Summary

This is a thesis about gravity, and a number of the interesting effects it has on the motion of stars when they get in close proximity to each other. Describing such motions turns out to be a complicated problem, and this at first seems to be a remarkable statement. After all, gravity is a force that we human beings are very familiar with from every day experience. It is the force that keeps us with our feet on the ground, makes apples fall from trees, traps the Moon in an orbit around the Earth, and the Earth itself in an orbit around the Sun. The fact that research in gravity is nonetheless challenging, is due to the fact that gravity is conceptually very different from the three other fundamental forces that are known in physics (the electromagnetic force, the strong nuclear force, and the weak nuclear force), which reflects itself in the radically different mathematics involved. It is for this reason that it required the genius of two of the greatest scientific minds in history to provide us with an understanding of how things fall.

Gravity according to Newton and Einstein

Sir Isaac Newton (1642-1727) was the first to provide mankind with an accurate description of gravity. He considered it to be an ‘action at a distance’, a mysterious and invisible tendency of all matter to pull all other matter in the Universe closer. In 1687 he published a mathematical formula with which the pull could be calculated. This Universal Law of Gravity is very successful: for the first time in history, mankind had the means to accurately calculate the trek of the planets around the Sun, to predict solar- and lunar eclipses, and to solve the mystery of the tides. Despite these successes, however, a number of things were not quite exactly described by this theory. For example, very precise measurements on the orbit of Mercury show that the planet orbits the Sun slightly faster than predicted by the Universal Law of Gravity. Also, Newton’s theory postulates that gravity is an instantaneous force, by which is meant that there is zero elapse of time between a cause and its gravitational effect. If, for example, the Sun were to magically disappear, the gravitational pull felt on the Earth would vanish at the exact same instant. This would indicate that the effects of gravity traversed the distance between the Sun and the Earth with an infinite velocity. This can not be, as it contradicts the Special Theory of Relativity. This is a theory, published in 1905 by Albert Einstein (1879-1955), which states that nothing can go faster than light. Gravity is not allowed to be an exception to this rule, and it thus became clear that Newton’s Law of Universal Gravity had to be modified.
It was that same Special Theory of Relativity that suggested how this should be done. The theory states that the laws of physics should be the same for all observers moving with a constant velocity with respect to each other. It should therefore be possible to write the equations of physics in such a way that they can be used by all such observers regardless of their relative constant velocity. Einstein realized that this idea should also apply to observers who are accelerating with respect to each other, and was able to link this principle to the force of gravity. After all, he argued, if we are standing in an elevator that is accelerating upward, we feel that we are pushed to the floor of the elevator just as we would if there was a force of gravity pulling us down. With this insight, Einstein postulated that the phenomena of acceleration and gravity are fundamentally equivalent to each other, and formulated the General Theory of Relativity in 1916.

In the General Theory of Relativity, gravity is described as the curvature of space and time. Just as a meridian deviates from a straight line because the globe is spherically shaped, the path of a mass in motion will not follow a straight line if space is curved; the deviation from the straight line is what we ascribe to gravity. The way that space and time are curved is, in turn, determined by the presence of mass and energy: the more mass is present in space, the more space and time are curved, and the more the paths of masses in motion deviate from straight lines. An example of this is shown in Figure 1, in which the presence of a heavy star curves space in such a way that the orbit of a planet is wrapped around the star. As a result, the planet follows a closed elliptical orbit instead of a straight line. Einstein published a formula that relates the curvature of space and time to the presence of mass and energy, and showed that when the curvature is not too extreme, the General Theory of Relativity exactly reduces to Newton’s Universal Law of Gravity.
The General Theory of Relativity is the most successful description of gravity that we have. It correctly predicts the orbit of the planet Mercury, the deflection of starlight when it grazes the Sun, the slowing down of time due to the presence of mass, and even the expansion of the Universe as a whole. All these predictions have, in a century of experiments, been accurately confirmed by observations. The theory makes two additional predictions that yet await experimental confirmation. The first is the existence of black holes: collapsed stars that are so massive that not even light can escape their gravitational pull and where time itself is slowed down to a standstill. Black holes are the most extreme examples of curved spacetime that we know.

The second unconfirmed prediction that the General Theory of Relativity makes is the existence of gravitational waves: microscopically small vibrations of space and time that are produced when two large masses move in each other’s close proximity. These vibrations travel through the Universe with the speed of light, and we can reveal their presence by closely observing the relative position of two masses. Just as two bobbers will wobble with respect to each other when a little wave of water disturbs the pond, two masses will wobble with respect to each other when a gravitational wave disturbs space and time. The relative motion of the two masses is usually extremely small (the wobbles are typically of the order of a millimoth of the size of a proton), and is biggest when the source of the gravitational wave has a very strong field of gravity. The production of the biggest gravitational waves is therefore expected to happen close to a very massive black hole.

Gravitational waves produced in this way contain a treasure trove of information about the black hole and allow us to test the General Theory of Relativity. It is for this reason that measuring gravitational waves is one of the biggest current challenges in physics. At this very moment, experiments such as VIRGO in Italy and LIGO in the United States are working to measure gravitational waves, and plans are in development to continue the effort underground in the upcoming Einstein Telescope, and in space in the upcoming LISA satellite experiment. In all cases, it is of absolute necessity to know in advance the exact shape of the gravitational waves in order to filter out the very small gravitational waves from the data collected by such experiments. This means that these must be calculated using the General Theory of Relativity.

It is not an easy task to calculate gravitational effects close to a black hole, as the mathematics needed to understand the General Theory of Relativity is very complicated. As such, solutions to the formulas are usually found only approximately. One of the ways that researchers do this is by first assuming that the gravitational field around the black hole is weak enough so as to describe it by Newton’s Universal Law; the effects due to Einstein’s curvature of space and time are subsequently added as corrections to Newton’s solutions. A disadvantage of such a method is that it becomes less accurate when the star gets very close to the black hole. After all, it is in that region that the curvature of space and time
Ptolemy (left) described the apparent motion of the planets, Sun, and Moon around the Earth by placing circles on top of circles, as is shown on a page (right) from his book *Almagest*.

is the most extreme and Newton’s gravity does not suffice anymore. This is unfortunate, as it is also exactly this region where the strongest gravitational waves are produced. In this thesis we present a novel method to calculate the gravitational waves produced when a star moves in close proximity to a black hole, in which we do not make the assumption that the gravitational field is weak. In our method, we assume instead that the orbit of the star around the black hole is simple enough to be described by a circle. By subsequently adding corrections to this circular orbit, we obtain the equations for more general orbits. The outcome of our calculations turns out to be akin to the system that the ancient Egyptian sage Ptolemy (90-168) proposed to describe the apparent motion of the planets, Sun, and Moon around the Earth (which he thought to be at the centre of the Universe). He placed the planets on circles, on top of which he placed smaller circles called epicycles, in the manner shown in Figure 2. Ptolemy’s system is, of course, not correct (he made the incorrect assumption that the Earth is at the centre of the Universe, and he had no knowledge of the Theory of Relativity), but our research has shown that the motion of a star around a black hole can be described in a way very similar to Ptolemy’s method. In our context, the corrections applied to a circular orbit are not themselves circles, but bear a more complicated shape that we have calculated accurately. We call our model *Relativistic Epicycles*.

In this model we never make any compromise on the strength of the gravitational field of the black hole: we do not assume it to be weaker than it really is. As a result, we expect our predictions for the orbit of the star to be accurate even when the star and the black hole are very close to each other. We found that this is indeed the case: as long as the orbit of the star does not deviate too much from a perfectly circular orbit, our results have an accuracy of more than 99%, and this regardless of how close the star is to the black hole.
...and the resulting gravitational waves

As the orbits of the star around the black hole could be accurately calculated, the next step in our research was to calculate the gravitational waves due to the star’s motion in the gravitational field of the black hole. This turns out to be a complicated mathematical challenge: the first steps were already taken in 1957 (half a century ago!), and the final formalism was only published in 2004. This formalism requires that the orbit of the star is known as a function of time, and exactly this is provided by our method of Relativistic Epicycles. The resulting formulas for the gravitational waves we have subsequently solved by using a computer program that we have written ourselves. The gravitational waves we calculated in this way agree very well with the ones that were already known in the literature, and new ones we can calculate effortlessly and rapidly. Here too we found that the accuracy of our method is excellent, with accuracies being of the order of 99% when the orbits do not deviate too much from perfect circles.

Finally, as the last step in our research, we have investigated the limitations of our method of Relativistic Epicycles. The main disadvantage of our method is that we need to assume that the orbit of the star is close to circular; we have indeed found that our results become less accurate when the orbits become more eccentric. However, our calculations have also shown that, as the system sends out gravitational waves, the star’s orbit becomes increasingly less eccentric and the predictions of our method are therefore rendered increasingly accurate. This means that the main disadvantage of our method is naturally nullified by the emission of gravitational waves! We therefore conclude that the method of Relativistic Epicycles is very well suited to describe the production of gravitational waves due to the motion of a star around a black hole, even when the star and black hole get in extremely close proximity.

Future research

There are numerous ideas for future research. For example, we could improve the accuracy of the Relativistic Epicycle even more by adding some more corrections on the orbit. Secondly, up to this point we have only taken into account the curvature of space and time due to the presence of the black hole, but it would be in order to also take into account the curvature due to the star. It would also be interesting to investigate whether the calculation of the gravitational waves could be done without invoking a computer, by replacing the outcome of the program by a mathematical formula. Finally, our calculations have shown that the method of Relativistic Epicycles also applies to the situation of electrically charged masses moving around a pulsar (which is a heavy star that is surrounded by a magnetic field that, like a lighthouse, periodically sends out flashes of light). The latter possibility is very interesting, as it allows the study of astrophysical objects by not just looking at the gravitational waves that they send out, but also at their electromagnetics waves. We have already taken the first steps in that direction, which will be the basis for future research.