out of the validity of the model-independent proof presented in Section 4 — while particular cases of Tri-maximal have mixing matrices independent of the masses (obviously this is the case for TB), in general it is possible to a Tri-maximal case where mixing angles depend on the masses. The perturbed model considered here is one such case, and as we will see it can admit viable leptogenesis. Including Eq. (58), the TB mixing receives small perturbations according to $U_\nu = U_{TB} \delta U$, where only the element $(\delta U)_{13}$ is relevant. Parametrising this term as:

$$|(\delta U)_{13}| = \left| \sqrt{\frac{3}{2}} \sin \theta_{13} e^{i\delta} \right| \sim \mathcal{O}\left(\frac{v_S}{\Lambda}\right), \quad (59)$$

where δ is the CP-violating Dirac phase in the standard parametrisation of the lepton mixing matrix, we write the other two mixing angles at NLO as

$$\sin^2 \theta_{23} = \frac{1}{2} (1 + \sqrt{2} \cos \delta \sin \theta_{13}), \quad \sin^2 \theta_{12} = \frac{1}{3} (1 + \sin^2 \theta_{13}). \quad (60)$$

In Fig. 3, we plot Eqs. (60) in red (with $\delta = 0$) and plot the results of a numerical analysis in green and blue points corresponding to the IH and NH neutrino spectrum respectively (in which we take $v_S/\Lambda \sim w/\Lambda = 0.007\text{--}0.23$, $\tan \beta = 2\text{--}50$ and we treat y , y_1 , y_2 and y_3 as random numbers with modulus between 0.1 and 2).

We expect that NNLO corrections affect these relations: we estimate that NNLO perturbations will be of the order of $\sin^2 \theta_{13}$ and therefore $\sin^2 \theta_{12}$ will receive non-negligible corrections.

We can impose an upper bound on v_S/Λ by requiring that the correction to the TB value of $\sin^2 \theta_{12}$ does not take it outside the experimental 3σ range: the maximal allowed deviation from the TB value is 0.05 and from there we impose the bound $v_S/\Lambda < \mathcal{O}(0.23)$.

We consider now $m_D^{R'}$ (the NLO Dirac neutrino mass matrix in the basis of diagonal and real RH neutrinos). We can write:

$$m_D^{R'} = m_D^R + m_D^{(1)} U_{TB} D', \quad (61)$$

and calculate the relevant product for leptogenesis, $m_D^{R'\dagger} m_D^{R'}$, keeping only the first terms in the expansion in the small parameter v_S^2/Λ^2 :

$$m_D^{R'\dagger} m_D^{R'} = m_D^{R\dagger} m_D^R + (D^* U_{TB}^T m_D^{(1)\dagger} m_D^R + \text{h.c.}), \quad (62)$$

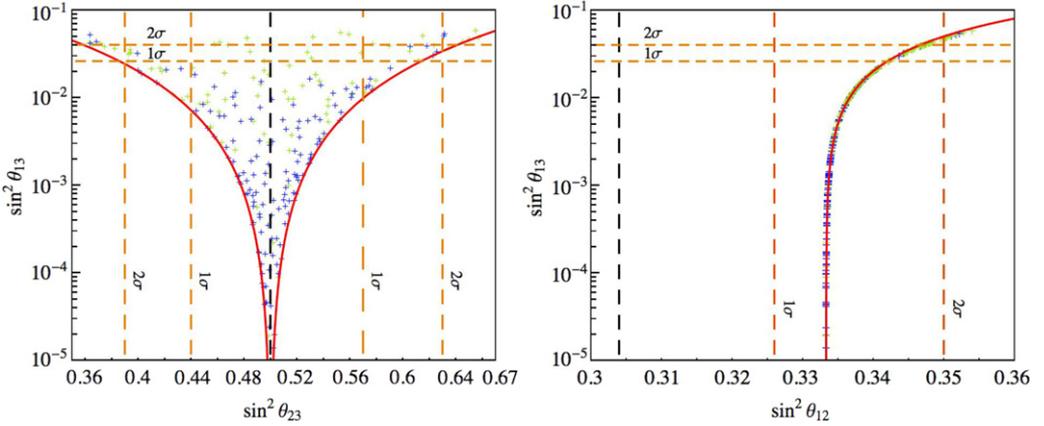


Fig. 3. Correlation between $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ (left panel) and $\sin^2 \theta_{13}$ and $\sin^2 \theta_{12}$ (right panel). Each panel compares the analytical approximations given in Eqs. (60) (red lines) with the numerical results. The green and blue points correspond to the IH and NH neutrino spectrum respectively. For the analytical expressions of $\sin \theta_{23}^2$ we have fixed the CP-Dirac phase δ at 0. In the numerical analysis the ratio $v_S/\Lambda \sim w/\Lambda$ has been taken in the window bounded from the constraints arising by y_τ and $\sin^2 \theta_{12}$, that is $0.007 < v_S/\Lambda < 0.23$. The value of $\tan \beta$ spreads between 2 and 50, while all the other free parameters, y, y_1, y_2 and y_3 , are treated as random numbers with absolute value between 0.1 and 2. The horizontal dashed orange lines corresponds to the bounds at 1 and 2 σ level, respectively, for $\sin^2 \theta_{13}$, the vertical dashed black line corresponds to the central values of $\sin^2 \theta_{23}$ and $\sin^2 \theta_{12}$ while the vertical dashed orange lines to their bounds at 1 and 2 σ level, respectively. The plots are cut in correspondence of the 3 σ level bound for $\sin^2 \theta_{13}, \sin^2 \theta_{23}, \sin^2 \theta_{12}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

where in the second term the only off-diagonal entries are the 13 and 31 ones. In the case of IH spectrum for the light neutrinos, the lightest RH neutrino is N_2 : in this case the summation in the numerator of Eq. (14) does not contain the term 13 and therefore ϵ_{N_2} is vanishing also at NLO. This, however, does not mean leptogenesis cannot be realized in this case. Since there is only a mild hierarchy between N_2, N_1 and N_3 and neither ϵ_{N_1} nor ϵ_{N_3} vanishes, leptogenesis will proceed through $N_{1,3}$ dynamics. In the NH case the RH neutrino mass spectrum follows the hierarchy $M_{N_3} < M_{N_2} < M_{N_1}$. There is a mild hierarchy between N_3 and N_2 while the hierarchy between N_3 and N_1 is large (around a factor 9). Consequently, the lepton asymmetry generated in N_1 decays will be, in general, erased by the lepton number violating interactions of N_3 . Only N_3 dynamics becomes relevant for the generation of a lepton asymmetry in this case. Note that if the hierarchy between N_1 and N_3 decreases (as could be in the case of a quasi-degenerate spectrum), so it becomes mild, N_1 dynamics should be taken into account. Henceforth, for simplicity, we will consider only the NH case for which, according to Eq. (14), the CP-violating parameter ϵ_{N_3} can be written as

$$\epsilon_{N_3} = \frac{1}{8\pi} \frac{1}{v_u^2 (m_D^{R\dagger} m_D^R)_{11}} \text{Im} [((m_D^{R\dagger} m_D^R)_{13})^2] f \left(\frac{M_1^2}{M_3^2} \right). \tag{63}$$

In the following figures we show a series of scatter plots related to the predictions of the model and the connections among low-energy observables and ϵ_{N_3} . The (blue) points correspond only to the NH neutrino spectrum (in which we take $v_S/\Lambda \sim w/\Lambda = 0.007-0.23$, $\tan \beta = 2-50$ and we treat y, y_1, y_2 and y_3 as random numbers with modulus between 0.1 and 2). Red lines correspond to analytical results.

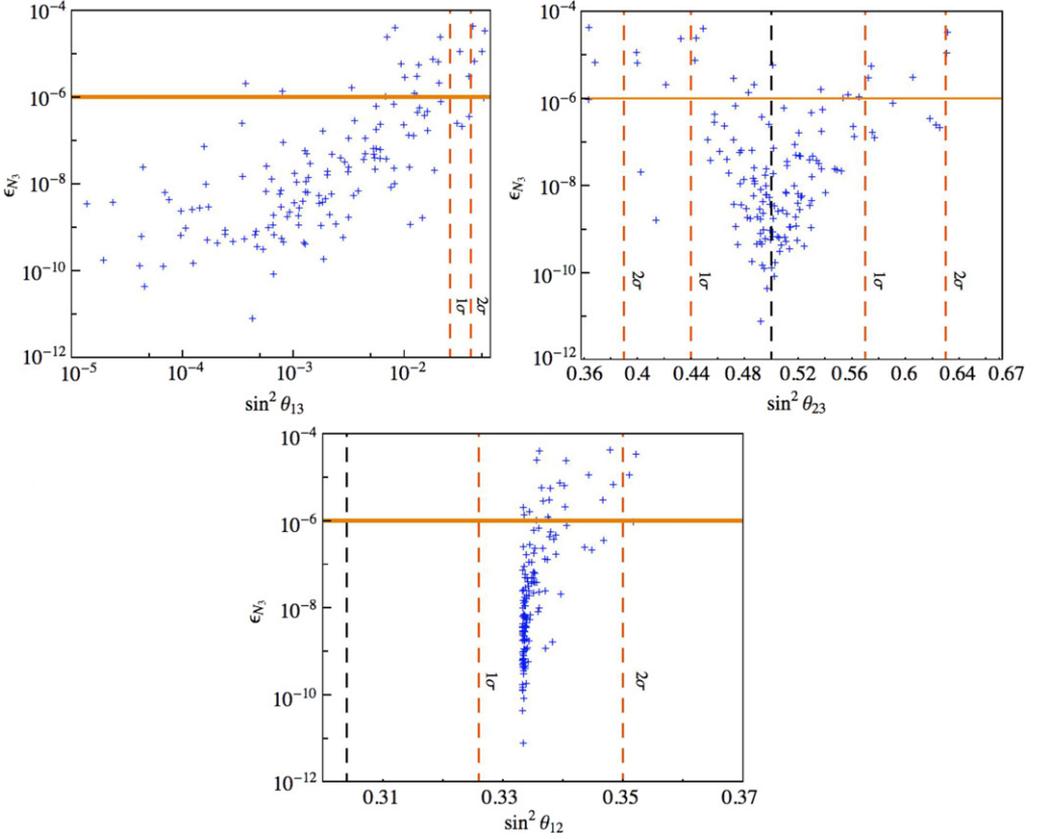


Fig. 4. Correlation between ϵ_{N_3} and $\sin^2 \theta_{13}$ (first row, left panel), $\sin^2 \theta_{23}$ (first row, right panel), $\sin^2 \theta_{12}$ (second row). The horizontal orange line corresponds to $\epsilon_{N_3} \sim 10^{-6}$, the vertical orange lines correspond to the bounds at 1 and 2σ level of $\sin^2 \theta_{13}$, $\sin^2 \theta_{23}$ and $\sin^2 \theta_{12}$ respectively while the vertical dashed black lines correspond to the central values of $\sin^2 \theta_{23}$ and $\sin^2 \theta_{12}$. The range presented in the plots covers the 3σ level bounds. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

In Fig. 4 we show all the correlations between the CP asymmetry parameter ϵ_{N_3} and the lepton mixing angles (expressed as $\sin^2 \theta_{13}$, $\sin^2 \theta_{23}$ and $\sin^2 \theta_{12}$). As expected by comparing Eq. (63) with Eq. (60), ϵ_{N_3} is correlated to all low-energy mixing angles.

The same information is contained in Fig. 5 where we show the deviations of θ_{13} , θ_{12} , θ_{23} from the respective TB value for the points that reach the necessary amount of CP asymmetry. The deviations are compared to the reference value $\lambda_C^2 \sim 0.05$, where λ_C is the Cabibbo angle. This comparison is particularly interesting because λ_C^2 is the typical order of magnitude of the corrections to the TB mixing allowed by neutrino data fit in models based on flavour symmetries and predicting TB mixing (in particular, this is a natural consequence in classes of flavour models that include GUTs [12]). Our numerical analysis shows indeed that the order of magnitude of $|\Delta\theta_{23}|$, $|\Delta\theta_{13}|$ is close to λ_C^2 while $|\Delta\theta_{12}|$ tends to be smaller. We note that only a few points reach the necessary amount of CP asymmetry ϵ_{N_3} when the deviations of the mixing angles from their TB values are smaller than λ_C^2 . By comparing the left and right panel of Fig. 5 we can get the lower bound for $\sin^2 \theta_{13}$ necessary to have successful leptogenesis in this model, that is

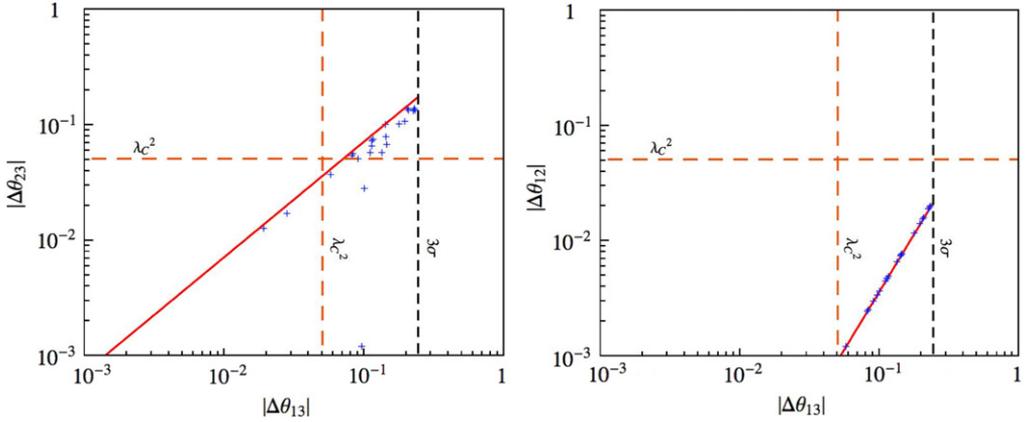


Fig. 5. Correlation between the deviation of θ_{23} (left panel) and θ_{12} (right panel) with θ_{13} from their TB mixing values for the points that satisfy $\epsilon_{N_3} \geq 10^{-6}$. The red lines represent the analytical results from Eqs. (60). The vertical and horizontal orange lines corresponds to $|\Delta\theta_{13}|, |\Delta\theta_{23}|, |\Delta\theta_{12}| \sim \lambda_C^2$ with λ_C the Cabibbo angle. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

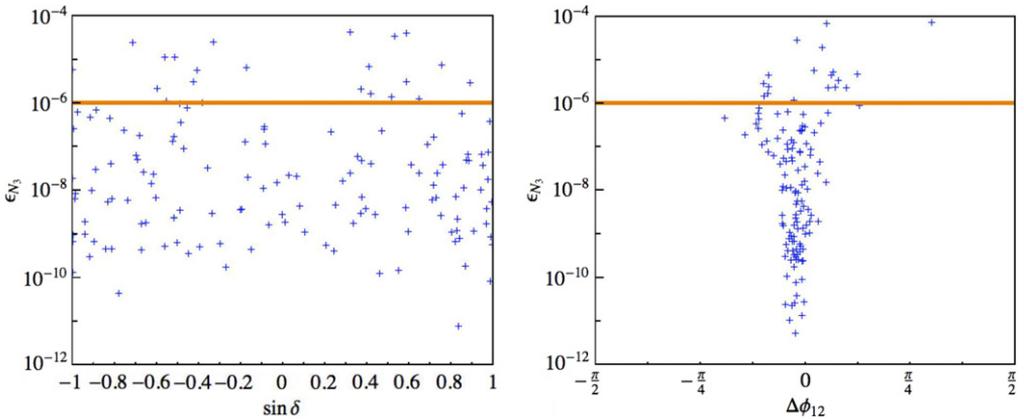


Fig. 6. The high-energy CP violation asymmetry parameter ϵ_{N_3} versus the low-energy lepton CP-Dirac phase δ and the difference of the light neutrino Majorana phases ϕ_1 and ϕ_2 . In both plots the horizontal orange line corresponds to $\epsilon_{N_3} \sim 10^{-6}$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

$\sin^2 \theta_{13} \sim 10^{-2}$. Future experiments will further constrain (and possibly rule out leptogenesis within this model). Double Chooz will probe $\sin^2(2\theta_{13})$ to 10^{-2} in the next five years and Triple Chooz will reach below that value [61].

Finally Fig. 6 wants to investigate the possible correlations between the high-energy CP asymmetry parameter ϵ_{N_3} and the low-energy CP-Dirac and Majorana phases. The left panel of Fig. 6 shows that the low-energy CP-Dirac phase δ (δ in the plot) is not correlated to ϵ_{N_3} . This result is not surprisingly: the phases that enter in ϵ_{N_3} are related to the phases present in M_R , while δ arises by the phases that appear in what we defined as $m_D^{(1)}$ in Eq. (61). On the contrary the right panel of Fig. 6 indicates that the difference between the Majorana phases ϕ_1 and ϕ_2 ($\Delta\phi_{12}$ in the plot) presents a correlation with ϵ_{N_3} . The reason is very simple: at LO the phenomenological

analysis of this model shows that the NH spectrum can be reproduced only if $\Delta\phi_{12}^0$ is small. Moreover at LO, $\Delta\phi_{12}^0$ coincides with the corresponding Majorana phase difference $\Delta\phi_{12}^R$ of the right-handed neutrinos. By perturbing the neutrino Dirac mass matrix we introduce new arbitrary phases that can vary in all the interval $(0, 2\pi)$. However the NLO contributions are responsible both of deviating the lepton mixing angles by the TB values and slightly modifying the neutrino spectrum, in the range allowed by the data fit. This means that in general at NLO the neutrino (complex) mass eigenvalues are given by

$$m_i \sim m_i^0 + \delta m_i, \quad (64)$$

where δm_i are complex parameters and m_i^0 the neutrino mass eigenvalues at LO. Requiring now that Δm_{12}^2 is still in the range indicated for Δm_{sol}^2 we have that $\delta m \sim |\delta m_{1,2}| \sim 10^{-3}$ eV for $|m_1^0|, |m_2^0| \sim \mathcal{O}(\sqrt{\Delta m_{sol}^2})$. A straight computation shows then that the Majorana phase $\Delta\phi_{12}$ satisfies

$$\tan \Delta\phi_{12} \sim \tan \Delta\phi_{12}^0 + \alpha \frac{\delta m}{\mathcal{O}(\sqrt{\Delta m_{sol}^2})}, \quad (65)$$

where $\Delta\phi_{12}^0$ is the LO Majorana phase difference and $\alpha \in (0, 1)$ a parameter that takes into account that the $\delta m_{1,2}$ phases run into the interval $(0, 2\pi)$. We can estimate the maximal deviation of $\Delta\phi_{12}$ by its LO value getting

$$\Delta\phi_{12} - \Delta\phi_{12}^0 \sim \frac{\pi}{10}. \quad (66)$$

Notice that the left panel of Fig. 6 shows that the majority of all the points are indeed inside the interval $(-\pi/8, \pi/8)$ in perfect agreement with our analytical results for a small LO $\Delta\phi_{12}^0 \leq 0.1$.

In summary our analysis shows that in the model considered it is possible to obtain correlations between low-energy observables and the high-energy CP-violating parameter, but it confirms that in general no correlation is present between high and low-energy CP-violating parameters (or in the case of the Majorana phases, negligible correlation).

7. Conclusion

In this paper we considered under rather general conditions the possibility of links between low-energy observables and high-energy parameters that are relevant for leptogenesis — in the most general case no such connections can be recovered.

When assuming exact TB mixing independently of any specific justification, we conclude that it is in general possible to obtain leptogenesis. Constraining the situation to the case of type I see-saw is insufficient to provide a link between the different type of parameters.

In the main part of this work, we considered the more natural case where exact mixing patterns originate from any flavour symmetry. We confirmed that the results of [33] concerning TB mixing apply to the case of unflavoured leptogenesis when there is only type I see-saw. We generalised this conclusion into a model-independent proof that is also valid for other flavour symmetry imposed mixing schemes if the mixing matrix consists purely of numbers — this includes Bi-maximal mixing, golden-ratio mixing, and some but not all cases of Tri-maximal mixing. We emphasise that the proof does not hold when there are also other types of see-saw (such as type II): in models in which there is interplay between different see-saws, it is possible to have leptogenesis without lifting the exact (TB) pattern. These interesting cases shall be considered in

detail in future work [60]. Still in the model-independent framework with only type I see-saw, we considered the most general NLO corrections that can lift TB mixing and how these corrections can enable leptogenesis.

From the model-independent proof we proceeded by considering several flavour symmetry models with exact TB mixing and only type I see-saw. As expected, in all cases the specific conditions led to vanishing CP asymmetry.

Finally we studied a specific example in which a flavour symmetry model has deviations from exact TB, which can then enable leptogenesis. In a general case there would be many parameters governing the deviation from TB mixing and thus an interesting link between observable mixing angles and leptogenesis may not even exist. We selected an example where the deviation is parametrised such that it is possible to obtain relatively simple analytical expressions relating the observable deviations from TB angles to the CP asymmetry. There is a clear and strong correlation between deviations from TB angles (such as the non-vanishing value of θ_{13}) and the value of the CP asymmetry parameter: particularly we note that if we insist in having viable leptogenesis in this model we can considerably constrain the allowed parameter space of the NH spectrum. Future experiments will probe the remaining parameter space.

To summarise, in order to have TB mixing scheme originating from a flavour symmetry and still have viable leptogenesis the model requires NLO corrections lifting the exact mixing, or alternatively it requires independent contributions to the CP asymmetries such as those that naturally arise from an interplay between different see-saws.

Note added in proof

While completing this paper we received Refs. [62,63], where the interplay of flavour symmetries and leptogenesis in the context of type I see-saw is also discussed. Both papers consider specific models based on the discrete group A_4 . Furthermore, Ref. [62] discusses a general model-independent approach which complements the distinct model-independent proof we provide here.

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