Seismic motion presents a challenge to all ground based gravitational wave detectors. Although future detectors may take advantage of seismically quiet sites, vibrations induced by seismic motion will still be many orders of magnitude higher than displacement limits required to reach the low-frequency sensitivity targets. The Initial Virgo detector achieved unprecedented low-frequency sensitivity by implementing vibration isolation technology such as the superattenuator, in the suspension of its main optical components [65]. The advanced era of gravitational wave detectors requires improved vibration isolation, not only for the main optical components, but also for the auxiliary optical systems.

This chapter will focus on a compact vibration isolator designed to suspend in-vacuum optical benches for Advanced Virgo. It is a multi-stage seismic attenuation system known as MultiSAS. MultiSAS achieves most of its attenuation performance through passive isolation; yet, an important feature of the design involves the active control of low frequency (< 5 Hz) resonances. The design and realization of MultiSAS, including its separate components will first be discussed followed by results of the system’s characterization campaign. A detailed account of the active control subsystem will then be presented, concentrating on a multi-channel feedback approach for vertical control, that is novel to the field.

5.1 Vibration isolation for Advanced Virgo

To reach the desired sensitivity of the Advanced Virgo detector, suppression of seismic noise induced vibrations of the main optical components is required. The residual motion of the interferometer arm mirrors, recycling mirrors and the beam-splitter must be suppressed by up to 10 orders of magnitude above 10 Hz. This is achieved with 8 m high suspension systems called superattenuators [65].

Auxiliary optical systems distributed around the detector (see Fig. 2.1) are essential
for the control, readout and alignment of the interferometer. The benches housing these optics also need to be isolated from seismic motion. In Section 2.2.3 it was described how the angular alignment of the interferometer will be measured with quadrant photodiodes. Unwanted motion of these photodiodes will mimic a misalignment of the cavities and hence introduce control noise. The most critical alignment modes, in terms of control noise, are the interferometer common and differential (Δ)-modes (see Section 2.2.3). These are sensed by the DC signals from the terminal bench photodiodes (B7 and B8), placing stringent demands on their seismic isolation systems. Furthermore, light that is scattered or diffused off the optical components can re-enter the interferometer. This light is modulated by the residual motion of the optics. Any non-linear behavior of this coupling can lead to the up-conversion of low-frequency seismic excitations (< 10 Hz) into the detection band (> 10 Hz) [116, 117].

Table 5.1 lists the isolation requirements in terms of residual rms motion and spectral displacement above 10 Hz. The translational rms requirements are driven by the tolerance on scattered and diffused light, while the rotational and spectral requirements are based on control noise limits. The most challenging demands are those for the rotational motion (pitch, roll and yaw) of the bench.

<table>
<thead>
<tr>
<th>Bench motion</th>
<th>Requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>δz</td>
<td>2.1 \times 10^{-12} \text{ m/√Hz}</td>
</tr>
<tr>
<td>δθ</td>
<td>3.3 \times 10^{-15} \text{ rad/√Hz}</td>
</tr>
<tr>
<td>z_{rms}</td>
<td>1 \times 10^{-6} \text{ m}</td>
</tr>
<tr>
<td>θ_{rms}</td>
<td>3.1 \times 10^{-8} \text{ rad}</td>
</tr>
</tbody>
</table>

**Table 5.1:** Requirements for the suspended terminal benches. Spectral requirements are valid above 10 Hz. All the translational and rotational degrees of freedom are represented by z and θ respectively.

MultiSAS, a compact seismic attenuation system was designed based on the required isolation performance and the limited space provided by the existing Virgo infrastructure. The system is based on compact inverted pendulums in combination with geometric anti-springs. Five of Advanced Virgo’s in-vacuum optical benches shown in Fig. 2.1 will be suspended by a MultiSAS: the end benches (SNEB and SWEB), the injection bench (SIB2), the power recycling pick-off bench (SPRB) and the detection bench (SDB2). An impression of MultiSAS supporting an end bench is given in Fig. 5.1.

An out of vacuum system, based on similar technology was designed for the seismic isolation of the Advanced Virgo external injection bench (EIB) and is known as EIB-SAS [118]. It is a single stage system, supporting an optical bench from underneath and utilizes similar isolation technology as that of MultiSAS. EIB-SAS was designed and constructed at Nikhef in 2010 - 2011, before being installed and intensively tested at the Virgo site [119, 120]. The knowledge gained from EIB-SAS proved extremely valuable during design and construction of MultiSAS.
5.1. Vibration isolation for Advanced Virgo

Figure 5.1: An impression of MultiSAS suspending an Advanced Virgo end bench in a MiniTower vacuum chamber. 1) Optical bench. 2) MultiSAS. 3) Transmission beam from end mirror. 4) MiniTower vacuum chamber. 5) Removable cupola. 6) Observant physicist.
Chapter 5. Vibration isolation

A prototype of MultiSAS was constructed at Nikhef starting at the end of 2011, with the mechanical assembly of the full system completed in the summer of 2012. A number of tests were performed, first on subsystems and then on the entire (out of vacuum) system. These tests were crucial to verify critical design strategies and isolation performance and prompted small design changes before MultiSAS was committed to a final production status. The next steps are the integration of MultiSAS in the proposed vacuum enclosure, followed by the production of five more MultiSAS systems and the commissioning in Advanced Virgo. The first of these systems (suspending SIB2) is scheduled to be installed in Advanced Virgo in the middle of 2014 and will form an integral part of the first Advanced Virgo commissioning phase.

5.2 Vibration isolation with anti-spring technology

There are two main strategies that can be pursued when it comes to seismic noise isolation: active and passive. Active isolation involves broadband monitoring of payload motion, the signals of which are used in a feedback control system, to produce a corrective force on the to-be-isolated object. Limitations of purely active systems are the sensitivity and bandwidth of sensors and the speed and precision of the feedback control system. An important advantage of active systems is their low compliance and flexibility of payload design. Passive isolation makes use of the mechanical second-order lowpass filter characteristics inherent to harmonic oscillators such as pendulums and mass-spring systems. MultiSAS combines both passive and active isolation techniques. Seismic attenuation is provided by passive filters, while active feedback is used to damp resonance frequencies. Applications of such technology has found widespread use in seismic attention systems for gravitational wave experiments, such as the Japanese detectors TAMA300 [121] and KAGRA [63], the 10 m prototype suspension AEI-SAS in Hannover [122] and the Australian initiative AIGO [123].

Consider a simple harmonic oscillator with a spring of mass $m$ and spring constant $k$ supporting a payload of mass $M$. It is assumed that the motion is confined to a single degree of freedom $x$, and that the mass of the spring is evenly distributed so that its center of mass moves according to $x_s = (x + x_0)/2$. Here the ground displacement is given by $x_0$. Neglecting the internal dynamics of the system, the equation of motion can then be described by

$$\left(M + \frac{m}{4}\right)\dddot{x} + \frac{m}{4}x_0 + k(1 + i\phi)(x - x_0) + \gamma \dot{x} = f,$$

where $\gamma$ is a viscous damping coefficient and $\phi$ is the loss angle, a measure of a spring’s anelasticity. It is defined as the phase angle in radians by which the response $x$ would lag behind a sinusoidal driving force [58]. A constant (in frequency domain) loss angle accounts for internal or structural damping by describing a force proportional to the displacement but in phase with velocity. Typically $\phi \ll 1$ and, in the case of pure structural damping, it is the inverse of the oscillator’s quality factor, $\phi = 1/Q$. A force
acting on the supported mass is described by $f$. Fourier transforming Eq. (5.1), one obtains

$$ - M_{\text{eff}} \omega^2 X - \frac{m}{4} \omega^2 X_0 + k(1 + i\phi)(X - X_0) + i\gamma \omega X = F, $$

where an effective mass has been defined as $M_{\text{eff}} \equiv M + \frac{m}{4}$. Substituting for the undamped resonance frequency $\omega_0 \equiv \sqrt{k/M_{\text{eff}}}$, the displacement and force transfer functions can be written as

$$ H_x = \frac{X}{X_0} = \frac{\omega_0^2(1 + i\phi) + \frac{m}{4M_{\text{eff}}} \omega^2}{\omega_0^2(1 + i\phi) - \omega^2 + i\frac{\gamma}{M_{\text{eff}}} \omega}, $$

$$ H_f = \frac{X}{F} = \frac{1}{\omega_0^2(1 + i\phi) - \omega^2 + i\frac{\gamma}{M_{\text{eff}}} \omega}. $$

Note that for the displacement transfer function we assume $F = 0$ and for the force transfer function, that $F$ is much larger than the forces imposed by the ground motion. Generally, for well engineered systems $\frac{m}{4M_{\text{eff}}} \ll 1$ and $\frac{\gamma}{M_{\text{eff}}} \ll 1$ so that the transfer function decreases with $1/\omega^2$ above the resonance frequency $\omega_0$. Well below the resonance frequency $H_x = 1$. As $\omega \to \infty$ the transfer function will level out to a value given by $\beta = \frac{m}{4M_{\text{eff}}}$. This is known as the center of percussion effect and can be tuned by adjusting the mass distribution of the oscillator with the use of counterweights, as will be demonstrated later. Note that the center of percussion effect is not visible in the force transfer function.

The lower the resonance frequency, the higher the attenuation factor at the frequencies of interest, above $\omega_0$. Attenuation requirements therefore place constraints on $\omega_0$. In basic mechanical oscillators, such low resonance frequencies can only be achieved with large, often impractical dimensions. For example, a 40 dB suppression at a few Hertz requires $f_0 < 100$ mHz. In the case of a simple pendulum, this would mean a length of $25$ m ($\omega_0 = 2\pi f_0 = \sqrt{g/l}$).

This section will describe two types of harmonic oscillators that are implemented in MultiSAS. Their design incorporates an anti-spring effect, allowing for low resonance frequencies, while still having compact dimensions. An anti-spring is a system that incorporates a negative spring constant. That means that once perturbed out of equilibrium the system will keep on moving away from its equilibrium point. In combination with a

Figure 5.2: A simple mass-spring harmonic oscillator. The spring of mass $m$ and spring constant $k$ supports a payload of mass $M$. Viscous damping is represented by the dashpot with damping coefficient $\gamma$.}

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5.2. Vibration isolation with anti-spring technology
positive spring constant the system’s overall stiffness can be tuned, effectively making it possible to achieve near zero stiffness.

5.2.1 The inverted pendulum

A platform supported by two simple inverted pendulums is shown on the left of Fig. 5.3. Each inverted pendulum is comprised of a rigid rod of mass \( m \) and length \( l \), supporting a mass \( M \). It is well known\(^1\) that such a system behaves as an anti-spring: once disturbed out of equilibrium, it will continue to fall over. By fixing the inverted pendulums to the ground with a flexible pivot with a positive angular spring constant \( k_\theta \), a quasi-stable system is obtained. The platform is also fixed to the pendulums by flexures at the top of the legs, the stiffness of which are neglected for now. These allow the platform to translate horizontally (in a small angle approximation). By adjusting the amount of supported mass it is possible to tune the resonance frequency given by \([124]\)

\[
\omega_0^2 \approx \frac{k_\theta}{M} - \left( M + \frac{m}{3} \right) \frac{g}{l} \frac{M}{M + \frac{m}{3}}.
\]  

The inverted pendulum transfer function is similar to Eq. (5.3), but with \( \beta = \frac{m}{6(M + m/3)} \) determining the saturation level of the displacement transfer function due to the center of percussion effect. For an inverted pendulum the center of percussion can be envisaged as follows \([124]\): when the base of the leg is displaced by translational ground motion at frequencies above resonance, the leg rotates around a center of percussion at point \( P \) which remains still. Below \( \omega = \omega_0/\sqrt{\beta} \), the center of percussion coincides with the top flexure. For \( \omega > \omega_0/\sqrt{\beta} \), the center of percussion shifts towards \( P' \) and away from the top flexure. As a result the top of the leg translates out of phase with the ground motion with an amplitude \( x \approx -\beta x_0 \).

To counteract this effect the mass of the pendulum rod can be designed as light as possible, thus reducing \( \beta \). This approach is recommended although limited for legs that bear heavy loads. Additionally, the high frequency center of percussion can be moved to again coincide with the bending point of the top flexure. This restores the leg’s high frequency rotational point back to that of the \( 1/\omega^2 \) dynamics. Tuning the center of percussion is achieved by adding counterweights on an extension of the pendulum rod that reaches below the flexible pivot. This can be seen on the right hand side of Fig. 5.3.

5.2.2 The geometric anti-spring

A geometric anti-spring (GAS) filter, as shown in the left panel of Fig. 5.4a, is a set of radially positioned blade springs, each supporting a load \( F_y \) \([125–127]\). They are radially

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\(^1\)Anybody who has tried to balance an broom, upside down, on their hand will immediately identify with this situation.
5.2. Vibration isolation with anti-spring technology

Figure 5.3: Left: A schematic of a platform of mass $2M$ supported by two basic inverted pendulums with legs of mass $m$. The inverted pendulum is attached to the floor via a flexible joint with a spring constant $k_\theta$. The ground and platform displacement are given by $x_0$ and $x$ respectively and the bending angle by $\theta$. MultiSAS features three inverted pendulums, typical values for MultiSAS are $l = 440$ mm, $m = 1.5$ kg and $M = 200$ kg (per inverted pendulum). Right: A cross-section of the inverted pendulum installed in MultiSAS.

1) Inverted pendulum leg.
2) Bottom flexure
3) Top flexure.
4) Counter weight.
5) Top stage support.

compressed in such a way that the horizontal force they exert on each other results in a tunable anti-spring effect along the vertical axis. In this way the vertical stiffness can be tuned to an arbitrarily low value by adjusting the compression distance. The right panel of Fig. 5.4a shows a schematic of a single GAS blade whose bending profile is described by $\theta(s)$, the angle between horizontal and the tangent of the blade. It is a function of $s$; the distance along the blade, starting from the clamp. The compressional force $F_c$ is reacted on by the opposing blade and produces a horizontal component represented by $F_x$. Once perturbed out of equilibrium the vertical component of this force $F_a$ will provide the anti-spring effect: a force pushing away from the working point, that increases the farther from equilibrium. This is balanced by the vertical force from the bending of the blade spring; a force restoring the keystone to its working point. To accommodate the high loads and stresses on the blades, they are made from heat treated maraging steel, a low carbon alloy with high tensile strength and low creep under stress [128].

The static properties of a GAS filter can be solved analytically for a simplified model, as was done by Cella et al. [129] and is derived from the potential energy of a massless elastic line [130]. It involves solving the differential equation

$$
\frac{d}{ds} \left[ EI(s) \frac{d\theta}{ds} \right] = F_x \sin \theta(s) - F_y \cos \theta(s),
$$

(5.6)
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(a) Left: Example of a GAS filter. Maraging steel blade springs of thickness $d$ and length $L$ are distributed radially around a base plate and clamped at a set angle $\theta_0$. The tips of all the blades are connected to a single central keystone at an angle $\theta_L$. The compressional distance $x_L$ can be tuned by adjusting the compression bolts that slide the blade clamps inwards or outwards. Right: A schematic of a single GAS blade. Compressional and load forces are given by $F_x$ and $F_y$ respectively. The distance along the blade is denoted by $s$ and $\theta(s)$ describes the tangential angle.

(b) Left: Finite element results of applied filter load versus the vertical position of the blade tip, for various compression distances $x_L$. A bi-stability state is evident for $x_L = 244.7$ mm. The solid red curve corresponds to a well-tuned GAS filter. Right: Resonance frequency versus vertical position of the blade tip for various compression distances. Tuning the GAS spring involves finding a compression distance $x_L$ that corresponds to the desired resonance frequency at the required load, whilst remaining stable.

Figure 5.4: Geometric anti-spring design and tuning.
with boundary conditions $\theta(0) = \theta_0$, $\theta(L) = \theta_L$, where $L$ is the length of the blade and $EI(s)$ the blade flexural rigidity at point $s$. This model neglects the force due to stress and strain in the blades and is therefore only valid for materials of Poisson ratio $\nu = 0$. Finite element models of the GAS blades were used by us to accurately reproduce the solutions to Eq. (5.6). In addition, finite element methods were used to extend the model to account for different material Poisson ratios and variations in the blade geometries.

The tunable spring constant properties of the GAS are evident in plots of finite element results for various compressional distances $x_L$. In the left panel of Fig. 5.4b the blade tip height $y_L$ is plotted as a function of the load supported by the GAS filter. The slope of these curves indicate the spring’s vertical stiffness at each point. Note that in this case a negative slope implies a positive stiffness. The solid red curve corresponds to a well-tuned GAS filter i.e. one that is stable and has a low vertical stiffness. If the blades are compressed even further, the system passes a critical point and becomes bi-stable. This can be seen on the right of Fig. 5.4b where the corresponding resonance frequency for each point is plotted as a function of the blade tip vertical position. For increasing compression it is possible to tune the GAS filters to increasingly lower frequencies, down to the critical point, after which the resonance frequencies become undefined.

The dynamic behavior of a GAS filter is governed by Eqs. (5.3) and (5.4) and will also suffer from the center of percussion effect. For a typical GAS filter $\beta \approx 0.001$. The value of $\beta$ can be tuned by adding so called magic wands [122]. A magic wand is a light-weight rigid tube attached to the filter base near the blade clamp via a flexible pivot. A counterweight is fixed to one end and the other end is attached to the keystone via a thin flexure. In this way the counterweight will follow the vertical movement of the keystone but in opposite phase. An illustration of a magic wand is given in Fig. 5.5. By tuning the mass and/or position of the counterweight, it is possible to reduce the $\beta$ value to below $10^{-4}$ as will be demonstrated in Section 5.5.1. The tube is made of silicon carbide for its rigidity and low mass, thus shifting any internal modes to high frequencies, where displacement amplitudes become very small. The lowest (banana) mode of the tube is expected to be above 250 Hz.

### 5.3 MultiSAS design

MultiSAS has been designed to achieve its seismic attenuation requirements within the limited space available in the existing Virgo facility. It is a hybrid system in which bulk isolation is provided by means of a chain of low natural frequency ($f_0 < 1$ Hz) mechanical oscillators, or filters. Filters are realized with pendulums for horizontal and GAS for vertical isolation. An active feedback control system is used only to damp the rigid body eigenmodes and to maintain long term position and orientation of the optical bench.

Based on the translational requirements for residual seismic motion, a single stage system
would have been sufficient to achieve the isolation goal. The horizontal seismic motion measured at Virgo is around 1 nm/√Hz at 10 Hz. A single stage mechanical filter with a resonance frequency at 0.5 Hz providing -60 dB of attenuation at 10 Hz, would yield sufficient isolation. However the rotational requirements, being much stricter, necessitate additional stages. MultiSAS is therefore equipped with two vertical stages realized with GAS filters and three horizontal stages consisting of one inverted pendulum and two conventional pendulum stages. Fig. 5.6 shows a schematic layout of the five attenuation stages realized in MultiSAS. Furthermore, MultiSAS was conceived based on the following design criteria:

- Minimize cross-couplings between degrees of freedom due to asymmetries in the mechanics. In particular the horizontal to tilt coupling is expected to dominate the residual pitch and roll motion of the bench.
- Reduce the transmission of seismic motion around the high frequency internal modes of the system.
- Utilize the full potential of the filters by allowing the mechanical tuning of their mass distributions.
- Parasitic stiffness of the cabling must be kept as small as possible.

5.3.1 Mechanical overview

An illustration of the MultiSAS design is shown in Fig. 5.7. Note that the coordinate system is chosen to be consistent with the Virgo coordinate convention where $z$ is in
5.3. MultiSAS design

the direction of the laser beam and $y$ is the vertical component. MultiSAS features the following components and subsystems:

- A horizontal pre-isolation stage known as the *top stage*. It is a rigid structure supported by three inverted pendulum legs with a length of 440 mm. The mass of the top stage is adjusted so that the lowest horizontal modes of MultiSAS are around 100 mHz. The yaw ($\theta_y$) mode of the top stage is at 800 mHz. This stage also provides an inertial platform from which to detect and damp the modes of the suspension chain that recoil to top stage horizontal motion.

- A vertical *top filter* stage consisting of a 12 blade GAS filter supported by the top stage. The top filter will be able to support a maximum load of 450 kg on a single wire connected at the GAS keystone and is tuned to a resonance frequency of about 200 mHz. Two weak blade springs known as fishing rods are connected to the filter keystone. Via a stepper motor they will control the DC position of the keystone without spoiling the seismic isolation.

- An *intermediate stage* suspended by the 694 mm long wire from the top filter. It consists of an 8 blade GAS filter tuned to around 300 mHz that can support up to 320 kg. It has an inverted base design which allows the positioning of the center of mass close to the bending point of the suspending wire in order to reduce the coupling between the horizontal and tilt degrees of freedom. The intermediate filter utilizes up to 4 magic wands to compensate for the moment of inertia of the GAS blades. A photo of the intermediate stage is shown in Fig. 5.8.

- The *optical bench* will be suspended from the intermediate filter keystone by a 760 mm long wire. The tilt modes of the bench will be set (by adjusting the connection point of the wire with respect to the bench’s center of mass) to around 200 mHz.
to decouple these modes from the horizontal pendulum modes of the chain. The single wire design introduces a second stage of decoupling between the pitch and roll modes of the bench and the translational degrees of freedom of the chain.

**MiniTower vacuum chamber**

The entire MultiSAS and optical bench will be incased by a vacuum system coined MiniTower. MiniTower is designed to provide both a rigid support for MultiSAS and a clean vacuum environment for the optical systems, and to eliminate acoustic coupling to the bench. An impression of MultiSAS supporting an end bench in a MiniTower is given in Fig. 5.1.
Figure 5.8: Intermediate filter as viewed from underneath, showing the 8 GAS blades attached in the center to the keystone. There are two magic wands installed, connected to the keystone and via a flexible pivot to the intermediate stage frame. The magic wand’s counter weights can be seen attached outside of the pivots.

5.3.2 Sensors and actuators

MultiSAS is monitored continuously by two types of motion sensors: linear variable differential transformers (LVDT) and inertial sensors called geophones.

A geophone is a velocity sensor that produces a voltage proportional to the velocity difference between its housing and an internally suspended reference mass. An electromotive force $\mathcal{E}$ (voltage) is induced in a pick-up coil (of resistance $r_c$) wound on the reference proof mass as it moves with respect to a permanent magnet attached to the housing of the geophone. The electromotive force is given by

$$\mathcal{E} = -N \frac{d\Phi}{dt} = -N \frac{d\Phi}{dx} \dot{x},$$

(5.7)

where $N$ is the number of coil turns and $\Phi$ the magnetic flux produced by the permanent magnet. We see that the voltage is proportional to the velocity difference $\dot{x}$ between coil and housing. The geophone sensitivity as defined by $G = \mathcal{E}/\dot{x}$ in terms of volts per meter per second. It can therefore be described by $G = -N \frac{d\Phi}{dx}$. Geophone design permits $\frac{d\Phi}{dx}$ and hence $G$ to be uniform over the proof mass displacement range.
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The frequency response of a geophone can be described in the Laplace domain by

\[
G_{\text{geo}}(s) = G \frac{r_d}{r_d + r_c \omega_0^2 + 2 \xi \omega_0 + s^2} \quad [\frac{V}{m/s}],
\]

(5.8)

where \(r_d\) is the damping resistance placed in parallel with \(r_c\), \(\xi\) the damping ratio, \(\omega_0\) the resonance frequency of the geophone’s mass spring system and \(s\) the Laplace parameter. For \(s \gg \omega_0\) the transfer function is flat providing a constant \(G_{\text{geo}} \approx G \frac{r_d}{r_d + r_c}\). For \(s \ll \omega_0\), \(G_{\text{geo}}\) approaches zero. For this reason geophones lose sensitivity at low frequencies where their reference mass looses its inertial properties.

The custom built MultiSAS LVDTs consists of three solenoidal coils, centered along the same axis with the central primary coil placed within the two outer secondary coils [131]. A bisection of an LVDT is shown on the right of Fig. 5.9. The primary coil is attached to the suspended object and is free to move along the measurement axis between the two secondary coils that are fixed to a reference frame. At the zero displacement position the center of the primary coil is situated equidistant from both secondary coils. A 10 kHz excitation current is driven through the primary coil which induces currents in both secondary coils. Because the secondary coils are wound in opposite directions (in a so-called Maxwell pair configuration) the equal yet opposite induced currents will cancel at the zero displacement position. The LVDT’s voltage output, after demodulation, is linear to displacement within 1%, over a range of ±4 mm, with a noise level of a few nm/√Hz. It should be noted that the measurement is always made with respect to the reference frame and hence contains information on the motion of both reference frame and suspended object. In the case of MultiSAS, seismic motion of the reference frame is therefore included in the raw LVDT measurements and will dominate the signal above the resonance frequency of the filters. Later, a technique known as sensor correction will be introduced to reduce these effects.

Each of the LVDTs are co-located with co-axial magnetic voice coil actuators that allow active control of the system. The geometry of the coil and magnetic yoke of these actuators are designed to deliver constant force (within 1%) over a 10 mm movement range. The actuators are capable of positioning the optical table within the resolution of the LVDTs. A bisection of a voice coil actuator is also shown on the left of Fig. 5.9.

**Top stage sensing and actuation**

The top stage houses three L4C horizontal geophones [132] positioned around a circle separated by 120 degrees. These will provide inertial sensing of the top stage for the dynamic control of the eigenmodes of the pre-isolator itself as well as all other modes of the chain that recoil to the horizontal motion of the top stage. Three LVDT position sensors mounted underneath the top stage (co-located with the geophones) will monitor its DC and low frequency position with respect to the reference frame that is rigidly connected to the base ring (and hence the ground). Voice coil actuators are co-located with the LVDTs to provide feedback forces for dynamic control. Fig. 5.10 shows a photo.
of the MultiSAS top stage, indicating the locations of a horizontal geophone, LVDT and voice coil.

**Bench sensing and actuation**

The rotational modes of the bench will not recoil to the top stage and are therefore not controllable by the top stage dynamic control. Four horizontal and four vertical LVDT sensors and co-located coil-magnet actuators will be placed at each corner of the bench, and referenced from the base of the vacuum chamber. From there, they will be used for the active damping and low frequency positioning of the bench. The actuators have been optimized for the weaker forces required by the control to minimize any re-injection of seismic noise from the vacuum chamber reference. One L22E vertical geophone [132] will be placed in the center of the bench to provide an inertial measurement of the bench’s vertical motion.

**DC positioning**

It will be required to provide intermittent (at intervals of several weeks), out of loop control of the DC position of the optical bench. This is to cope with large scale temperature changes and to correct any long term drift of the chain’s vertical position due to residual creep of the GAS blades. For this reason, DC positioning of the bench in all 6 degrees of freedom is foreseen by a series of stepper motor controlled adjustment blade
springs or counter weights. All degrees of freedom will in this way be controllable to within 1 \( \mu \text{m} \) / 1 \( \mu \text{rad} \). Three motorized adjustment blade springs act on the top stage in the horizontal plane to move the entire MultiSAS chain in the horizontal and yaw degrees of freedom within the range set by the horizontal stops (± 8 mm); see Fig. 5.10. Motorized ‘fishing rod’ blade springs are mounted on each of the vertical filters. Two stepper motor controlled counterweights underneath the bench will be used to perform fine leveling of the tilt degrees of freedom. The tilt error signal will be provided by a commercial dual axis tilt meter which has a resolution better than 1 \( \mu \text{rad} \).

### 5.4 System modeling

Modeling of the attenuation system is an important part of understanding its dynamic characteristics and a useful tool for the design of control strategies. Here, MultiSAS is modeled as a rigid body system. That means that it is assumed that each component of the system has no internal vibrational modes and we concentrate on the modes only associated with the coupled vibrations of the rigid components. This assumption is generally accurate when studying the low frequency (< 10 Hz) dynamics of such systems.
This corresponds to the frequency range in which active feedback will be implemented, allowing us to use these models to analyze control strategies. Furthermore, we assume that the vertical, horizontal and yaw modes are uncoupled. In this way, we can divide the modeling problem into three separate models, each one responsible for describing vertical, horizontal and yaw motion. In this section only the vertical and horizontal models will be presented. The yaw motion could not be properly measured in the current setup and is therefore left for subsequent studies.

5.4.1 Vertical model

The vertical dynamics of MultiSAS can be reduced to a double mass spring system shown in Fig. 5.11. The assumption is made that the mass of the springs (GAS blades) are negligible with respect to the suspended masses. The generalized coordinates describing the degrees of freedom are the vertical displacement of the intermediate filter of mass \( m_1 \) given by \( y_1 \), and the vertical displacement of the bench payload with mass \( m_2 \) denoted by \( y_2 \). The GAS filters are characterized by the corresponding spring constants \( k_1 \) and \( k_2 \), with viscous damping coefficients \( \gamma_1 \) and \( \gamma_2 \). The actuation force \( f_y \) for the resonance frequency damping is applied to the intermediate stage to evoke a control displacement \( u_y \). Finally the top stage vertical motion is given by \( y_0 \) and can be considered equivalent to the vertical ground motion.

![Figure 5.11: Schematic of MultiSAS vertical model.](image)

The Euler-Lagrange approach is used to solve the equations of motion. Given the potential energy \( U \) and the kinetic energy \( T \) and defining the Lagrangian \( L = T - U \), the
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equations of motion will follow from the Euler-Lagrange equation [133]

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}_i} \right) - \frac{\partial L}{\partial y_i} = Q_i. \tag{5.9}$$

Here each of the generalized coordinates \(y_1\) and \(y_2\) are accounted for by \(i = [1, 2]\) and external forces enter into the equation through the generalized forces \(Q_i\). Non-conservative forces\(^2\) such as friction and other sources of dissipation or damping require special treatment in the Euler-Lagrange method. In the special case of viscous damping it is useful to introduce the Rayleigh dissipation function \(R\) and describe the generalized forces in terms of their conservative contribution only, denoted here by \(Q^*_i\) [134]. Eq. (5.9) then becomes

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}_i} \right) - \frac{\partial L}{\partial y_i} + \frac{\partial R}{\partial \dot{y}_i} = Q^*_i. \tag{5.10}$$

The MultiSAS vertical kinetic energy, potential energy and dissipation are described by

\[
T = \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2, \\
U = \frac{1}{2} k_1 (y_1 - y_0)^2 + \frac{1}{2} k_2 (y_2 - y_1)^2, \\
R = \frac{1}{2} \gamma_1 (\dot{y}_1 - \dot{y}_0)^2 + \frac{1}{2} \gamma_2 (\dot{y}_2 - \dot{y}_1)^2,
\]

and the conservative contribution of the external forces become \(Q^*_1 = f_y = k_1 u_y\) and \(Q^*_2 = 0\). We then arrive at the following equations of motion for \(y_1\) and \(y_2\) respectively

\[
m_1 \ddot{y}_1 = -k_1 (y_1 - y_0) + k_2 (y_2 - y_1) - \gamma_1 (\dot{y}_1 - \dot{y}_0) + \gamma_2 (\dot{y}_2 - \dot{y}_1) + f_y^*, \tag{5.12}\\
m_2 \ddot{y}_2 = -k_2 (y_2 - y_1) - \gamma_2 (\dot{y}_2 - \dot{y}_1). \tag{5.13}
\]

To facilitate a transformation to a state space description, the additional first order differential equations \(v_1 = \dot{y}_1\) and \(v_2 = \dot{y}_2\) are defined. It is now possible to reduce the equations of motion from two second order, to four first order differential equations.

\(^2\)Conservative forces, also known as potential forces, can be described as the gradient of a potential \(f_i = -\Delta U\). Forces arising from gravitation \((U = mgy)\) or ideal springs \((U = \frac{1}{2}kx^2)\) are conservative.
These can be rewritten into a state space representation given by

\[
\begin{bmatrix}
\dot{y}_1 \\
\dot{y}_2 \\
\dot{v}_1 \\
\dot{v}_2
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\frac{-k_1 + k_2}{m_1} & \frac{k_2}{m_2} & \frac{-(\gamma_1 + \gamma_2)}{m_1} & \frac{\gamma_2}{m_2} \\
\frac{-k_1}{m_1} & \frac{k_2}{m_2} & \frac{-\gamma_1}{m_1} & \frac{\gamma_2}{m_2}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
v_1 \\
v_2
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
k_1 \\
k_2
\end{bmatrix}
\begin{bmatrix}
y_0 \\
v_0
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
\gamma_0 \\
\gamma_0
\end{bmatrix} u_y
\]

\[
\begin{bmatrix}
y_{vdt} \\
y_{geo}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
v_1 \\
v_2
\end{bmatrix}
\]

The dynamics of the MultiSAS vertical system (in control terms often referred to as the plant) can now be described by the state space matrices \([A_p, B_p, C_p, D_p = 0]\), state vector \(x_p = [y_1, y_2, v_1, v_2]^T\) and input \(u_y\). Disturbances to the system from ground motion are contained in \(B_{u_{td}}\). The state space model also provides transfer function predictions of input forces to output measurements; see Eq. (1.45). Measurements were made for the MultiSAS vertical transfer by applying lowpass filtered white noise to the vertical voice coil on the top filter. The filter corner frequency was set at 5 Hz. The displacement \(y_1\) was recorded with the top filter LVDT and the velocity \(v_2\) with the geophone on the payload. Note that the raw LVDT measurement is in fact \(y_1 - y_0\), here \(y_{vdt}\) refers to the sensor corrected LVDT signal. Sensor correction is discussed in Section 5.6.1. It is assumed that the force displacement is much larger than the external (seismic) displacement \((u_y > u_d)\) such that the last term in Eq. (5.14) can be neglected. The measurement and model results are shown in Fig. 5.12. The parameters used in the model were chosen to best fit the measurements given the constraint that the known total mass is 420 kg. They are given in Table 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass [kg]</td>
<td>(m_1)</td>
<td>(m_2)</td>
<td>315</td>
</tr>
<tr>
<td>Spring constant [N m^{-1}]</td>
<td>(k_1)</td>
<td>1036</td>
<td>(k_2)</td>
</tr>
<tr>
<td>Damping coefficient [N s m^{-1}]</td>
<td>(\gamma_1)</td>
<td>20</td>
<td>(\gamma_2)</td>
</tr>
</tbody>
</table>

**Table 5.2:** Parameters for the vertical MultiSAS state space model.

There is agreement between the modeled and measured transfer functions. The noise seen above 5 Hz is from the loss of correlation between the excitation noise and the measured signals. At around 620 mHz a small glitch is visible. This is due to the coupling of the dummy payload pitch and roll modes to the vertical motion of the top keystone. One can also notice a difference between measurements and model in the low-frequency damping properties. This is evident in the sharper modeled peaks and steeper
Figure 5.12: Comparison between measured and modeled force to displacement transfer functions for the vertical stages of MultiSAS. Modeled results given with no structural damping ($\phi = 0$, dashed black curve) and with structural damping ($\phi = 0.05$, dot dashed cyan curve). The insets plot the ratio between modeled and measured transfer functions in the range 0.1 to 1 Hz. (a) Top filter measured LVDT displacement over forced displacement. (b) Measured bench geophone velocity over forced displacement.
modeled phase transitions, and emphasized by the insets showing the ratio between measured and modeled transfer function magnitudes. This can be attributed to additional structural damping in the GAS blades, where the model only regards viscous damping. Structural damping is more prevalent the lower the resonance frequency. It can partially be accounted for in the model by adding a small complex term in the spring constant such that $k' = k(1 + i\phi)$. This effectively adds a damping term that is proportional to the displacement but in phase with the velocity. For the MultiSAS vertical model $\phi \approx 0.05$ rad is sufficient to account for the structural damping. However, to simplify subsequent computations and control schemes this term is further neglected.

Fig. 5.12 also demonstrates the attenuation performance of MultiSAS. At 5 Hz the top filter already provides -40 dB of vibration suppression while the transmission to the velocity of the bench is -60 dB. Translated to bench displacement this is equal to an attenuation of roughly 90 dB ($20 \log_{10}[10^{-60/20}/(2\pi f)]$).

### 5.4.2 Horizontal model

To model the horizontal dynamics of MultiSAS a number of assumptions were made to simplify the problem. The model developed here is a useful tool in the understanding of MultiSAS dynamics and a starting point for a more elaborate framework. Computational assistance from Maple, a symbolic math program, was used to arrive at the final results. The following assumptions were made:

- The three inverted pendulums can be described by a single pendulum.
- There is no coupling from the vertical and yaw degrees of freedom to horizontal motion. This is based on the assumption of having a symmetric design and perfect construction.
- The top stage undergoes no tilt, roll or vertical motion when translated horizontally. In the small angle regime this would seem a valid assumption. The tilt motion of the intermediate stage is the only rotational mode accounted for in the model.
- Similarly, the rotational motion of the payload will not couple to a horizontal response in the chain. Sufficiently low bending stiffness of the final wire and correct placement of the suspension point with respect to the bench’s center of mass should validate this assumption.
- The horizontal dynamics are isotropic. Under this assumption the problem can be reduced from a three to a two dimensional system and is founded on a perfectly symmetric design.
- Viscous damping is neglected but structural damping will be included as complex stiffness terms.
Chapter 5. Vibration isolation

- The masses of the wires are negligible with respect to the masses of the top and intermediate stages and payload.

MultiSAS is described by the inverted pendulum with an effective horizontal stiffness $k_{IP}$, supporting the top stage of mass $m_0$, which in turn supports the suspension chain. The latter consists of a top wire of length $l_1$ suspending the intermediate stage of mass $m_1$ and moment of inertia $J$, followed by the bottom wire suspending the payload of mass $m_2$ on a wire of length $l_2$. At the intermediate stage the wire bending points are not exactly at the center of mass but rather shifted vertically by a distance $d_1$ and $d_2$ for the top and bottom wires respectively. A schematic of the MultiSAS horizontal model is given in Fig. 5.13.

![Figure 5.13: Schematic of the horizontal model of MultiSAS.](image)

The horizontal translational motion of the ground, top stage, intermediate stage and payload are given by $x_{gr}$, $x_0$, $x_1$ and $x_2$ respectively. Vertical motion of the top stage is neglected but the respective vertical displacements of the intermediate stage and payload are denoted by $y_1$ and $y_2$. The tilt angle of the intermediate stage is represented by $\alpha$ while the angles of the suspension wires with respect to the vertical are given by $\delta_1$ and $\delta_2$. Finally the bending angles of the wires at the intermediate stage are defined as $\beta_i \equiv \alpha - \delta_i$ where $i = [1, 2]$ correspond to the top and bottom wires respectively. The wire angular bending stiffness $k_i$, is considered identical at both ends of the wire.
system modeling

The Euler-Lagrange approach will again be used to derive the equations of motion with the generalized coordinates \( x_0, x_1, x_2 \) and \( \alpha \). In this case the kinetic and potential energies can be described by

\[
T = \frac{1}{2} J \dot{\alpha}^2 + \sum_{i=0}^{2} \frac{1}{2} m_i \dot{x}_i^2,
\]

\[
U = \frac{1}{2} k_{1p} (x_0 - x_{gr})^2 + \sum_{i=1}^{2} \left( m_i g y_i + \frac{1}{2} k_i (\beta_i^2 + \delta_i^2) \right),
\]

where the last term in the sum of the potential energy accounts for the angular bending energy in the top and the bottom of each wire. Note also that the inverted pendulum stiffness has been converted from an angular to a linear stiffness \( k_{1p} = k_\theta / l_{1p}^2 \). In the small angle approximation the vertical displacements can be written as

\[
y_1 = \frac{1}{2} (l_1 \delta_1^2 + d_1 \alpha^2),
\]

\[
y_2 = y_1 + \frac{1}{2} (l_2 \delta_2^2 + d_2 \alpha^2),
\]

where the small angle approximation \( \cos \theta \approx 1 - \theta^2 / 2 \) has been applied. The first term between brackets accounts for the center of mass vertical position moving due to the pendulum motion and the second term is an additional contribution due to the displaced bending points with respect to the intermediate stage center of mass. The angles \( \delta_i \) are related to the generalized coordinates via

\[
\delta_i = \frac{x_i - x_{i-1} - d_i \alpha}{l_i}.
\]

It is more convenient to write the masses in terms of the tension forces they create on the wires so that for the top wire \( T_1 = (m_1 + m_2)g \) and the bottom wire \( T_2 = m_2g \). Deriving the equations of motion via the Euler-Lagrange equation produces

\[
\begin{bmatrix}
 m_0 & 0 & 0 & 0 \\
 0 & m_1 & 0 & 0 \\
 0 & 0 & m_2 & 0 \\
 0 & 0 & 0 & J
\end{bmatrix}
\begin{bmatrix}
 \ddot{x}_0 \\
 \ddot{x}_1 \\
 \ddot{x}_2 \\
 \ddot{\alpha}
\end{bmatrix}
+
\begin{bmatrix}
 k_{1p} + b_1 & -b_1 & 0 & -c_1 \\
 -b_1 & b_1 + b_2 & -b_2 & c_2 - c_1 \\
 0 & -b_2 & b_2 & -c_2 \\
 c_1 & c_2 - c_1 & -c_2 & b_1 d_1^2 + b_2 d_2^2 + c_1 l_1 + c_2 l_2
\end{bmatrix}
\begin{bmatrix}
 x_0 \\
 x_1 \\
 x_2 \\
 \alpha
\end{bmatrix}
= \begin{bmatrix}
 k_{1p} \\
 0 \\
 0 \\
 f
\end{bmatrix} u_x, \tag{5.19}
\]
where the mass matrix $M$ and stiffness matrix $K$ have been defined and the displacements from the horizontal control force $f_x$ are described by $u_x$. To simplify the above equation the following substitutions are made

$$b_i = \frac{l_i T_i + 2 k_i}{l_i^2},$$

and

$$c_i = \frac{1}{2} b_i l_i + b_i d_i - \frac{1}{2} T_i. \quad \text{(5.20)}$$

The values for the various parameters can now be substituted into these equations or calculated. The bending stiffness of the wires was derived from their flexural rigidity by

$$k_i = \sqrt{EI_i T_i (1 + i \phi)}, \quad \text{(5.21)}$$

where the wire’s properties are contained in the material’s Young’s modulus $E$ and the second moment of area, given by $I = \pi r_i^4 / 4$ for a cylindrical rod of radius $r_i$. The structural damping is accounted for in the complex coefficient $\phi$. The inverted pendulum stiffness is obtained from its resonance frequency $f_{IP}$ by

$$k_{IP} = (m_0 + m_1 + m_2)(2\pi f_{IP})^2 (1 + i \phi). \quad \text{(5.22)}$$

Finally the moment of inertia of the intermediate stage is assumed to be that of a thin disk of radius $R$: $J \approx m_1 R^2 / 4$. The parameters of the model and their corresponding values are given in Table 5.3 and are equivalent to the designed mechanical dimensions.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the top stage [kg]</td>
<td>$m_0$</td>
<td>100</td>
</tr>
<tr>
<td>Mass of the intermediate stage [kg]</td>
<td>$m_1$</td>
<td>105</td>
</tr>
<tr>
<td>Payload mass [kg]</td>
<td>$m_2$</td>
<td>315</td>
</tr>
<tr>
<td>Inverted pendulum resonance freq. [Hz]</td>
<td>$f_{IP}$</td>
<td>0.11</td>
</tr>
<tr>
<td>Top wire - Length [m]</td>
<td>$l_1$</td>
<td>0.7</td>
</tr>
<tr>
<td>- Radius [mm]</td>
<td>$r_1$</td>
<td>1.5</td>
</tr>
<tr>
<td>Bottom wire - Length [m]</td>
<td>$l_2$</td>
<td>0.8</td>
</tr>
<tr>
<td>- Radius [mm]</td>
<td>$r_2$</td>
<td>1.25</td>
</tr>
<tr>
<td>Intermediate mass radius [m]</td>
<td>$R$</td>
<td>0.6</td>
</tr>
<tr>
<td>Maraging steel Young’s modulus [GPa]</td>
<td>$E$</td>
<td>186</td>
</tr>
<tr>
<td>Structural damping coefficient [N m$^{-1}$]</td>
<td>$\phi$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Table 5.3: Parameters for the horizontal MultiSAS model.**

The state space representation of the horizontal model is easily derived from the mass and stiffness matrices. When the state vector is given by $x_H = [x_0, x_1, x_2, \alpha, \dot{x}_0, \dot{x}_1, \dot{x}_2, \ddot{\alpha}]$ consisting of $n = 4$ generalized coordinates, the state space equations become

$$\begin{aligned}
\dot{x}_H &= \left[ \begin{array}{cc}
\Theta_{n \times n} & I_n \\
-M^{-1}K & \Theta_{n \times n}
\end{array} \right] x_H + \left[ \begin{array}{c}
\Theta_{n \times 1} \\
\kappa/m_0
\end{array} \right] u_x, \\

y_H &= C_H x_H,
\end{aligned} \quad \text{(5.23)}$$
where $\kappa = [k_{1p} \ 0 \ 0 \ 0]^T$ from the final term in Eq. (5.19). Setting $C_H = [I_{n \times n} \ \theta_{n \times n}]$ will simulate a measurement of each of the generalized coordinates.

Simulated force transfer function results of the modeled horizontal MultiSAS are shown in Fig. 5.14. The model predicts the three horizontal modes associated with: the inverted pendulum resonance where all three masses move in common motion (~0.1 Hz), the top stage moving in anti-phase to the suspension chain (~0.75 Hz) and the fastest mode (~1.8 Hz) where the top stage and bench move in opposite phase to the intermediate stage. A fourth mode corresponding to the rotational resonance of the intermediate stage is found at ~1.3 Hz in the simulated results of $\alpha/u_x$. This mode couples weakly to the other degrees of freedom, evident in the relatively small glitches that are produced in the modeled $x_0$ and $x_2$ results. The MultiSAS horizontal attenuation performance is demonstrated by these simulations. Transmission of ground motion is reduced at the level of the bench motion $x_2$ by 150 dB (a factor of $3 \cdot 10^{-7}$) at 10 Hz.

![Figure 5.14](image)

**Figure 5.14**: Left: Comparison between measured (solid red curve) and modeled (dashed blue curve) forced displacement transfer functions for the horizontal top stage motion. Parameters for modeled results are given in Table 5.3. Right: Modeled forced displacement transfer functions for the horizontal payload (bench) motion (solid green curve) and the intermediate stage tilt motion (dashed magenta curve).

The modeled top stage motion is verified against a forced displacement transfer function measurement by applying a lowpass filtered white noise excitation at the horizontal voice coils and measuring the subsequent response on the LVDTs. These results are also shown in the left panel of Fig. 5.14 (solid red curve). There is agreement between the modeled and measured results. Above 5 Hz the measurements are distorted due to the diminishing signal to noise ratio. The additional structures around 0.5 Hz are most likely the result
of tilt motion of the payload (during these measurements represented by a dummy mass) recoiling to the top stage. These couplings were neglected in the model as this effect is expected to be reduced once the prototype bench has been installed. The appropriate wire suspension position close to the bench’s center of mass will drive the tilt modes to lower frequencies and reduce the couplings.

5.5 System characterization

The MultiSAS prototype underwent a number of characterization and performance tests to verify its dynamic behavior and fulfillment of design criteria. All tests described here (and in subsequent sections) were not performed in vacuum. This was done in order to gain a full understanding of the system before committing it to a vacuum system where access is less trivial. To emulate an optical bench a dummy payload consisting of two steel disks, each 1 meter in diameter and weighing 150 kg, were suspended from the intermediate filter.

5.5.1 Vertical transfer functions

The isolation performance of the GAS filters makes it challenging to measure their characteristics over a large frequency span and dynamic range. Common measurement techniques fail because sensors are not sensitive enough to measure residual motion of the isolated object. On the other hand, the signals may clip around the filter resonance frequency due to dynamic range restrictions. Forced excitation in combination with a series of accelerometers with varying sensitivity helped to overcome these issues.

Top stage and filter

The forced excitation of the base ring was achieved with piezo actuators. Three actuators were installed to support the base ring under each of the three inverted pendulum legs. By applying a (high voltage) in-phase signal to each of the piezo actuators the entire MultiSAS could be excited vertically. Increasing the amount of input displacement made it possible to detect vibrations on the isolated objects with a sufficiently high signal to noise ratio. Note that both the GAS filters were first tuned and tested separately in a dedicated setup. The tests described here present those performed on the fully assembled system. The base ring was excited with a swept sine signal that increased the frequency of excitation at set intervals. An accelerometer with relatively low sensitivity was placed on the base ring, while a second accelerometer was placed on the top or intermediate stage. The latter was fastened as close as possible to the center of mass of the respective objects to reduce tilt to vertical coupling. For the top filter test, the sine wave amplitudes were kept small around the resonance frequency to prevent the accelerometer signals...
from clipping, and increased at high frequencies, where the isolation was largest and the signal to noise ratio would otherwise be too small.

The results of the piezo actuated vertical transfer function measurements are shown in Fig. 5.15. The dashed black curve represents the base to top stage transfer and shows that the inverted pendulum can be considered vertically stiff up to around 50 Hz. Above 50 Hz a number of peaks and notches appear. These are a result of the finite stiffness of the inverted pendulum legs and flexures in the vertical direction causing the top stage to ‘bounce’ on its supports. The solid green curve is the transmission from the base ring to the intermediate stage. The curve follows the expected $1/f^2$ before leveling out to $6 \cdot 10^{-3}$ (−45 dB) above 2 Hz. Note that there are no magic wands installed on the top filter and that the filters are tuned to slightly different resonance frequencies with respect to the measurements done in Section 5.4.1.

**Figure 5.15:** MultiSAS vertical piezo actuated swept sine transfer functions for separate stages. The total transfer function is a projection obtained by multiplying the top and intermediate stage transfer functions.

**Intermediate filter**

Similar techniques to those implemented for the top stage were used to determine the intermediate filter transfer function. In these tests the intermediate stage was excited using the voice coil in the top filter. This bypasses the top filter entirely making it possible to isolate the intermediate filter properties. The results of the intermediate
filter transfer function results are presented by the dash-dotted blue curve in Fig. 5.15 and will be described in more detail in Section 5.5.2.

**Total vertical transfer function**

Despite the forced actuation it was still not possible to perform measurements from the base ring all the way to the suspended bench. Instead, the separate measurements from the base ring to intermediate stage and intermediate stage to the bench, were combined to predict the full MultiSAS vertical transmission performance. This is done by multiplying the top and intermediate filter transfer functions. These results are represented in Fig. 5.15 by the solid red curve. Here we see that an attenuation, from base ring to bench, of a factor $10^{-5}$ is achieved at 10 Hz and a factor of 1 million at 30 Hz.

The seismic noise level at the Virgo site at 10 Hz has been measured and amounts to around $10^{-9} \text{ m}/\sqrt{\text{Hz}}$. This would result in a residual vertical motion of the bench at 10 Hz of $10^{-14} \text{ m}/\sqrt{\text{Hz}}$ which is well within the translational requirements of the end benches ($2.1 \cdot 10^{-12} \text{ m}/\sqrt{\text{Hz}}$). As mentioned earlier the additional filter stages were foreseen to achieve the more stringent rotational requirements. Unfortunately, the performance of the rotational dynamics could not be measured with the current setup, but will be performed once the optical bench prototype and accompanying sensing system have been installed.

### 5.5.2 Magic wand performance

The swept sine tests were performed on the intermediate filter with and without the magic wands installed. The results are shown in Fig. 5.16 in terms of displacement transfer functions. Modeled responses of the respective configurations based on Eq. (5.3) are also plotted. It is clear that the resonance of the filter is tuned to 300 mHz above which it shows the expected $1/f^2$ drop in transmission. The resonance frequency is poorly sampled so the modeled structural damping loss angle $\phi$ is chosen to best fit the resonance flanks. Without magic wands the transfer function plateaus out at $10^{-4}$ (-60 dB). Installation of the magic wands adjusts the weight distribution of the blade springs and lowers the value of $\beta$, allowing the filter to better follow the $1/f^2$ model. As a result, an attenuation of $10^{-4}$ (-80 dB) is achieved around 50 Hz. Above 50 Hz the transmission of both configurations starts to increase again as a result of the first higher order vertical mode of the system. This mode reaches its peak at 135 Hz and is associated with the keystone of the intermediate filter oscillating on the non-rigid wire supporting the suspended mass.
Figure 5.16: MultiSAS intermediate filter swept sine transfer function with and without magic wands installed. Modeled transfer functions based on Eq. (5.3) are also included. The magic wands allow the filter to better reproduce the $1/f^2$ characteristic of an ideal harmonic oscillator. The peak at 135 Hz is the first higher order vertical mode associated with the filter keystone mass.

5.6 Control system

We have seen that a harmonic oscillator can function as a mechanical second-order lowpass filter. At the resonance frequency, however, seismic motion can be amplified by up to several orders of magnitude, depending on the quality factor of the respective oscillator. For MultiSAS an amplification of 20 dB is typical. The requirements for the suspended bench residual motion demand full suppression of these modes. This is achieved by an active feedback control system.

5.6.1 Coordinate system transformation and pre-filtering

Coordinate system transformation involves combining the real sensor signals $\mathbf{y}$, into virtual sensors $\tilde{\mathbf{y}}$, that describe the motion of the bench in the $xyz$ coordinate system referred to the Virgo optical coordinate frame. For example, the three horizontal LVDT signals from the top stage are combined to produce the $x$, $z$ displacements and yaw ($\theta_y$) rotation. This transformation is done via the sensing matrix $\mathbf{S}$. The feedback signals are then computed in the usual way, only now for the virtual actuator $\tilde{\mathbf{u}}$, in the $xyz$
Chapter 5. Vibration isolation

coordinate system. Again a transformation is required to obtain the correct real actuator signal \( \mathbf{u} \) and this is done with driving matrix \( \mathbf{D} \). Mathematically these relations can be described by

\[
\tilde{\mathbf{y}} = \mathbf{S}\mathbf{y}, \quad \text{and} \quad \mathbf{u} = \mathbf{D}\mathbf{u}.
\] (5.24)

The top stage control is shown schematically in Fig. 5.17. Sensing matrix \( \mathbf{S} \) is derived geometrically from the physical positions of each sensor, while \( \mathbf{D} \) is obtained by measuring transfer functions between actuators and sensors, described in Section 5.6.2. The numerical values for \( \mathbf{S} \) for the top stage LVDT sensing matrix are given by

\[
\begin{bmatrix}
\tilde{x}
\tilde{z}
\tilde{\theta}_y
\end{bmatrix} =
\begin{bmatrix}
-0.5773 & 0 & 0.5773 \\
0.333 & -0.667 & 0.333 \\
0.725 & 0.725 & 0.725
\end{bmatrix}
\begin{bmatrix}
\text{Hor0} \\
\text{Hor1} \\
\text{Hor2}
\end{bmatrix}.
\] (5.25)

This matrix transforms the LVDT signals, Hor0, Hor1 and Hor2 into the \( xyz \) coordinate system measured in \( \mu m \) or \( \mu rad \). The information from the geophones is diagonalized in an identical way to the \( xyz \) signals to obtain inertial measurements in the same degrees of freedom.

**Sensor correction**

The LVDT displacement measurement is made with respect to the reference frame and is therefore sensitive to the (seismic) motion of this frame. A technique known as sensor correction involves subtracting the measured seismic motion from the LVDT signal [135]. It is performed on the top stage vertical and horizontal LVDT signals where a Trillium 240 seismometer, placed on the ground near the MultiSAS setup, provides the reference correction. Sensor correction is applied in a bandwidth up to a few Hz, above which the self noise of the LVDTs begins to dominate its signal.

**Sensor blending**

Sensor blending is used to cope with the incongruity between LVDT and geophone sensitivity. Where both LVDT and geophone signals are available for the same reconstructed degree of freedom, the two measurements are blended into one super sensor. A super sensor combines the low frequency LVDT signals with high frequency geophone data. This is done by applying a lowpass filter, with frequency response \( L(s) \) to the LVDT signal and a complimentary highpass filter \( H(s) = 1 - L(s) \) to the geophone signal, before combining the two [136]. This ensures that there is no phase distortion around the blending point. Typically, fifth order Butterworth filters are used with respective corner frequencies around 0.3 Hz.
5.6. Control system

Figure 5.17: Schematic of the top stage control scheme for the horizontal degrees of freedom, showing the sensing matrix $S$ transforming the real LVDT and geophone signals $y$ into virtual sensors $\tilde{y}$. Sensor correction is performed on the $x$ and $z$ LVDT measurements before blending of the geophone and LVDT signals. Via the single-input single-output (SISO) compensator filter $C(s)$ (discussed in Section 5.7.1) the virtual feedback actuation signals $\tilde{u}$ are calculated before being transformed back to the real actuators coordinates $u$ via the driving matrix $D$. Analog hardware such as geophone preamplifiers, LVDT demodulators and voice coil drivers have been omitted for clarity.

Geophone calibration

The geophones have a frequency dependent sensitivity response that is unique for each sensor. It is given by Eq. (5.8) and is a function of a geophone’s properties such as natural frequency, coil resistance and damping ratio. Often it suffices to use the values provided by the manufacturer but a cross check with a Trillium 240 is always performed and where necessary used to refine the calibration. An additional $1/s$ is included in the geophone digital filters to apply a velocity to displacement transformation.

5.6.2 Actuator diagonalization

Obtaining a suitable driving matrix is an iterative process that can take several steps before arriving at a satisfactory result. The method described below aims to obtain a driving matrix such that when one degree of freedom is excited, only movement in that
degree of freedom will be detected. At DC the horizontal actuators apply forces only in the horizontal direction and the vertical actuator only in the vertical direction. Therefore the resulting matrices will have dimension 3×3. For example, the desired horizontal motions (in μm or μrad) \( \ddot{u}_x, \ddot{u}_z \) and \( \ddot{u}_{\theta_y} \) will translate to real actuator signals through

\[
\begin{bmatrix}
\text{Act0} \\
\text{Act1} \\
\text{Act2}
\end{bmatrix}
= \begin{bmatrix}
V_{\text{Act0},x} & V_{\text{Act0},z} & V_{\text{Act0},\theta_y} \\
V_{\text{Act1},x} & V_{\text{Act1},z} & V_{\text{Act1},\theta_y} \\
V_{\text{Act2},x} & V_{\text{Act2},z} & V_{\text{Act2},\theta_y}
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_x \\
\ddot{u}_z \\
\ddot{u}_{\theta_y}
\end{bmatrix},
\] (5.26)

where the ActX values give the voltages that need to be sent to the actuators and the elements of matrix \( \mathbf{D} \) have units \( \text{V/μm} \) or \( \text{V/μrad} \). Matrix \( \mathbf{D} \) is determined according to the following procedure:

1. A first guess for \( \mathbf{D} \) is a 3×3 unity matrix. This is a poor guess as couplings are expected for each actuator with all degrees of freedom, but it is the starting point of the diagonalization procedure.

2. Each virtual actuator is separately driven by a sine wave with a frequency \( f_D \) well below the resonance frequencies, typically around 30 mHz. Here one can assume the system to be stiff and the movement of the top stage follows the actuators with zero phase. Note that during the first iteration virtual and real actuators are equivalent because \( \mathbf{D} = I \). The magnitude of the transmission between measured and driven signal at \( f_D \) is calculated and populates matrix \( \mathbf{E} \) as shown in Eq. (5.27). Each column is obtained separately for each virtual actuator.

\[
\mathbf{E} = \begin{bmatrix}
|x/\ddot{u}_x|_{f_D} & |x/\ddot{u}_z|_{f_D} & |x/\ddot{u}_{\theta_y}|_{f_D} \\
|z/\ddot{u}_x|_{f_D} & |z/\ddot{u}_z|_{f_D} & |z/\ddot{u}_{\theta_y}|_{f_D} \\
|\theta_y/\ddot{u}_x|_{f_D} & |\theta_y/\ddot{u}_z|_{f_D} & |\theta_y/\ddot{u}_{\theta_y}|_{f_D}
\end{bmatrix},
\] (5.27)

3. The driving matrix is updated by multiplying it with the inverse of \( \mathbf{E} \): \( \mathbf{D}_{\text{new}} = \mathbf{E}^{-1}\mathbf{D} \). If the system was perfectly diagonalized then \( \mathbf{E} \) would be the identity matrix (the excitation \( \ddot{u}_x \) would equal the measured response \( x \)). However, for a poorly diagonalized system \( \mathbf{E} \) will have non-diagonal components and its inverse provides a correction factor to the previous \( \mathbf{D} \).

4. Step 2 is repeated to verify the new driving matrix. Typically, after the first trial there are still around 20 percent admixtures in coupled coordinates and steps 2 and 3 are repeated two or three times to obtain a sufficiently diagonalized system.

To test the effectiveness of the diagonalization, broadband white noise is injected in one degree of freedom and the transfer function between the measured and injected noise is determined. A perfect diagonalization would result in zero transfer from the injected
noise to other degrees of freedom. Results from such a test for white noise from DC to 8 Hz, injected in the $x$ and $\theta_y$ direction are shown in Fig. 5.18. At 30 mHz a decoupling of 10% or less is observed. However, at resonance frequencies the coupling becomes much stronger but never exceeds the on-diagonal coupling. Although not desirable, these off-diagonal couplings do not present a problem for the active control of MultiSAS.

### 5.6.3 Vertical control and the state observer

A state observer can be a useful tool in the development of control strategies as it allows states of a system to be estimated based on measurements from different types of sensors, monitoring a limited number of states. The use of observer based control in the context of seismic attenuation for gravitational wave detectors has been proposed before, for different types of suspension systems [137–139]. Here the vertical control of MultiSAS is approached from a state observation point of view.

**Motivation**

MultiSAS foresees the use of an LVDT vertical displacement sensor on the top filter and a geophone on the bench monitoring vertical motion. There is a strong case for the use of both types of sensors in the control of MultiSAS. The LVDT has an excellent low frequency performance with a resolution of a few nm/$\sqrt{\text{Hz}}$ across a broad frequency range. It provides error signals for the DC and low frequency (< 0.1 Hz) control. Geophones...
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on the other hand, typically loose sensitivity below 0.1 Hz, yet have superior performance in comparison to the LVDTs at frequencies above roughly 0.5 Hz. In addition, being inertial sensors they are not limited by noise from a reference frame and will not re-inject seismic noise. A standard technique in a situation where LVDT and geophone measurements of the same object are available is sensor blending, as described earlier. In the MultiSAS vertical motion case at hand, the LVDT registers displacements between the top and intermediate stages, while the geophone monitors the bench motion, the dynamics between the two are governed by Eq. (5.14). Standard sensor blending techniques can therefore not be implemented. State observer can however be generated based on Eq. (5.14) with both LVDT and geophone measurements as inputs.

Another crucial element of MultiSAS control is the fact that all of the feedback (with the exception of the rotational modes of the bench) is performed on the top stage. This has the advantage of allowing the mechanics of the system itself to be used to filter out any noise introduced by the sensors. For example, the geophone directly monitors the payload but the force is applied to the intermediate stage; therefore, any injected geophone noise is filtered by the intermediate GAS.

Kalman filter with non-Gaussian noise

The theoretical background for state estimation with a Kalman filter was introduced in Section 1.4.2. It was shown that an optimal estimator could be constructed given a plant state space model and known input and measurement noises. It was assumed that the noise terms have zero mean, are Gaussian distributed and mutually uncorrelated. In reality it is often the case, as it is with MultiSAS, that the relevant noise sources are not Gaussian distributed. Here a method is presented to extend the state-space model to include the non-Gaussian properties of the disturbance and measurement noises. The method utilizes shaping filters that produce colored noise from a white noise input. The extended state space model can then be manipulated with the usual tools, to generate a Kalman state observer $K_{est}$ and subsequent feedback controllers [34, 43, 140, 141]. A schematic of the MultiSAS vertical state observer is shown in Fig. 5.19.

The measurement noise of the LVDT and geophone are readily determined. By locking the LVDT firmly in its zero position while recording the demodulated output, its noise floor could be measured. The LVDT noise is flat with a low frequency $1/f$ characteristic (due to electronic noise) with a corner frequency at 0.03 Hz. A single order shaping filter $W_L$ is sufficient to model the LVDT noise and is described by a state space system with states $x_L$, zero mean flat input noise $n_1$ and output $w_L$, such that

\[
\dot{x}_L = A_L x_L + B_L n_1, \\
w_L = C_L x_L + D_L n_1. 
\]  

(5.28)

At frequencies below a few hundred Hz, the geophones are limited by the electronic noise of their preamplifier [97]. Preamplifier noise can be measured by shorting the input with
Figure 5.19: Schematic of the MultiSAS vertical state observer $K_{\text{est}}$. The MultiSAS dynamics are described by the plant $P$. The shaping filters $W_L$, $W_g$ and $W_d$ account for the colored frequency response of the LVDT, geophone and disturbance noises respectively. The feedback control signal is denoted by $u_y$ and the displacements measured by the LVDT and geophone by $y_{\text{LVDT}}$ and $y_{\text{geo}}$ respectively.

A resistor of the same resistance as the geophone coil. The resulting geophone noise can be described by a second order shaping filter $W_g$ represented by $[A_g, B_g, C_g, D_g]$ with states $x_g$, zero mean flat input noise $n_2$ and output $w_g$. The measured and modeled LVDT and geophone noises are plotted in Fig. 5.20.

The disturbance noise $w_d$ is a term that enters into the input of the plant $u$. In the case of MultiSAS seismic motion of the top stage acts on the system by applying additional forces to the intermediate mass. Neglecting other input disturbances such as DAC noise or voice coil non-linearities we can model $u_d$ by the measured seismic noise in the laboratory. The seismic noise will be modeled by a sixth order shaping filter $W_d$ with a corresponding state space representation given by $[A_d, B_d, C_d, D_d]$ with states $x_d$, zero mean white input noise $d$ and output $u_d$. The disturbance noise shaping filter and measured seismic noise are plotted in Fig. 5.20.

It is clear that, based on these noise properties, an ideal combination of the LVDT and geophone data would require the blending of the two signal with a cross over frequency around 0.3 Hz. Note that the geophone noise tapers down at low frequencies. This is not a property of the geophone sensitivity but simply the effect of the highpass filters used to eliminate large offsets as a result of dividing out the geophone sensitivity which approaches zero as $\omega$ does. The disturbance noise shown in Fig. 5.20 is seen to be larger than the best respective sensor.

The state space equations governing the dynamics of the MultiSAS vertical stages given by Eq (5.14), can now be extended to include the shaping filters [140]. They are then
Figure 5.20: The modeled shaping filters for the measurement and disturbance noises. The curves show their frequency response to a zero mean white noise input. The respective measured spectra are also plotted.

given by

\[
\begin{bmatrix}
\dot{x}_p \\
\dot{x}_d \\
\dot{x}_L \\
\dot{x}_g
\end{bmatrix} =
\begin{bmatrix}
A_p & B_p C_d & 0 & 0 \\
0 & A_d & 0 & 0 \\
0 & 0 & A_L & 0 \\
0 & 0 & 0 & A_g
\end{bmatrix}
\begin{bmatrix}
x_p \\
x_d \\
x_L \\
x_g
\end{bmatrix} +
\begin{bmatrix}
B_p \\
0 \\
0 \\
B_g
\end{bmatrix} u +
\begin{bmatrix}
B_p D_d & 0 & 0 & 0 \\
0 & B_d & 0 & 0 \\
0 & 0 & B_L & 0 \\
0 & 0 & 0 & B_g
\end{bmatrix}
\begin{bmatrix}
d \\
n_1 \\
n_2
\end{bmatrix},
\]

(5.29)

and

\[
\begin{bmatrix}
y_{lvd} \\
y_{geo}
\end{bmatrix} =
\begin{bmatrix}
C_{p1} & 0 & C_L & 0 \\
C_{p2} & 0 & 0 & C_g
\end{bmatrix}
\begin{bmatrix}
x_p \\
x_d \\
x_L \\
x_g
\end{bmatrix} +
\begin{bmatrix}
0 & D_L & 0 & 0 \\
0 & 0 & D_L & 0 \\
0 & 0 & 0 & D_g
\end{bmatrix}
\begin{bmatrix}
d \\
n_1 \\
n_2
\end{bmatrix},
\]

(5.30)

where \( C_{p1} \) and \( C_{p2} \) are the rows of \( C_p \) corresponding to the \( y_{lvd} \) and \( y_{geo} \) outputs respectively. The symbol \( \emptyset \) denotes zero matrices of the relevant sizes.

An optimal approach to MultiSAS control with multiple sensors is to perform sensor blending via a state observer to obtain optimized estimates of the state variables associated with Eq. (5.29). These estimates can subsequently be used in the feedback control
To be sure the system under consideration is in fact observable and controllable, a verification of the observability and controllability criteria can be performed on the state-space model. This corresponds to determining the rank of the respective observability and controllability matrices defined by Eqs. (1.46) and (1.47). The rank must equal the order of the system and in the case of observability implies that each state of the system can be inferred from the outputs. The dual concept of controllability implies that each state can be moved from an initial to a final position by the system’s inputs. The vertical MultiSAS model given by \([A_p, B_p, C_p, D_p = 0]\) is reassuringly found to be observable and controllable.

The optimal estimate of state \(x\) denoted by \(\hat{x}\) will be obtained here with a Kalman filter. The state equations of Eqs. (5.29) and (5.30) can be rewritten as

\[
\dot{x} = Ax + B_u u + B_n n_p,
\]

where \(n_p = [d n_1 n_2]^T\) is defined to be the process noise and \(n_m\) is the measurement noise defined by \(n_m = D_n n_p = [D_L n_1 D_g n_2]^T\). Of importance for the Kalman filter equations are the process and measurement noise covariance matrices \(Q_n, R_n\) and \(N_n\) defined as

\[
Q_n \equiv E(n_p n_p^T) = \mathbb{I}_{3\times3},
\]

\[
R_n \equiv E(n_m n_m^T) = \begin{bmatrix} D_L^2 & 0 \\ 0 & D_g^2 \end{bmatrix},
\]

\[
N_n \equiv E(n_p n_m^T) = \begin{bmatrix} 0 & 0 \\ D_L & 0 \\ 0 & D_g \end{bmatrix}.
\]

A steady state Kalman filter can now be constructed. The Kalman observer gain matrix is given by \(L = (PC^T + B_n N_n)R^{-1}\), where the covariance matrix \(P\) is derived by solving the algebraic Riccati equation

\[
[A - B_n N_n R_n^{-1} C]P + P[A - B_n N_n R_n^{-1} C]^T - PC^T R_n^{-1} CP + B_n [Q - N_n R_n^{-1} N_n^T] B_n^T = 0.
\]

The resulting observer equations are given by

\[
\dot{\hat{x}} = A\hat{x} + B_u u + L(y - \hat{y}),
\]

\[
\hat{y} = C\hat{x}.
\]

The state observer relates measurements \(y\) and input \(u\) to estimates of the states \(\hat{x}\) and outputs \(\hat{y}\) in an optimal way based on the respective noise levels of the measurements and input.
Of interest to the MultiSAS vertical control issues is the state observer for \( y_1 \): the displacement of the intermediate stage. This is also the point where a control force will be applied via the top stage vertical voice coil actuator. The frequency response of the Kalman filters for the estimate of \( y_1 \) is shown in Fig. 5.21. At low frequencies the \( y_1 \) estimate is based solely on the LVDT signal. This is expected due to the superior sensitivity of the LVDT with respect to the geophone at these frequencies. Towards increasing frequency the geophone signal is blended with that of the LVDT and from 0.5 Hz starts to dominate the \( y_1 \) estimate. This corresponds to the cross over in sensor noise as seen in Fig. 5.20. The form of the geophone filter reflects the dynamics of the plant by gaining magnitude towards higher frequencies where the bench motion is attenuated by the system’s mechanics. This drops off again above 4 Hz as the signal to noise ratio in the geophone becomes too small. The estimate of \( y_1 \) based on the input \( u \) is largely neglected due to the high level of disturbance noise introduced by seismic motion.

![Figure 5.21: Frequency response of the Kalman filters for the \( y_1 \) estimator. The LVDT signal contributes the most at low frequencies while the geophone contribution dominates above 0.5 Hz. The contribution from input \( u \) is small due to the high level of disturbance noise.](image)

The Kalman filter’s ability to observe all states is demonstrated with time domain data in Fig. 5.22. MultiSAS was excited vertically by a sinusoidal input \( u_y \) with an amplitude of 10 \( \mu \)m and a frequency of 0.1 Hz. The (sensor corrected) LVDT, geophone and input signals were collected and the estimates \( \hat{y} \) and \( \hat{x} \) were calculated offline. The output estimates \( \hat{y} \) will include the dynamics of the measurement noise shaping filters, as these
are included in the measurement matrix $C$ of Eq. (5.30), while the estimates $\hat{x}$ are state observations governed by the system and disturbance noise dynamics only. The top panel of Fig. 5.22 plots the LVDT signal, the estimated LVDT output and the estimate of state $y_1$. The system takes some time to respond to the input signal but after roughly 30 seconds the sinusoidal excitation is clearly visible. The estimated output and state are very similar because the LVDT noise is not significant at these levels of displacement. The difference between the measured LVDT and its estimate is the contribution, largely at higher frequencies, from the geophone. The low frequency components, in particular the 0.1 Hz excitation response, are contributed mainly by the measured LVDT signal. The two central panels present data from the estimates of the unmeasured states, $v_1$ and $y_2$. Finally, the measured and estimated geophone signals and corresponding estimate of state $v_2$ are shown in the bottom panel of Fig. 5.22. The measured and estimated outputs are very similar owing to the filter’s belief that the LVDT will not be able to contribute to the estimated output: at low frequencies the LVDT may have better sensitivity but the output will be dominated by sensor noise from the geophone anyway. In the state estimate, on the other hand, the filter rejects low frequency geophone signal in favor of (low frequency) LVDT measurements made at the top filter.

5.7 Control performance

The focus here will be on the control of the vertical degrees of freedom presenting in-loop results of a multiple-input single-output (MISO) controller based on a state observer. This is the first time that such results have been presented in the context of seismic attenuation for Virgo. For the purpose of the vertical controls it was useful to damp the horizontal degrees of freedom of the top stage as well, in order to reduce couplings between the various degrees of freedom. These results will also briefly be discussed.

5.7.1 Top stage horizontal control

The horizontal control was performed with inertial damping. This is a simple form of derivative control in which the feedback signal is proportional to the velocity of the isolated object. Each degree of freedom is sensed and controlled separately; hence, the control loops are considered to be single-input single-output (SISO) regulators. The feedback signals are band-passed to avoid injection of control noise at high frequencies ($> 10$ Hz) where the top stage is passively isolated, and at low frequencies ($< 0.01$ Hz) where dynamic control is not needed. The resulting feedback controller is given by

$$C(s) = sG \cdot H_L(s) \cdot H_H(s)$$

$$= sG \cdot \frac{2s^2}{s^2 + 2sLs + 2s_L^2} \cdot \frac{s^2}{s^2 + 2s_H + 2s_H^2},$$

(5.34)
where \( H_L(s) \) is a second order lowpass filter defined by a corner frequency \( s_L \) and \( H_H(s) \) a highpass filter with corresponding corner frequency \( s_H \). The additional \( s \) in Eq. (5.34) transforms the displacement measurements into a velocity signal and \( G \) represents the feedback gain. The corner frequencies of the high and lowpass filters should be chosen well below and above the to be damped resonances respectively. For the MultiSAS top stage horizontal control \( s_H = 0.05 \text{ rad/s} \) and \( s_L = 20 \text{ rad/s} \) were chosen, which correspond to 0.008 Hz and 3.2 Hz respectively. A schematic outline of the top stage horizontal control is given in Fig. 5.17 where the SISO regulators are given by Eq. (5.34).
The controller gain $G$ could be selected separately for each degree of freedom. For $\theta_y$ it was set to 1, and for $x$ and $z$ to 10. The lower frequency resonances of $x$ and $z$ required a larger gain where $sG$ would otherwise be less than unity. Results of top stage horizontal control performance is shown in Fig. 5.23 for the $x$ and $\theta_y$ degrees of freedom. These data are observed by the blended super sensors which are the same sensors used in the feedback control. It should be noted that these are hence in-loop sensor results.

It is evident that the resonance frequencies are damped by at least an order of magnitude. The ground motion of the $x$ direction as observed by the Trillium 240 seismometer is included in Fig. 5.23a and used to calculate the transfer functions shown in Fig. 5.23b. The pre-isolation performance of the top stage is observed and amounts to -40 dB attenuation at 3 Hz. Above 3 Hz the top stage motion is dominated by acoustic noise, an issue often associated with out of vacuum passive attenuation systems [120]. These effects are expected to disappear once the system is installed in the vacuum enclosure. The peak at $\sim$0.5 Hz is a coupling of the poorly suspended dummy mass tilt motion recoiling to the top stage. This is also expected to diminish once the actual optical bench is installed. However, as demonstrated here, this mode can be effectively suppressed by the horizontal controls of the top stage.

### 5.7.2 Vertical proportional integral derivative control

The first step in the control of the MultiSAS vertical degrees of freedom is the design of a proportional integral derivative (PID) controller. Here the sensor corrected vertical top stage LVDT signal is used in the feedback loop and the actuation signal applied to the co-located vertical voice coil actuator. Both are situated under the keystone of the top filter. This is the traditional approach to resonance frequency damping and will function as a performance base line against which to later compare the MISO control. The vertical PID controller can be described by

$$C(s) = G \left( G_D s + G_P + \frac{G_I}{s} \right) \cdot H_L(s) \cdot H_H(s),$$  \hspace{1cm} (5.35)

where $G_D$, $G_P$ and $G_I$ are the derivative, proportional and integral gains respectively with $G$ representing an overall gain. The feedback signals are again high- and lowpass filtered by $H_H(s)$ and $H_L(s)$ from Eq. (5.34). For the MultiSAS vertical PID controller the values $G_D = 1$, $G_P = 0.5$ and $G_I = 0.05$ were chosen. At low frequencies ($s < 0.05$ rad/s) the integration term dominates and produces a feedback signal proportional to the integrated value of the vertical position. As the frequency increases the proportional term takes over applying a force proportional to the measured displacement. This is useful for keeping the vertical position around zero (or an arbitrary set position) at frequencies below the resonance. At increasing frequencies ($s > 0.5$ rad/s) the derivative term begins to dominate and produces a feedback signal proportional to the vertical velocity, essentially implementing inertial control, a technique effective in damping resonance frequencies.
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(a) Left: open and closed loop horizontal motion of the top stage. The ground motion as measured by the Trillium 240 is also included. Right: open and closed loop rotational (yaw) motion of the top stage.

(b) Ground to top stage transfer function in open and closed loop operation. Horizontal top stage motion $x$ over ground motion as observed by the Trillium 240. Roughly 40 dB pre-isolation is gained at 3 Hz. Above 3 Hz top stage motion is dominated by acoustic noise.

Figure 5.23: Top stage horizontal control results based on inertial damping.

The effectiveness of the control scheme can be tested by injecting white noise at the input, i.e. at the voice coil, while at the same time implementing the feedback control and monitoring the system response. In this way, the signal levels are increased above any sensor noise floors. In addition, the results can be plotted as forced displacement
transfer functions. Fig. 5.26a shows the forced displacement transfer function results of the vertical PID controller for various values of the overall controller gain. On the left the transmission of input noise to LVDT displacement is shown. On the right the same transmission to the bench velocity measured by the geophone is presented. We see that the control is successful in damping the resonance frequencies as seen at both the top stage LVDT and the bench geophone. However, due to a notch in the top stage response, only a small signal is measured by the LVDT and hence a weak feedback force is applied around WTT moz. As a result, the bench motion is left largely intact at these frequencies. This limits the effectiveness of the PID in reducing the rms motion of the payload. A certain amount of noise injection is evident above 4 Hz. Increasing the gain further aggravates this issue.

5.7.3 Vertical linear quadratic gaussian control

Linear quadratic Gaussian (LQG) control combines the linear quadratic regulator (LQR) outlined in Section 1.4.3 with the Kalman state observer discussed in Section 5.6.3 [34, 139, 141]. The term Gaussian comes from the fact that the Kalman observer assumes Gaussian input noise. The LQR assumes that all the states of the system are known and produces a gain matrix $K$ that optimizes a quadratic cost function (see Eq. (1.56)). However, in real control problems all of the states are not always easily accessible. The Kalman observer provides an optimal means to estimate all the states based on the measurements that are available, and on knowledge of the system dynamics and noise sources. Finally, the separation principle makes it possible to compose an optimal LQG controller by first determining the LQR and Kalman observer separately before combining them. LQG control for MultiSAS utilizes multiple measurement inputs to produce a single feedback signal. It is therefore considered to be a MISO regulator. A schematic of an LQG controlled system is shown in Fig. 5.25.

An LQG regulator can be tuned by adjusting the weighting matrices $Q_{LQR}$ and $R_{LQR}$ of the LQR, and $Q_n$ and $R_n$ of the Kalman filter. Generally, one of the matrices in each pair is set to unity and the other scaled accordingly. In the case of the MultiSAS LQR, $R_{LQR} = 1$ and $Q_{LQR} = \text{Diag}[Q, 0, 0, Q]$, where $Q$ can be tuned and weighs the $y_1$ and $v_2$ state estimates evenly. Increasing $Q$ places more emphasis on minimizing the system states, rather than the input signal. This has a similar effect to increasing the overall gain of the controller.

The Kalman filter weighting matrices are given in Eqns. (5.31) - (5.32). To facilitate the ability to tune the disturbance noise with respect to the measurement noise the process covariance matrix is redefined here as

$$Q_n = \begin{bmatrix} Q_d & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (5.36)$$

where $Q_d$ is the variance of the white noise input to the disturbance noise shaping filter.
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(a) Vertical transfer functions in open and closed loop to test the performance of the PID controller. The gain is varied from 0 (open loop) to 2. The left panel shows the LVDT displacement over forced displacement results. Right panel the geophone velocity over forced displacement results.

(b) Vertical transfer functions in open and closed loop to test the performance of the LQG controller. The parameters $Q$ and $Q_d$ are varied. The left panel shows the LVDT displacement over forced displacement results. Right panel the geophone velocity over forced displacement results.

Figure 5.24: MultiSAS vertical forced displacement transfer functions in open and closed loop configurations.
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![Figure 5.25: Schematic of the MultiSAS vertical control scheme with a MISO regulator consisting of a state observer $K_{est}$ and LQR gain matrix $K$. The MultiSAS dynamics are described by the plant $P$. The shaping filters $W_L$, $W_g$, and $W_d$ account for the colored frequency response of the LVDT, geophone and disturbance noises respectively.]

By increasing $Q_d$ with respect to $R_n$ and the other diagonal elements of $Q_n$, the Kalman filter places a stronger belief in the measurements, rather than the system model.

Forced displacement transfer functions with LQG control are shown in Fig. 5.26b for various values of $Q$ and $Q_d$. The LQG regulator is successfully able to damp the vertical resonances. Examining the bench motion as observed by the geophone, we see a significant improvement over the PID control. Because the LQG controller is designed to minimize both LVDT and geophone signal, the bench motion is actively suppressed. However, there is evidence of additional noise injected in the frequency range from 1 to 5 Hz. This effect is somewhat reduced by increasing $Q_d$. As $Q_d$ becomes large the injected noise decreases, suggesting that a stronger belief in the measurement (with respect to the model) reduces injected noise. Forced displacement measurements however, increase signal levels well above their noise floors making an increased $Q_d$ a plausible solution. Under normal conditions, this may not be the case.

For the controlled MultiSAS to meet its requirements, the control system must suppress rms motion without interfering with the passive isolation performance above 10 Hz. Therefore, the injection of noise in the 1 - 5 Hz range is not troublesome. In fact, the improved reduction of bench rms motion will be highly advantageous.

5.7.4 Comparative results

The performance of the control system was tested with MultiSAS in its free running state with only environmental disturbances i.e. without forced excitation. These results
are shown in Fig. 5.26. In the top panel the LVDT displacements are shown for the open and closed loop configurations alongside the ground motion as measured by the Trillium 240 seismometer. In the bottom panel the projection of bench displacement from the geophone is presented. In addition to the spectral density the rms motion of the bench is also plotted (black curves). The resonance peaks are clearly visible in the open loop results (solid red curves). These are effectively damped by both the PID (dotted blue curve) and LQG (dash-dotted green curve) control schemes. In both the LVDT and geophone results we see the injection of control noise by the LQG controller above 1 Hz. However in the critical range above 10 Hz, no injection could be identified. In general the LQG control method outperforms the PID control below 1 Hz. This is most evident in the LVDT measurements below 0.1 Hz. However, a more aggressive PID proportional and integral gain may be able to improve the PID results.

A critical measure for control performance is the low frequency rms motion. Observing the rms curves on the right of Fig. 5.26 at 0.1 Hz the PID control is seen to reduce rms motion by a factor of 3 in comparison to the open loop system. The LQG controller improves on this by an additional factor of 2, bringing the rms motion down to 0.5 μm. This is within the translational requirements for the optical bench of 1 μm. Geophone results below 0.1 Hz are limited by the instrument’s sensitivity. Measuring above 10 Hz was not feasible with the current setup as acoustic coupling acting directly on the payload dominates its residual motion above a few Hz.

The increase in displacement registered by the LVDT towards low frequencies is a result of thermal effects induced by the air conditioning in the cleanroom. As a result, MultiSAS is subjected to temperature variations of ±0.3 °C over periods of roughly 10 minutes. This is expected to reduce significantly once MultiSAS has been installed in the vacuum system where thermalization of the top stage can occur only by conduction and radiation. The associated time constants are much larger than is the case for out of vacuum convection thermalization.

5.8 Summary

MultiSAS is a vibration isolation system designed to suspend in-vacuum optical benches for Advanced Virgo. Its design is based on passive isolation from multiple stages of mechanical oscillators such as (inverted) pendulums and geometric anti-springs. Active feedback is used for the dynamic control of the system’s resonance frequencies in a bandwidth up to about 5 Hz. The control system is based on input from a series of differential displacement (LVDT) and inertial (geophone) sensors and the actuation forces are applied by magnetic voice coil actuators.

It was shown that the system could be effectively modeled with Lagrangian mechanics. This was done by assuming uncoupled dynamics between horizontal, vertical and yaw motion and developing separate models for each. The vertical and horizontal models
5.8. Summary

(a) The LVDT displacement signal and the vertical ground displacement measured by the T240.

(b) The bench displacement projected from the geophone signal and the equivalent rms motion. At frequencies below 0.1 Hz the signal is dominated by measurement noise and above 2 Hz by acoustic noise coupling to the suspended mass.

Figure 5.26: MultiSAS vertical control results with environmental disturbances only. Closed loop performance plotted for the PID controller with a gain $G = 2$ and the LQG controller designed with $Q = 100$ and $Q_d = 10$. 
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were presented here and were shown to coincide with measured transfer functions.
The characterization campaign of the vertical performance of MultiSAS was described showing measured transfer functions of the GAS filters. Magic wands on the intermediate filter were shown to improve high frequency performance by an additional factor of 10. The total projected transmission of vertical ground vibrations to the optical bench is expected to be $10^{-6}$ at 30 Hz, well within the spectral translation requirements. The horizontal and rotational characterization tests could not be presented here but are currently underway.

MultiSAS implements a feedback control system to damp the resonance frequencies. Conventional techniques were effectively implemented to control the horizontal motion of the top stage, and vertical motion of the suspension chain. A novel approach to the vertical control problem, by using Kalman filters and linear quadratic regulators, was addressed in more detail. This approach proved effective in damping resonance frequencies and outperformed conventional techniques in reducing low frequency rms motion. More attention can now be paid to optimizing both conventional techniques and the Kalman filter approach.

The following steps for the MultiSAS prototype will be the installation of the system inside the MiniTower vacuum chamber. The currently suspended dummy payload will be replaced by an optical bench prototype with its dedicated sensing and actuation systems. Then the performance of the entire system can be fully tested and characterized. The attenuation performance above 10 Hz is expected to reduce residual motion of the bench well below the sensitivity of conventional sensors so special techniques will be adapted. Higher order modes have only briefly been addressed in this chapter but their dynamics may spoil performance above 10 Hz so they will need to be carefully investigated. The control system plays an important role in achieving low frequency rms requirements. The control schemes presented here may be improved upon, and extended to include all the controllable degrees of freedom.