English Summary

Coordination control of linear systems

Coordinated linear systems are decentralized linear systems with a specific structure: They consist of three or more subsystems, one of which plays the role of a coordinator for the other subsystems. The different roles of the subsystems are reflected by the information structure: The coordinator may influence the other subsystems, while the other subsystems may not influence the coordinator or each other. A coordinated linear system represents one branch in a hierarchical linear system, with a directed tree as information structure.

Possible applications of coordinated linear systems include decentralized systems with an inherent hierarchical structure, such as traffic networks or power networks, but also other decentralized systems which allow for a hierarchical modeling approach, such as groups or formations of autonomous vehicles. Moreover, unstructured linear systems can often be decomposed into several subsystems with a hierarchical structure in order to reduce the complexity of the corresponding control problem.

The research described in this thesis focuses on the following questions:

(1) How can unstructured linear systems, or structured systems with a non-hierarchical information structure, be decomposed or transformed into coordinated linear systems? Is a given coordinated linear system ‘as decentralized as possible’, or is a further decomposition within the hierarchical structure possible?

(2) Which part of the system is controllable by using which input – is a local controller sufficient, or is coordination required to achieve the control objective? Are all measurements which are needed for the implementation of the control law locally observable, or is communication of these measurements necessary?

(3) Given a coordinated linear system, can we find a control law which achieves the control objective but also respects the hierarchical information structure? How does the performance of such a control law compare to unstructured control laws? Can the performance be improved by allowing event-based feedback from the subsystems to the coordinator?

In Chapters 1 and 2 the topic of coordinated linear systems is motivated and introduced, and relevant concepts and results from the classical theory of systems and control are summarized. In Chapter 3 the definition and some basic properties of coordinated linear systems are described and compared to related classes of decentralized systems.
Summary

Question (1) is discussed in Chapter 4: Explicit procedures for the transformation of an unstructured or non-hierarchical linear system into a coordinated linear system are given. Possible definitions of minimality of a coordinated linear system are introduced in order to formalize the concept of a system which is ‘as decentralized as possible’.

The refinement of the classical concepts of controllability and observability with respect to the different subsystem states, inputs and outputs, relates to question (2) and is described in Chapter 5. We distinguish between independently and jointly controllable subspaces to identify both the intentional and unintentional effects of the coordinator on the subsystems. Decompositions of the subsystem state spaces according to these refined concepts give insight into the question which parts of each system are locally controllable or observable, and where coordination or communication is necessary.

In the generality of its formulation, question (3) is easier posed than answered: A mathematical analysis of the closed-loop performance is only possible for particular control problems. Chapters 6 and 7 deal with the restriction of question (3) to LQ (linear-quadratic) control problems: In Chapter 6 we construct a linear feedback for a coordinated linear system which minimizes a quadratic cost function but also respects the information structure. The extension of the set of admissible control laws to linear feedbacks which may also make use of piecewise-constant approximations of the states of other subsystems is discussed in Chapter 7. A consequence of the hierarchical structure of the system is that the local control problems for the subsystems can be solved independently of each other; only the control problem for the coordinator depends on the rest of the system.

In order to illustrate the theory developed in this thesis, two case studies are worked out in Chapter 8: One case study concerns a formation of three autonomous underwater vehicles – due to the costs and limitations of underwater communication it is beneficial to give one vehicle the role of the coordinator, who regularly sends his position to the other two vehicles. The formation problem is an example of the LQ problem discussed in Chapter 6. To improve robustness with respect to disturbances and communication problems, the other two vehicles may send feedback to the coordinator if they are unable to follow the reference signal.

The other case study concerns the coordination of ramp metering devices at neighboring on-ramps of a highway: If the overall demand exceeds the capacity of the highway, ramp metering leads to queues at the on-ramps. If the metering devices are only controlled locally then the waiting times at neighboring on-ramps can be very different – an effect that can be avoided using coordination.