In electroweak interactions, an up-type quark $u$ can change its flavour to a down-type quark $d$ via the emission of a charged $W$ boson: $u \rightarrow W^+ d$. In the Standard Model (SM) the couplings of these interactions are described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Flavour physics is the field of particle physics that studies these flavour changing interactions. $B$ mesons contain one $b$ or $\bar{b}$ quark and because of their relatively high mass and long lifetime, $B$ meson decays are particularly well suited to study flavour changing interactions and the structure of the CKM mechanism. Some $B$ meson decays are observed to be not invariant under the combined operation of the charge (C) and parity (P) operators, i.e. they are CP violating. One of the main subjects of this thesis is the search for CP violation in a specific type of $B^0_s$ meson decays, namely the decay of a $B^0_s$ meson into a $J/\psi$ meson and a $\phi$ meson, also indicated as $B^0_s \rightarrow J/\psi \phi$ decays. In the SM, CP violation is expected to be tiny in this decay channel, implying that a positive signal indicates the presence of physics beyond the SM. In this chapter, a general introduction to CP violation is given and the physics of neutral $B$ meson decays is discussed.

1.1 History of CP Violation

CP symmetry is the symmetry of the combined operations of P and C, where the charge operator C inverts the charge of particles, and P is the parity operator which inverts all spatial coordinates. Processes that are not invariant under the CP operation are called CP-violating processes. In 1956, Chien-Shiung Wu performed a historical experiment utilizing the decay of $^{60}$Co, which demonstrated that P symmetry was violated in the weak interactions [1]. Contrary to expectations, she found that neutrinos occur in one single helicity state only, implying maximal P violation.

Experiments with pion decays [2] revealed that the C symmetry is also maximally violated by the weak interactions. However, no evidence was found that the combined symmetry CP is violated. Indeed, CP seems to be preserved in strong and electromagnetic interactions [3]. But in 1964, James Cronin and Val Fitch performed an experiment using neutral kaon decays [4], in which they observed the (until that time assumed to be) CP-odd $K_L$ particles
decaying to CP-even $\pi^+\pi^-$ final states with a branching fraction of about 0.2%. In other words, they observed that CP is violated in kaon oscillations, but in contrast to C and P violation, the magnitude of the CP violation is small. Cronin and Fitch received the Nobel Prize for their experiment in 1980.

In 1973, Makoto Kobayashi and Toshihide Maskawa postulated that a hypothetical additional third quark family could explain the observed small amount of CP violation through an imaginary phase in their proposed Cabibbo-Kobayashi-Maskawa (CKM) matrix [5, 6]. At that time, only three quarks were known to exist and a fourth quark had been postulated to complete the second quark family. This fourth quark was discovered in November 1974 [7, 8]. The bottom quark, the lightest member of the third quark family and predicted by Kobayashi and Maskawa, was discovered in 1977 at Fermilab [9], for which they were awarded the 2008 Nobel Price, sharing the price with Yoichiro Nambu. Finally, the top quark was discovered in 1995, completing the third quark family [10, 11].

CP violation outside of the kaon system was discovered in the first decade of this century by the so-called $B$-factories: the BaBar and Belle experiments. Both these experiments found CP violation in the $B^0$ meson system [12, 13]. The search for CP violation continued at the Tevatron at Fermilab, where the D0 experiment found evidence of CP violation in the mixing of $B$ mesons [14]. The Tevatron was shut down in 2011 and currently the LHCb experiment at CERN plays a leading role in the search for CP violation. In 2012, LHCb reported the first evidence of CP violation in charged meson decays [15].

The subject of this thesis is the search for time-dependent CP violation in the interference between mixing and decay in $B^0_s \to \psi J/\psi$ decays at LHCb. But in order to explain this type of CP violation and how to search for it, it is necessary first to explain how CP violation is incorporated in the SM.

### 1.2 CP Violation in the Standard Model

CP violation is incorporated in the SM by the CKM matrix, which appears in the description of the charged current interactions. This matrix arises when diagonalizing the quark mass terms (also known as the Yukawa terms) in the SM Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = M^d_{ij} d^I_i L d^F_J R + M^u_{ij} u^I_i L u^F_J R \ .$$

(1.1)

Here, $(u, d)$ indicates the quark type, $(i, j)$ stands for the quark generation, $(L, R)$ means left-handed or right-handed chirality states and $I$ indicates the interaction basis. To obtain mass terms, the mass matrices are diagonalized via unitary transformations $V_L$ and $V_R$, which yields:

$$\mathcal{L}_{\text{Yukawa}} = \bar{d}^I_i L M^d_{ij} d^F_J R + \bar{u}^I_i L M^u_{ij} u^F_J R$$

$$= \bar{d}^I_i L V^d_{iJ} V^d_{iJ} M^d_{ij} V^d_{iJ} d^F_J R + \bar{u}^I_i L V^u_{iJ} V^u_{iJ} M^u_{ij} V^u_{iJ} u^F_J R$$

$$= \bar{d}^I_i L (M^d_{ij})_{\text{diag}} d^F_J R + \bar{u}^I_i L (M^u_{ij})_{\text{diag}} u^F_J R \ .$$

(1.2)
where in the second equality unitarity is used \((V^\dagger V = 1)\), while the third equality is obtained by defining
\[
\begin{align*}
    d_{iL} &= (V^d_L)_{ij} d^L_{jL}, & d_{iR} &= (V^d_R)_{ij} d^I_{jR}, \\
    u_{iL} &= (V^u_L)_{ij} u^L_{jL}, & u_{iR} &= (V^u_R)_{ij} u^I_{jR}.
\end{align*}
\]
By definition, these quark states represent the mass eigenstates. Rewriting the charged current interaction terms in the Lagrangian leads to
\[
\mathcal{L}_{\text{kinetic}} = \frac{g}{\sqrt{2}} u^I_{iL} \gamma^\mu W^-_\mu d^I_{jL} + \frac{g}{\sqrt{2}} d^I_{iL} \gamma^\mu W^+_\mu u^I_{iL},
\]
where the CKM matrix is given by
\[
(V_{\text{CKM}})_{ij} = (V^u_L V^d_L)^{ij}.
\]
By convention, the interaction and mass eigenstates are chosen to be the same for the up-type quarks, but are rotated for down-type quarks. This implies, that for the down-type quarks they differ by a unitary transformation:
\[
\begin{align*}
    u^I_i &= u_j, \\
    d^I_i &= (V_{\text{CKM}})_{ij} d_j.
\end{align*}
\]
From Eq.\ref{eq:1.4}, it follows that the Standard Model Lagrangian is invariant under CP if, and only if, \(V_{ij} = V^*_{ij}\). Therefore, a non-vanishing complex phase in the CKM matrix can generate CP violation in the SM. Kobayashi and Maskawa realized, that the presence of a third quark family allows for the existence of such an imaginary degree of freedom. This can be seen from the fact that a general complex \(n \times n\) CKM matrix has \(2n^2\) real parameters. The unitarity constraints \(V^\dagger V = 1\) reduce this number of free parameters by \(n^2\). Subsequently, due to the freedom to choose the phase of the \(2n\) quark fields, \(2n - 1\) relative phases are not observable. This leaves \(2n^2 - n^2 - (2n - 1) = (n - 1)^2\) degrees of freedom.

For a general orthogonal \(n \times n\) matrix such as the CKM matrix, there are \(\frac{1}{2}n(n-1)\) independent rotation (or Euler) angles. Therefore, out of the \((n-1)^2\) original degrees of freedom, \(\frac{1}{2}n(n-1)\) are Euler angles and the remaining \(\frac{1}{2}(n-1)(n-2)\) degrees of freedom are independent complex phases. Indeed, for \(n = 3\), as Kobayashi and Maskawa proposed, there is one imaginary degree of freedom in the corresponding \(3 \times 3\) CKM matrix, offering the possibility to describe the observed CP violation.

### 1.3 Unitarity Triangles

Writing out the quark generation index \(i, j = \{u, c, t\}, \{d, s, b\}\), the CKM elements from Eq.\ref{eq:1.6} are written in matrix form as follows:
\[
V_{\text{CKM}} = \begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix}.
\]
Since the CKM matrix is unitary, six orthogonality and three unitarity relations exist. Two of the orthogonality equations are particularly interesting in flavour physics, since the corresponding matrix elements appear in decays involving $B$ mesons:

\[ V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0 \quad (B^0 \text{ system}) \]  
\[ V_{us}V^*_{ub} + V_{cs}V^*_{cb} + V_{ts}V^*_{tb} = 0 \quad (B^0_s \text{ system}) \]  

The orthogonality equations define so-called unitarity triangles in the complex plane. The sides of these unitarity triangles correspond to the various terms in Eq. 1.8 and Eq. 1.9, and their magnitudes are observables that can be related to the (relative) rate of certain $B$ meson decays. In addition, the internal angles of the unitarity triangles are invariant under rephasing of the quark fields and are physical observables. A convenient representation of the two unitarity triangles associated with Eq. 1.8 and Eq. 1.9 is shown in Fig. 1.1, where a phase convention is adopted so that one of their sides is on the real axis, and this side is subsequently normalized to unit length.

The apex of the unitarity triangle for the $B^0_d$ system (UT) is located at $-V_{ud}V^*_{ub}/V_{cd}V^*_{cb}$ and the interior angles are defined as

\[ \alpha \equiv \arg \left[ -\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}} \right], \quad \beta \equiv \arg \left[ -\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}} \right], \quad \gamma \equiv \arg \left[ -\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} \right]. \]  

Similarly, for the unitarity triangle in the $B^0_s$ system (UT$_s$), the apex is located at $-V_{us}V^*_{ub}/V_{cs}V^*_{cb}$ and the interior angles in the UT$_s$ are

\[ \alpha_s \equiv \arg \left[ -\frac{V_{tb}V^*_{ts}}{V_{ub}V^*_{us}} \right], \quad \beta_s \equiv \arg \left[ -\frac{V_{cb}V^*_{cs}}{V_{tb}V^*_{ts}} \right], \quad \gamma_s \equiv \arg \left[ -\frac{V_{ub}V^*_{us}}{V_{cb}V^*_{cs}} \right]. \]  

As will be shown in Sec. 5.7, the angle $\beta_s$ is related to $B^0_s - \bar{B}^0_s$ mixing through the appearance of the CKM element $V_{ts}$ to leading order, and can be measured using $B^0_s \to J/\psi \phi$ decays.

### 1.3.1 Wolfenstein Parameterization

As was mentioned before, for three quark families, the CKM matrix can be parameterized using four real parameters. The Wolfenstein parameterization is inspired by the observation that $|V_{cb}| \gg |V_{ub}|$ and $|V_{cb}| \sim |V_{us}|^2$, and takes the following form [16]:

\[
V_{\text{CKM}} = V_{\text{CKM}}(O(\lambda^3)) + O(\lambda^4) = \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2} \lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4). \]  

The four real parameters are $A, \rho, \eta$ and the expansion parameter $\lambda$, with $\lambda \sim 0.225$.

To obtain the imaginary contribution to the coupling $V_{ts}$, which is important in the
Chapter 1. CP Violation and Physics of $B$ Mesons

$$\begin{align*}
\beta & = \arg(-1) + \arg(V_{td}V_{tb}^*) - \arg(V_{td}V_{tb}^*) = -\arg(V_{td}) \\
\gamma & = \arg(-1) + \arg(V_{ud}V_{ub}^*) - \arg(V_{cd}V_{cb}^*) = \arg(V_{ub}^*) = -\arg(V_{ub}) \\
\beta_s & = \arg(-1) + \arg(V_{cb}V_{cs}^*) - \arg(V_{tb}V_{ts}^*) = \pi - \arg(V_{ts}^*) = \pi + \arg(V_{ts})
\end{align*}$$

Figure 1.1: (a) Unitarity triangle in the $B^0_d$ system, historically known as 'The Unitarity Triangle' (UT). (b) Unitarity triangle in the $B^0_s$ system. In both pictures the sides are scaled so that one of the sides is on the real axis and equal to one. In the SM, the magnitude of the angle $\beta_s$ is small, but in this picture it is shown enlarged for the sake of clarity.

$B^0_s \to J/\psi \phi$ decay, a higher order expansion of the CKM matrix in $\lambda$ is needed:

$$V_{\text{CKM}} = V_{\text{CKM}}(O(\lambda^3)) + \begin{pmatrix}
-\frac{1}{8}\lambda^4 & 0 & 0 \\
\frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & -\frac{1}{2}\lambda^4(1 + 4A^2) & 0 \\
\frac{1}{2}A\lambda^5(\rho + i\eta) & \frac{1}{2}A\lambda^3[1 - 2(\rho + i\eta)] & -\frac{1}{2}A^2\lambda^4
\end{pmatrix} + O(\lambda^6).$$

From this parameterization it follows that the apex of the UT is located at

$$-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} = (\bar{\rho}, \bar{\eta}) \simeq (1 - \lambda^2/2)(\rho, \eta),$$

which is in the first quadrant as shown in Fig. 1.1(a). Similarly, in the case of $\text{UT}_s$, the apex is at

$$-\frac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*} = (\bar{\rho}_s, \bar{\eta}_s) \simeq \frac{-\lambda^2}{1 - \lambda^2/2}(\rho, \eta),$$

located in the third quadrant as shown in Fig. 1.1(b). Using the fact that $\lambda \sim 0.225$, it turns out that this figure is not to scale and $\beta_s$ is actually very small, $O(\lambda^2)$.

Using the angles defined in Eq. 1.10 and Eq. 1.11, it follows that, in the phase convention implied by the Wolfenstein parameterization (see Eq. 1.12),

$$\begin{align*}
\beta & = \arg(-1) + \arg(V_{td}V_{tb}^*) - \arg(V_{td}V_{tb}^*) = -\arg(V_{td}) \\
\gamma & = \arg(-1) + \arg(V_{ud}V_{ub}^*) - \arg(V_{cd}V_{cb}^*) = \arg(V_{ub}^*) = -\arg(V_{ub}) \\
\beta_s & = \arg(-1) + \arg(V_{cb}V_{cs}^*) - \arg(V_{tb}V_{ts}^*) = \pi - \arg(V_{ts}^*) = \pi + \arg(V_{ts})
\end{align*}$$
1.4 Mixing of Neutral $B$ Mesons

### Table 1.1: Latest results of global fits to the $UT(s)$ angles from the CKMfitter Group, as of the Moriond 2012 conference [17].

<table>
<thead>
<tr>
<th>$UT(s)$ angle</th>
<th>value ($^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$91.1^{+4.3}_{-4.3}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$21.85^{+0.80}_{-0.77}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$67.1^{+4.3}_{-4.3}$</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>$1.044^{+0.044}_{-0.046}$</td>
</tr>
</tbody>
</table>

This implies that the CKM matrix can be written in the following way:

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5) \ . \quad (1.16)$$

The Wolfenstein parameterization will be used throughout this thesis, unless explicitly stated otherwise.

1.3.2 CKM Constraints from Experiments

The CKM mechanism in the SM can be tested experimentally by measuring the lengths of the sides and the interior angles of the unitarity triangles. The lengths of the sides of the $UT(s)$ yield an indirect determination of the angles in the CKM matrix. Direct measurements of the angles can be obtained from CP-violating observables in $B$ decays. The combination of these direct and indirect measurements provides a test of the consistency of the CKM mechanism.

The current constraints on the location of the apex of the UT and $UT_s$ are shown in Fig. 1.2 and the resulting $UT(s)$ angles are given in Table 1.1. Notice that the location of the apex is defined by only two parameters, $\rho(s)$ and $\eta(s)$, while there are various physical observables that lead to different constraints, as indicated by the different colored regions in Fig. 1.2. It follows that currently all measurements are consistent with the CKM description of the weak interactions in the SM. That is, both the apex in the UT and the $UT_s$ lie within the uncertainty bounds of all measurements. More accurate measurements are needed to reveal potential tensions in the CKM mechanism and possible deviations from the Standard Model.

1.4 Mixing of Neutral $B$ Mesons

Neutral $B$ mesons have the property that they can oscillate to their antiparticle. This process is called mixing. The flavour eigenstates of the $B^0$ meson and the $B^0_s$ meson are:

$$|B^0\rangle = |\bar{b}d\rangle \ , \ |\bar{B}^0\rangle = |\bar{b}d\rangle \ ,$$
$$|B^0_s\rangle = |\bar{b}s\rangle \ , \ |\bar{B}^0_s\rangle = |\bar{b}s\rangle \ . \quad (1.17)$$
Figure 1.2: (a) UT constraints. (b) UTs constraints. Both graphs are the latest results (as of the Moriond 2012 conference) from the CKMfitter Group [17]. Notice that the UTs is drawn to scale here, unlike the schematic picture in Fig. 1.1 (b).

For the remainder of this section $B$ will denote any neutral $B$ meson, i.e. either a $B_d^0$ or a $B_s^0$ meson.

Since $B$ mesons can oscillate between $|B\rangle$ and $|\bar{B}\rangle$ states, the time evolution of the linear combination $|B(t)\rangle = a(t) |B\rangle + b(t) |\bar{B}\rangle$ can effectively be described by a two-dimensional Schrödinger equation:

$$i \frac{d}{dt} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \mathbf{H} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix},$$

where the Hamiltonian can be written as

$$\mathbf{H} = \mathbf{M} - i \frac{\Gamma}{2},$$

with the Hermitian matrices $\mathbf{M}$, the so-called mass matrix, and $\Gamma$, the decay matrix. Assuming CPT invariance, the masses and decay times of $B$ mesons are equal. This implies that $M_{11} = M_{22} = M$ and $\Gamma_{11} = \Gamma_{22} = \Gamma$. In addition, for the off-diagonal elements, responsible for mixing between $|\bar{B}\rangle$ and $|B\rangle$ eigenstates, $M_{21} = M_{12}^*$ and $\Gamma_{21} = \Gamma_{12}^*$, due to hermiticity [18]. Hence the Hamiltonian can be written as

$$\mathbf{H} = \begin{pmatrix} M - \frac{i}{2} \Gamma & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M - \frac{i}{2} \Gamma \end{pmatrix}.$$  

In the off-diagonal elements, $M_{12}$ is dominated by short-distance (virtual) processes, whereas $\Gamma_{12}$ is dominated by real intermediate states (long-distance processes), to which both the $B$ and the $\bar{B}$ mesons can decay [19] [20].
By diagonalizing the Hamiltonian, the solutions to the Schrödinger equation for the mass eigenstates are found:

\[ |B_H(t)⟩ = e^{-(im_H + \frac{1}{2}\Gamma H)t}|B_H(0)⟩ \quad \text{and} \quad |B_L(t)⟩ = e^{-(im_L + \frac{1}{2}\Gamma L)t}|B_L(0)⟩ \]

with eigenvalues \( m_H - \frac{1}{2}\Gamma H \) and \( m_L - \frac{1}{2}\Gamma L \), respectively.

In turn, these time-dependent mass eigenstates are linear combinations of the weak flavour eigenstates:

\[ |B_{H,L}(t)⟩ = e^{-im_{H,L}t}|B_{H,L}(0)⟩ \]

with eigenvalues \( m_{H} - \frac{i}{2}\Gamma_H \) and \( m_{L} - \frac{i}{2}\Gamma_L \), respectively.

Solving the eigenvalue equation for either \( \lambda_H = H_0 + \sqrt{H_{21}H_{12}} \) with eigenvector \((p, -q)\)

or \( \lambda_L = H_0 - \sqrt{H_{21}H_{12}} \) with eigenvector \((p, q)\) the ratio \( \frac{q}{p} \) is found to be

\[ \frac{q}{p} = -\sqrt{\frac{H_{21}}{H_{12}}} = -\sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12} - \frac{i}{2}\Gamma_{12}}} \].

Expressing the time-dependent flavour eigenstate in Eq. (1.21) in terms of mass eigenstates gives

\[ |B(t)⟩ = |B_L(t)⟩ + \frac{|B_H(t)⟩}{2p} \].

Inserting the time dependence of the mass eigenstates yields

\[ |B(t)⟩ = \frac{e^{-(im_L + \frac{1}{2}\Gamma_L)t}|B_L(0)⟩ + e^{-(im_H + \frac{1}{2}\Gamma_H)t}|B_H(0)⟩}{2p} \].

Using the definition

\[ g_{±}(t) = \frac{1}{2}(e^{-(im_L + \frac{1}{2}\Gamma_L)t} ± e^{-(im_H + \frac{1}{2}\Gamma_H)t}) \],

it is finally found that

\[ |B(t)⟩ = g_+(t)|B⟩ + \frac{q}{p}g_-(t)|\bar{B}⟩ \].

This equation expresses the time-dependent composition in flavour eigenstates of a state \( |B(t)⟩ \), initially produced as a \( |B⟩ \) state.

Using these expressions, the probability to observe a \( |B⟩ \) state in a measurement at time \( t \), provided that the original particle was produced as a \( |B⟩ \) state, is given by

\[ |⟨B|B(t)⟩|^2 = |g_+(t)|^2 \],

while the probability to observe a \( |B⟩ \) state at time \( t \) that was produced as a \( |\bar{B}⟩ \) state is

\[ |⟨\bar{B}|B(t)⟩|^2 = \left| \frac{q}{p} \right|^2 |g_-(t)|^2 \],

where

\[ |g_{±}(t)|^2 = \frac{e^{-\Gamma t}}{2} \left( \cosh \left( \frac{\Delta \Gamma t}{2} \right) ± \cos(\Delta m t) \right) \].
and
\[ \Delta m = m_H - m_L = 2\text{Re}\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)} \]  \hfill (1.30)  
\[ \Delta \Gamma = \Gamma_L - \Gamma_H = -4\text{Im}\sqrt{(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)} \]  \hfill (1.31)

Moreover, it is useful to define the average mass \( \bar{m} \equiv \frac{m_H + m_L}{2} \) and average width \( \bar{\Gamma} \equiv \frac{\Gamma_H + \Gamma_L}{2} \). By definition \( \Delta m > 0 \), but \( \Delta \Gamma \) can, a-priori, have either sign. LHCb recently determined the sign of \( \Delta \Gamma \) to be positive \cite{21} and this will be discussed in more detail in Sec. 6.7.3.

Writing the ratio of \( M_{12} \) and \( \Gamma_{12} \) in terms of its magnitude and phase, the convention-independent phase difference \( \phi_{M/\Gamma} \) is defined by
\[ \frac{M_{12}}{\Gamma_{12}} \equiv -\frac{M_{12}}{\Gamma_{12}} e^{i\phi_{M/\Gamma}} \]  \hfill (1.32)

Using the definition of the mixing phase
\[ \phi_M = \text{arg}(M_{12}) \]  \hfill (1.33)
\( \phi_{M/\Gamma} \) is written as
\[ \phi_{M/\Gamma} = \phi_M - \text{arg}(-\Gamma_{12}) \]  \hfill (1.34)

In terms of the Hamiltonian matrix elements, \( \Delta m \) and \( \Delta \Gamma \) can then be written as
\[ \Delta m = 2|M_{12}| \left[ 1 + \mathcal{O}\left( \frac{\Gamma_{12}}{M_{12}}^2 \right) \right] \]  \hfill (1.35)  
\[ \Delta \Gamma = 2|\Gamma_{12}| \cos \phi_{M/\Gamma} \left[ 1 + \mathcal{O}\left( \frac{\Gamma_{12}}{M_{12}}^2 \right) \right] , \]  \hfill (1.36)

where \( |\Gamma_{12}/M_{12}| \ll 1 \) was used.\(^1\)

Finally, from Eq. 1.22 \( \frac{q}{p} \) can be written as
\[ \frac{q}{p} = -e^{-i\phi_M} \sqrt{|M_{12}| + \frac{i}{2}\Gamma_{12}|e^{i\phi_{M/\Gamma}}|} \]  \hfill (1.37)  
As before, in the limit \( |\Gamma_{12}/M_{12}| \ll 1 \), this can be written as
\[ \frac{q}{p} = -e^{-i\phi_M} \left[ 1 - \frac{a_{fs}}{2} \right] , \]  \hfill (1.38)
with the so-called flavour-specific CP asymmetry
\[ a_{fs} \simeq \frac{\Gamma_{12}}{M_{12}} \sin \phi_{M/\Gamma} , \]  \hfill (1.39)

\(^1\)This follows from experiments that show \( \Delta m \gg |\Delta \Gamma| \) and theoretical calculations that show \( |\Gamma_{12}| \ll \Delta m \) \cite{22}.
where all terms of order \( \left( \frac{\Gamma_2}{12 M_2} \right) \) are neglected. The departure of this parameter from zero is a measure of the amount of CP violation in mixing, which is discussed in Sec. 1.7. This parameter can be measured with flavour-specific \( B \) decays. An example of these kind of decays are semileptonic \( B \) decays, hence this parameter is also referred to as the semileptonic CP asymmetry \( a_{sl} \) [14].

### 1.5 Decay of Neutral \( B \) Mesons

After production and mixing, neutral \( B \) mesons can decay in several hundreds of modes, with branching fractions in the range \( O(10^{-1} - 10^{-10}) \) [23]. The time-dependent decay amplitudes of flavour eigenstates to a final state \( f \) are defined as

\[
A_f \equiv \langle f | T | B \rangle \quad \overline{A}_f \equiv \langle f | T | \overline{B} \rangle \quad A_{\overline{f}} \equiv \langle f | T | \overline{B} \rangle \quad \overline{A}_{\overline{f}} \equiv \langle f | T | B \rangle,
\]

with \( T \) the transition matrix [24]. The decay rate of a \( B \) meson decaying to a final state \( f \) is therefore

\[
\Gamma_{B \to f}(t) = |A_f(t)|^2 = |\langle f | T | B(t) \rangle|^2.
\]

Defining the parameter

\[
\lambda_f = \frac{q}{p} \frac{A_f}{\overline{A}_f},
\]

and using Eq. 1.26, this is rewritten as

\[
\Gamma_{B \to f}(t) = |A_f|^2 |g_+(t) + \lambda_f g_-(t)|^2.
\]

The corresponding decay amplitude is

\[
A_f(t) = A_f(0)[g_+(t) + \lambda_f g_-(t)]
\]

and is graphically represented in Fig. 1.3, which shows an example where the total amplitude for a \( B \) meson decay to a final state, accessible to both \( B \) and \( \overline{B} \), consists of two contributions: a direct decay, and a decay after mixing. Equation 1.43 is expanded as

\[
\Gamma_{B \to f}(t) = |A_f|^2 \left( |g_+(t)|^2 + |\lambda_f|^2 |g_-(t)|^2 + 2Re[\lambda_f g_+(t) g_-(t)] \right).
\]

From the definition of \( g_{\pm}(t) \) it follows that

\[
g_{+}(t) g_{-}(t) = \frac{e^{-\Gamma t}}{2} \left( - \frac{\Delta \Gamma t}{2} + i \sin \Delta m t \right)
\]

Using this and Eq. 1.29, the decay rate finally becomes

\[
\Gamma_{B \to f}(t) = |A_f|^2 (1 + |\lambda_f|^2) \frac{e^{-\Gamma t}}{2} \left( \cosh \frac{\Delta \Gamma t}{2} - D_f \sinh \frac{\Delta \Gamma t}{2} + C_f \cos \Delta m t - S_f \sin \Delta m t \right).
\]
Chapter 1. CP Violation and Physics of B Mesons

Figure 1.3: The amplitude of a B meson decaying to a final state $f$ consists of two contributions: the direct decay ($A_f g_+(t)$) at the bottom and the decay after mixing ($A_f \lambda_f g_-(t)$) via the upper path.

where \[ D_f = \frac{2 \text{Re}[\lambda_f]}{1 + |\lambda_f|^2}, \quad C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f = \frac{2 \text{Im}[\lambda_f]}{1 + |\lambda_f|^2}. \tag{1.48} \]

Equivalently, for the other possible combinations of $B$ and $\bar{B}$ meson decays to a final state $f$ or $\bar{f}$, the decay rates are

\[
\Gamma_{B\rightarrow f}(t) = |A_f|^2 \left(1 + |\lambda_f|^2\right) e^{-\Gamma t} \cdot 
\left(\cosh \frac{\Delta \Gamma t}{2} - D_T \sinh \frac{\Delta \Gamma t}{2} - C_T \cos \Delta mt + S_T \sin \Delta mt\right) \tag{1.49}
\]

\[
\Gamma_{B\rightarrow \bar{f}}(t) = |A_f|^2 \left(1 + |\lambda_f|^2\right) e^{-\Gamma t} \cdot 
\left(\cosh \frac{\Delta \Gamma t}{2} - D_f \sinh \frac{\Delta \Gamma t}{2} - C_f \cos \Delta mt + S_f \sin \Delta mt\right) \tag{1.50}
\]

\[
\Gamma_{\bar{B}\rightarrow \bar{f}}(t) = |\bar{A}_f|^2 \left(1 + |\bar{\lambda}_f|^2\right) e^{-\Gamma t} \cdot 
\left(\cosh \frac{\Delta \Gamma t}{2} - D_T \sinh \frac{\Delta \Gamma t}{2} + C_T \cos \Delta mt - S_T \sin \Delta mt\right), \tag{1.51}
\]

with $\bar{\lambda}_f = \frac{\lambda_f}{q} \bar{A}_f$ and

\[
D_T = \frac{2 \text{Re}[\bar{\lambda}_f]}{1 + |\bar{\lambda}_f|^2}, \quad C_T = \frac{1 - |\bar{\lambda}_f|^2}{1 + |\bar{\lambda}_f|^2}, \quad S_T = \frac{2 \text{Im}[\bar{\lambda}_f]}{1 + |\bar{\lambda}_f|^2}. \tag{1.52}
\]

1.6 Measurement of Relative Phases

The overall phase of an amplitude can be written as the combination of a phase that flips sign under the CP transformation and a phase that is invariant. The former is referred to as the weak phase $\phi_{\text{weak}}$, as it originates from weak interactions; the latter typically arises from strong final state interactions. For processes with only one contributing amplitude, the phase of this amplitude is not observable. However, when two amplitudes contribute,
the magnitude of the total amplitude can differ between a process and its CP conjugated process. This can be shown by writing the total amplitude as

$$A = A_1 + A_2,$$

where, without loss of generality, the phase of $A_1$ can be chosen to be zero, and the relative weak and strong phase difference between $A_1$ and $A_2$ are indicated by $\phi$ and $\delta$, respectively:

$$A_1 = |A_1|, \quad \frac{A_2}{A_1} = \frac{|A_2|}{|A_1|} e^{-i\delta} e^{i\phi}.$$  \hfill (1.53)

In that case the total rate equals

$$|A|^2 = |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + 2 |A_1||A_2| \cos(\delta + \phi),$$  \hfill (1.54)

whereas the CP-conjugated rate is

$$|\overline{A}|^2 = |A_1|^2 + |A_2|^2 + 2 |A_1||A_2| \cos(\delta - \phi).$$  \hfill (1.55)

The CP asymmetry then reads

$$A_{CP} \equiv \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} = \frac{-2 \sin \delta \sin \phi}{|A_1|^2/|A_2|^2 + |A_2|^2/|A_1|^2 + 2 \cos \delta \cos \phi}.$$  \hfill (1.56)

This shows that there can be an observable non-zero CP asymmetry, provided there is both a strong and a weak phase difference between the contributing amplitudes.

### 1.7 Classification of CP Violation

Because Eq. 1.45 has three contributions, there are three ways to break the CP symmetry. Therefore, the following types of CP violation can be distinguished:

1) **CP violation in decay**

CP violation in decay occurs when the decay rate of a $B$ meson to a final state $f$ differs from the rate of a $\overline{B}$ meson to a final state $\overline{f}$. This type of CP violation occurs when

$$\left| \frac{A_f}{A_{\overline{f}}} \right| \neq 1.$$  \hfill (1.57)

2) **CP violation in mixing**

CP violation in mixing occurs when the probability to oscillate from a $B$ meson to a $\overline{B}$ meson is different from the probability to oscillate from a $\overline{B}$ to a $B$ meson. For this to happen, it is required that

$$|\langle B|\overline{B}(t)\rangle|^2 = |\frac{p}{q}|^2 |g_-(t)|^2 \neq |\frac{q}{p}|^2 |g_-(t)|^2 = |\langle \overline{B}|B(t)\rangle|^2.$$  \hfill (1.58)
which follows from Eq. 1.27. It follows that this requirement is satisfied when \(|q/p| \neq 1\), or, equivalently when

\[
a_- \equiv \left( 1 - \left| \frac{p}{q} \right|^2 \right) \neq 0.
\]

(1.59)

3) **CP violation in interference between decays with and without mixing**

There is a third type of CP violation which can occur even if there is neither CP violation in mixing nor CP violation in decay. This type of CP violation is caused by the interference between decays with and without mixing and can be observed in decays to a final state that is accessible to both \(B\) and \(\bar{B}\) mesons.

A special case occurs when the final state is a CP eigenstate: \(CP|f_{CP}\rangle = \pm |f_{CP}\rangle\). Then, by definition, the final state can be reached by a direct decay to the final state \(B \rightarrow f_{CP}\), and via mixing and subsequent decay \(B \rightarrow \bar{B} \rightarrow f_{CP}\). The CP asymmetry between a \(B\) meson decaying to a final state \(f_{CP}\) and its CP conjugated process is defined as

\[
A_{CP}(t) = \frac{\Gamma_{B \rightarrow f}(t) - \Gamma_{\bar{B} \rightarrow \bar{f}}(t)}{\Gamma_{B \rightarrow f}(t) + \Gamma_{\bar{B} \rightarrow \bar{f}}(t)}
\]

(1.60)

\[
= \frac{a_- \cosh \frac{\Delta \Gamma t}{2} - a_- D_f \sinh \frac{\Delta \Gamma t}{2} + a_+ C_f \cos \Delta m t - a_+ S_f \sin \Delta m t}{a_+ \cosh \frac{\Delta \Gamma t}{2} - a_+ D_f \sinh \frac{\Delta \Gamma t}{2} + a_- C_f \cos \Delta m t - a_- S_f \sin \Delta m t},
\]

where \(a_-\) was defined in Eq. 1.59 and \(a_+\) is defined as \(a_+ \equiv \left( 1 + \left| \frac{p}{q} \right|^2 \right)\). Note that, as opposed to the two previous types of CP violation, this CP asymmetry depends on \(t\), because of the dependence on decay time in \(B\) mixing.

The two amplitudes that contribute to the total decay amplitude are the direct decay of a \(B\) meson to the final state and the decay where the \(B\) meson first oscillates before decaying, as indicated in Fig. 1.4. Since now the total decay amplitude is the sum of two amplitudes, their relative weak phase difference can be determined by comparing the decay process and its CP conjugate process. This type of CP violation can be observed in \(B^0_s \rightarrow J/\psi \phi\) decays, or more generally, in \(B^0_s\) decays via \(b \rightarrow c\bar{s}s\) transitions.

### 1.8 CP Violation in Interference between Mixing and \(b \rightarrow c(\bar{s}s)\) Transitions

In \(B^0_s\) decays that occur through \(b \rightarrow c\bar{s}s\) transitions to a CP eigenstate, see for example Fig. 1.4, CP violation can manifest itself through interference between decays with and without mixing. Here, \(|\lambda_f| = \left| \frac{\frac{A_f}{p}}{\frac{A_f}{q}} \right| = 1\), if one assumes that there is no CP violation in mixing, i.e. \(|q/p| = 1\), and that penguin contributions (see Sec. 5.9) can be ignored, i.e.

\[2\]In this particular case, the origin of the strong phase difference is the mixing dynamics. For example, in the simplified case where \(\Delta \Gamma = 0\), from Eq. 1.25 it follows that mixing generates a phase difference of exactly 90° between \(g_+(t)\) and \(g_-(t)\).
\[
\frac{A_f}{A_f} = 1. \text{ This leads to } C_f = 0, a_+ = 2 \text{ and } a_- = 0, \text{ causing the decay time-dependent CP asymmetry from Eq.1.60 to simplify to }
\]

\[
A_{CP}(t) = \frac{-S_f \sin \Delta m t}{\cosh \frac{\Delta \Gamma t}{2} - D_f \sinh \frac{\Delta \Gamma t}{2}}. \tag{1.61}
\]

In the case of \( B_s^0 \to J/\psi \phi \) decays, an extra complication occurs because the final state \( J/\psi \phi \) is an admixture of CP eigenstates with eigenvalues \( \eta_f = \pm 1 \). The CP asymmetry then becomes

\[
A_{CP}(t) = \frac{-\eta_f \sin \phi_s \sin \Delta m t}{\cosh \frac{\Delta \Gamma t}{2} - \eta_f \cos \phi_s \sinh \frac{\Delta \Gamma t}{2}}. \tag{1.62}
\]

In this equation, \( \phi_s = \phi_M - 2 \phi_{c(c\bar{s})} \) is the relative weak phase difference, see Sec.1.6, where \( \phi_{c(c\bar{s})} \) is the phase of the \( b \to c(\bar{c}s) \) transition. The precise derivation of \( S_f = \eta_f \sin \phi_s \) and \( D_f = \eta_f \cos \phi_s \) is explained in more detail in Chap.5. In that chapter, it is also shown that in the SM, the parameter \( \phi_s \) is related to the angle \( \beta_s: \phi_s^{SM} = -2 \beta_s \), which is expected to be small. However, the value of \( \phi_s \) can be enhanced by New Physics (NP) models, indicating that the \( B_s^0 \to J/\psi \phi \) decay mode is an important probe for New Physics. Before going into the details of the analysis of \( B_s^0 \to J/\psi \phi \) decays in Chap.5 and Chap.6 it is time to introduce the LHCb experiment.