We report on detailed small-angle neutron scattering measurements with polarization analysis from the flux-line lattice in the anisotropic superconductor YBa$_2$Cu$_3$O$_{7-\delta}$. When the field was applied at an angle to the principal axes we have observed spin-flip neutron diffraction consistent with local transverse field components. From a detailed study of the angle dependence of the magnitude of the spin-flip neutron-scattered intensity we have found that the transverse-field components are larger than predicted by an anisotropic London model for reasonable mass anisotropy values. The transverse-field components are consistent with a reduced c-axis coherence length at low temperature.

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It is well known that many properties of high-temperature superconductors (HTS’s) are anisotropic due mainly to the CuO$_2$ planes. With a field greater than $H_{c1}$ applied parallel to the c axis, the screening currents associated with the flux-line lattice (FLL) flow in the plane perpendicular to the applied field. In an ideal isotropic superconductor this would be true for all angles of the applied field relative to the crystal axes. When the field is rotated away from the c direction in an anisotropic superconductor, the situation is complicated due to the reduced kinetic energy if current flows in an easy direction. The predicted current plane is tilted from perpendicular to the applied field and towards the basal plane.

The tilted current flow results in local transverse-field components as proposed by Dorer and Bömmel and later considered for the general case with $H_{c1} \ll H \ll H_{c2}$ by Thiemann et al. By a flux quantization argument, one may show that the flux-line cores lie along the direction of the average magnetic field. Hence, there is a zero spatial average of the local fields transverse to the flux lines. These fields are distinct from any transverse magnetization and can therefore only be detected by a microscopic probe. The variation of both the longitudinal and transverse fields in the FLL gives rise to small-angle neutron scattering (SANS), but the transverse fields also flip the spin of the diffracted neutrons. In this paper we present the first SANS measurements with polarization analysis from the mixed state of YBa$_2$Cu$_3$O$_{7-\delta}$. Our technique is quite different from neutron depolarization which gives a measure of larger scale field gradients by the depolarization of the transmitted beam. Such effects were undetectably small in our experiments. We have observed spin-flip neutron diffraction consistent with the predicted transverse-field components and make a comparison between our experimental observations and the anisotropic London model which we now describe.

For a moderately anisotropic superconductor such as YBCO with large $\kappa$ and a coherence length comparable with the interplane spacing, an appropriate theory is the anisotropic London model, particularly close to $T_c$. This theory has the advantage that it provides a simple and successful description of the magnetic response of anisotropic superconductors. However, the model is phenomenological, as it does not include the underlying physical origin of the anisotropy.

In the anisotropic London model, the effective mass of the charge carriers depends on their direction of motion. The normalized effective masses are written as $m_a$, $m_b$, and $m_c$ and are normalized to the “average mass” $M_{av} = (M_a M_b M_c)^{1/3}$ such that $m_i = M_i / M_{av}$ and $m_i m_i m_i = 1$. The anisotropy may be quantified as two ratios of the effective masses: $\gamma_{ca} = m_c / m_a$ and $\gamma_{ab} = m_a / m_b$. The supercurrent response is larger in the b direction than the a direction due to the Cu-O chains. The result is a shorter penetration depth $\lambda_\parallel$ for fields in the c direction which are varying along a than the corresponding penetration depth $\lambda_\perp$ for fields varying along b. Hence the FLL is distorted. We have shown in previous SANS measurements on our sample under similar conditions to here that $\gamma_{ca} = 4.5(6)$ and $\gamma_{ab} = 1.18(2)$. We will now examine the predictions of an anisotropic model using the nomenclature of Thiemann et al., extending their analysis to the case of biaxial anisotropy, i.e., $m_a \neq m_b$. In the mixed state, the magnetic field distribution $B(r)$ may be represented by the Fourier series

$$B(r) = \sum_q b(q) \exp(iq \cdot r),$$

(1)

where $q$ is the set of reciprocal lattice vectors. The average field is in the $z$ direction and the transverse fields $B_x(r)$ and $B_y(r)$ are in the $xy$ plane, defined in Fig. 1. In an isotropic...
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tunneling, and Bitter decoration. Using torque magnetization of the FLL is given by

\[ \text{torque} = \frac{\partial M}{\partial H} \]

SANS, techniques on a number of different crystals: Bitter decoration shown in Fig. 2.

\[ g \approx \text{value in the range} \]

used in this study we have found, by torque magnetometry, infrared reflectivity, Josephson tunneling, and Bitter decoration. Using torque magnetometry and \( \mu \text{SR} \), on the same sample as used in this SANS study, we have found \( \gamma_{ab} = 1.19(1) \) and \( \gamma_{ca} = 6.0(4) \). From these values using Eqs. (2) and (3) we can calculate the magnitude of the SF scattering in the anisotropic London model.

Other techniques may also be used to deduce the mass anisotropy: they include muon spin rotation (\( \mu \text{SR} \)), infrared reflectivity, Josephson tunneling, and Bitter decoration. Although small \( \approx 60 \text{ neV} \) this noticeably changes the energy, this results in a change of the neutron kinetic energy. In Fig. 4 we plot the intensity of the NSF scattering, we will examine some details of the spin-flip diffraction pattern. In Fig. 2(b) it can be seen that for \( q_x = 0 \) there is zero SF scattering. This is expected from Eqs. (2) and (3) as both \( b_x(q) \) and \( b_y(q) \) are zero for \( q = (q_x, 0) \). The \( b_x \) result arises from the Maxwell equation \( \nabla \cdot B = 0 \), which may be expressed in reciprocal space as \( q \cdot b(q) = 0 \). From mirror symmetry of the FLL about \( y = 0 \) \( b_x(q_x, 0) \) is also zero, so there should be no SF scattering for \( q_x = 0 \).

When the neutron spin is flipped in a magnetic field there is a change in the potential energy of the neutron equal to the Zeeman splitting for the neutron. From conservation of energy, this results in a change of the neutron kinetic energy. Although small \( \approx 60 \text{ neV} \) this noticeably changes the angle \( \omega \) through which the sample must be rotated to bring the scattering vector to a reciprocal lattice point. The change is given by \( \Delta \omega / \omega = \pm 1.91/2 \pi \) and is positive for a decrease in kinetic energy. In Fig. 4 we plot the intensity of the NSF and SF scattering as a function of the rocking angle \( \omega \). By changing the incident neutron polarization, SF scattering with either kinetic energy gain or loss was probed. The observed splitting \( \Delta \omega \) in the peak intensity \( \omega \) is due to the Zeeman splitting of the neutron potential energy in the sample region which means \( k_{\text{in}} \neq k_{\text{out}} \) as shown in the inset.

Before presenting a study of the angle dependence of the SF scattering, we will examine some details of the spin-flip diffraction pattern. In Fig. 2(b) it can be seen that for \( q_y = 0 \) there is zero SF scattering. This is expected from Eqs. (2) and (3) as both \( b_x(q) \) and \( b_y(q) \) are zero for \( q = (q_x, 0) \). The \( b_x \) result arises from the Maxwell equation \( \nabla \cdot B = 0 \), which may be expressed in reciprocal space as \( q \cdot b(q) = 0 \). From mirror symmetry of the FLL about \( y = 0 \) \( b_x(q_x, 0) \) is also zero, so there should be no SF scattering for \( q_x = 0 \).

When the neutron spin is flipped in a magnetic field there is a change in the potential energy of the neutron equal to the Zeeman splitting for the neutron. From conservation of energy, this results in a change of the neutron kinetic energy. Although small \( \approx 60 \text{ neV} \) this noticeably changes the angle \( \omega \) through which the sample must be rotated to bring the scattering vector to a reciprocal lattice point. The change is given by \( \Delta \omega / \omega = \pm 1.91/2 \pi \) and is positive for a decrease in kinetic energy. In Fig. 4 we plot the intensity of the NSF and SF scattering as a function of the rocking angle \( \omega \). By changing the incident neutron polarization, SF scattering with either kinetic energy gain or loss was probed. The observed splitting is consistent with the predicted value of \( \pm 0.15^\circ \) for an applied field of 0.5 T.

We now examine the angular dependence of the SF and NSF scattering for the four spots near the horizontal plane (we ignore the vertical spots because we did not have the facility to rock through the Bragg condition for them). A reliable measure of the intensity of the spin-flip diffraction is the ratio of the integrated SF and NSF intensities. This ratio is independent of many possible systematic experimental errors. The quantization axis for the neutron spin is the average magnetic field direction \( z \). Just as in NMR, varying perpen-

\[ \text{distortion}^2 = \left( \frac{\tan \phi}{\tan 30^\circ} \right)^2 = \gamma_{ab} \cos^2 \theta + \frac{1}{\gamma_{ca}} \sin^2 \theta, \]
FIG. 5. Plot of the ratio of the theoretical and experimental SF to NSF scattering as a function of the applied field direction. The experimental intensities were evaluated as the sum of the counts in a small area of the detector centered on the NSF spot position, integrated over the rocking curve. The theoretical plots are calculated using $\gamma_{ab} = 1.19$ and for $\gamma_{ca} = 4$, 6, and 8.

The ratio $r_{FS}$ of the SF and NSF integrated intensities is therefore given by

$$r_{FS} = \frac{b_x^2 + b_y^2}{b_z^2} = \frac{m_{ab}^2 (q_x^2 + q_y^2 q_z^2)}{m_{ca}^2 q_y^4}, \quad (8)$$

which may easily be evaluated as a function of $\theta$ and the anisotropy parameters $\gamma_{ab}$ and $\gamma_{ca}$. In Fig. 5 we plot the experimental ratio of the SF and NSF integrated intensities and compare with the prediction of Eq. (8). The theoretical lines in Fig. 5 are plotted for a range of $\gamma_{ca}$ values predicted from various techniques$^{13-17}$ as well as that deduced from the FLL distortion. The experimental ratio is greater than that predicted by the anisotropic London theory for any reasonable values of $\gamma_{ab}$ and $\gamma_{ca}$.

It should be noted that distortion of the FLL is governed by the high-temperature mass anisotropy, at the point where the FLL becomes pinned. However, the transverse-field components will be governed by the low-temperature anisotropy. The data could be fitted by the anisotropic London model with a much larger out of plane anisotropy ($\gamma_{ca} \approx 50$), which would correspond to a much greater confinement of supercurrent flow to the Cu-O planes as the $c$-axis coherence length $\xi_c$ becomes much smaller than the interplanar spacing. However, under such conditions the layered crystal structure becomes important and a continuum London theory should not apply. Nonlocal effects$^{20,21}$ can also give deviations from the London model, although it is not clear that they would result in an increased anisotropy $\gamma_{ca}$ in YBCO.

In any case, we conclude that the anisotropic London model does not give an accurate description of YBa$_2$Cu$_3$O$_7$-$\delta$, at low temperature where the superconductivity appears more two dimensional than at high temperatures. It would be of interest to study this over a range of temperatures.

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