CHAPTER 1

Introduction

Credit risk can be defined as a possibility that a contractual counterparty does not meet its obligations stated in the contract with the result that the creditor suffers a financial loss. Risk management for credit risky assets is a vast field, and presenting all the major aspects in detail is beyond the scope of this thesis. I focus on one specific class of problems: capturing dependence within portfolios of defaultable counterparties. The class of models used for this purpose can be roughly divided in two.

First, we have factor models combined with a conditional independence assumption. In a nutshell, this means that we postulate the existence of a latent factor driving the credit behavior within a portfolio. Usually, this factor is interpreted as the state of the credit market. Intuitively, in “good” times we see less companies entering default, and in “bad” times the effect is reversed. I contribute here by showing how the existing methodologies can be improved to provide more accurate forecasts of the default rates. Apart from the novelty in the theoretical approach, this result has two important practical aspects. First, I show that the credit market has its own dynamics, only mildly corresponding to those of the general state of the economy. Second, I extend the basic Hidden Markov Model\(^1\) by observable covariates to improve the fit to the data – this translates to more appropriate assessment of the capital buffer, which a financial institution needs to cope with credit-related losses. Third, I present a case study on quantile forecasting, which is a prediction method encouraged by the current banking regulations. I show, that if the model specification differs from the actual Data Generating Process (DGP), estimates of high quantiles of the default rate distribution can vary rapidly – potentially leading to either over- or under-estimation of the required capital buffers.

\(^1\) A generic class of state space models combining flexibility with analytical tractability. More detailed discussion is deferred to Chapter 2
Second, we can model the risky assets through their joint cumulative distribution functions (cdf) and more specifically via copulas. Copulas allow us to separate the marginal behavior from the joint dependence structure of the random vector of interest. This allows for an accurate modeling of joint extreme co-movements of the assets, which are of primary importance in credit risk modeling (as default itself is an extreme event). In this context I introduce a new flexible family of skewed-$t$ distributions, which allows for capturing asymmetric behavior in the tails of the distributions. I derive analytical formulas for the tail dependence coefficients, which makes it possible to quantify the risk of extreme co-movements under such models. This distribution class is later applied to the problem of modeling Economic Capital for a portfolio of credit risky assets. The results of the conducted study highlight the importance of region and industry diversification of credit risk.

Both of these models are static in a sense that they focus on the unconditional distribution of the asset portfolio. A different approach is taken in the final chapter, where we propose a dynamic extension of an Archimedean copula class, suited for modeling time-varying tail dependence in a parameter-driven manner. I show, how the familiar apparatus of Importance Sampling can be used to estimate such a model and how the dynamic version can help gain deeper insight into the economic properties of the series in question.

The remainder of this introductory chapter is structured in the following manner. In Section 1.1 I present the historical approach to credit risk assessment. Section 1.2 discusses the most popular industry models for credit risk management, with emphasis on their shortcomings, which continue to motivate research in the field. The regulatory environment, formed by the two Basel directives, is introduced in Section 1.3. Also, I provide a review of the existing literature in the field. This allows me to position my results against the research area in general. This chapter is concluded by two sections explaining in more detail the motivation behind the interest in models forming the two main parts of this thesis: Hidden Markov Models (Section 1.4) and copulas (Section 1.5).

1.1. A BIT OF HISTORY

Although credit risk management has bloomed in the last decade or so, it was not by any means the start of its existence. We review classes of models belonging to the traditional approach. For a more detailed exposition, the reader is referred to Caouette et al. [1998].
Historically, the first ones to emerge were **expert systems**: the credit decision is left to a local branch of the bank. While the choice of possible criteria is vast, one the most common approaches is the "five Cs of credit". Those factors, subject to evaluation by an expert, are:

1. **Character** A measure of the firm’s reputation, willingness to pay back its obligations and repayment history.

2. **Capital** The equity contribution of the company owners and the leverage (equity to debt ratio) of the firm.

3. **Capacity** The firm’s ability to repay its obligations, based on the volatility of the earnings.

4. **Collateral** Should a company in question default, a banker has claims on the collateral pledged by the borrower.

5. **Cycle Conditions** The state of the business cycle is an important factor in determining the credit risk exposure on a loan, especially if the company operates in an industry prone to economic-cycle effects.

In addition the "five Cs", experts tend to take into account the level of interest rates. Although many banks still use the expert systems, the role of such models is decreasing. This is mostly due to criticism coming from two angles: first, there is a problem of **consistency**: What are the common factors to analyze across different classes of borrowers except for the "five Cs"? The second problem is **subjectivity**: what are the optimal weights applied to the five factors described above? Potentially, those weights can vary between borrowers to an arbitrary degree. Consequently, comparability of rankings and decisions is very difficult.

As an attempt to resolve this consistency problem, computerized expert systems have been introduced. The most common type of those are **credit scoring systems**. The underlying idea is similar in the previous case: identify key factors affecting the default probability and combine them with weights to form a quantitative score. In some cases the result is the default probability itself, in others –a number allowing for ordering the loans according to quality.

Based on the functional form of the mapping used to translate the factor values into a score, we can distinguish four types of multivariate credit scoring models: linear probability, logit, probit and discriminant analysis. The most widely known example belongs to the last class. The Altman Z-score model – introduced in Altman [1968] – is a classification
model for corporate borrowers, which uses several ratios describing the company’s financial parameters. Once the score has been calculated, its value is compared to a cutoff point to make a credit decision. Cutoff points trade off the risk of granting a loan to a “bad” customer to the cost of increasing the chance of denying credit to a “good” customer.

Much like the other historical models, this one has come under a certain amount of criticism as well. First, the model is linear in the variables, while the relationship between the variables in question may be highly nonlinear. Second, most of the factors entering the formula are accounting ratios. As this type of data usually appears at discrete intervals, there is a risk of missing out on the shocks to the firm’s stability. Finally, there is a problem of the model not sending a warning signal fast enough. This has become a major concern with the increase in complexity of the financial markets in the last two decades. In addition, certain economists have raised the problem of economic interpretation of the values comprising the output of the model.

A class of models belonging to the group, but somewhat distinct from logit or probit models, is formed by artificial neural networks (NN). The underlying idea behind those was to form an automated decision-making system, that incorporates human knowledge by analyzing a database of decisions made. An NN simulates the human learning process and learns the nature of the relationship between input and output data by repeatedly sampling the input/output information sets. Since this field of Artificial Intelligence research is huge – and growing – we refer the reader to one of many books describing the topic from a formal perspective, e.g. Haykin [1998].

A number of studies have been dedicated to the problem of NN performance in the credit risk context: Hawley et al. [1990], Kim and Scott [1991] and Altman et al. [1994], among others.

A major issue with neural networks is their lack of transparency – intermediate steps of the system cannot be checked by e.g. banking supervision, so accountability is lacking. Although NN models perform well in prediction tasks, they do not reveal the relative importance of the variables, so model interpretation is cumbersome in this approach.

Finally, a widespread approach commonly employed in market, which has been developed by regulators: rating systems. Banks use the external, as well as internal ones, which allow a finer subdivision of a rating category. According to Fadil [1997], 50 percent of U.S. banks have developed rating systems of their own, including the top 50 finan-

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2In the sense: the variables defined in the model are entered into the formula as they, without transformation of any sort.

3provided by rating agencies such as Moody or Standard & Poor
cial institutions. Internal ratings are used for 96 percent of large and middle market loans.

Following the incentives provided by Basel II (see Section 1.3), more banks can be expected to adopt internal rating systems. As in the previous cases, certain points of concern can be raised here, including the integrity of the rating system, consistency of ratings across different financial industries and the evolution and disclosure of optimal practices. See Griep and Stefano [2001] for a more elaborate discussion of the topic.

1.2. INDUSTRY MODELS

Deriving from both the historical models of the previous section and academic research, different financial institutions have come up with default prediction models of their own. Below we present a brief overview of those. For an extensive discussion the reader is referred to Saunders and Allen [2002].

Given how well the option pricing theory has been developed after the introduction of the Black-Scholes formula, it is not surprising there have been attempts to apply the same type of apparatus to credit risk. The seminal work in this field is Merton [1974]. His crucial contribution was to observe a formal equivalence between issuing a loan bearing credit risk and writing a put option on the assets on the borrowing firm. The default event occurs if the value of the firm’s assets falls below the market value of the liabilities at a given time horizon. The default probability is the area under the probability distribution of the assets to the left of the default point, while distance to default (DD) is the number of standard deviations between current asset value and the default point.

In subsequent years, his ideas have been extended in many directions – most notably, in the form of Structural Models. Prominent examples include KMV Credit Monitor, Moody’s Public Firm Model and JP Morgan CreditMetrics. The fundamental idea is to treat the loan as an option on the company’s assets. In order to price it correctly, we need to determine asset volatility – however, in practice only the volatility of the equity can be observed. This is where the uncertainty comes in structural models: if we knew the asset price, default events could be predicted with certainty.

In the KMV approach, a structural default model is applied to the credit history database in order to determine an empirical cdf of the assets by examining default likelihood for any given DD level. Moody’s approach amounts to incorporating ratings and financial variables along with a risk-neutral expected default frequency in an artificial NN. Credit-
ItMetrics is based on the Value at Risk for a given loan. We cannot observe the asset’s market value nor its standard deviation directly, but they can be estimated based on the borrower’s credit rating, the rating transition matrix, recovery rates and credit spreads in the loan market.

The main advantage of those models is the utilization of stock data, which permit prediction and are sensitive to changes in the firm’s financial conditions. On the mathematical side, the underlying models rely on jump-diffusion processes – see Hanson [2007] for an introduction to this class of processes.

Reduced form models assume, that the observed yield on a risky debt can be decomposed into a risk-free rate and a risk premium. This relationship is utilized to compute default probabilities, recovery rates and risky debt prices. The credit spread consists of the risk-neutral probability of default multiplied by the loss given default. In the KPMG’s Loan Analysis System, this information is utilized to compute the price of untraded loans. In case of the Kamakura’s Risk Manager the KPMG approach is extended to using the liquidity premium and carrying costs, which are included in bond spreads, to determine the credit spread. CreditPortfolio aims at incorporating business cycle effects and take a forward-looking view at VaR through a construction of a conditional migration matrix within a portfolio. The latter process has three main components: explanatory processes (identification of relevant macro variables), speculative default rate processes (stochastic relationship between macro variables and default rates for different industry segments) and shifting factors (transforming unconditional into conditional migration matrices). The main improvement of this class of models over structural ones, like KMV, is the relatively low computational cost and better fit to observed data. A disadvantage is the lack of an economic model explaining default causality.

The crucial difference between those two approaches lies in the predictability of the default event. In structural models, if we know the asset process exactly, the default event can be predicted with certainty. On the other hand, the stochastic nature of the default process is inherent in reduced form models and perfect prediction is not possible.

Another trend in recent literature is the use of actuarial survival models applied in the credit risk context. The key to applying actuarial mortality models is making the link between “death and the default event - see Kavvathas [2001]. Again, modeling dependencies requires special attention in credit risk in insurance, multiple lives are typically assumed to be statistically independent. Li [2000] developed a method of gluing single exposure using a Gaussian copula and the methodology of this seminal work has been widely used in CDO pricing.
1.3. BASEL I & II

The phrase Basel Accords (Basel I and Basel II) refers to the banking supervision recommendations on banking laws and regulations issued by the Basel Committee on Banking Supervision (BCBS). The organization has a secretariat with the Bank of International Settlements (BIS)\(^4\) in Basel.

The Basel Committee consists of representatives from central banks and regulatory authorities of the G10 countries, Luxembourg and Spain. It does not have the authority to enforce recommendations, although most member countries tend to implement the Committee’s policies.

The first directive (Basel I) issued in 1988, was of crucial importance to the industry, as it aimed to unify credit risk capital requirements across the world’s major banking industries. Banks assets were classified and grouped in five categories according to credit risk, carrying risk weights of zero,\(^5\) ten, twenty, fifty, and up to one hundred percent (most corporate debt). Banks with international presence were required to hold capital equal to 8% of the risk-weighted assets – irrespective of the borrower quality or credit rating. This lack of distinction among commercial loans was the primary source of criticism of Basel I, as it led to mis-pricing of commercial lending risk.

The intended goal of the new Basel Capital Accord of 2002 (Basel II) is to correct the mis-pricing problems inherent in Basel I and make bank capital requirements more flexible by incorporating risk-sensitive credit exposure measures. Following the exposition of Hammes and Shapiro [2001], we can point to several factors motivating the introduction of Basel II:

1. Changes in the credit market at a structural level
2. Improving credit market efficiency
3. Inflation of debt levels during expansion periods in the economy, combined with problems with servicing debts, which occur during recessions (“the bad credits” syndrome).

Basel II follows a multi-step procedure in modeling credit risk. First, we have the standardized model. This approach is conceptually similar to Basel I, but is more risk sensitive, since the obligors are divided into

\(^{4}\)which is sometimes (incorrectly) referred to as the organization responsible for the Accords

\(^{5}\)home country sovereign debt – note, that this occurred a decade before the 1998 Russian crisis
risk classes of varying default risk. The risk weights are defined via external credit ratings assigned by rating agencies. Credit risk mitigation is taken into account by adjusting the transaction’s Exposure At Default (EAD) to reflect a number of financial and economic factors. The assessments of creditworthiness are based on the estimates from the credit rating agencies. There is increasing concern about the quality of these estimates: there is no actual competition in the credit rating industry, which is dominated by few major players.\footnote{Since the obligors are free to choose a rating agency, there is a risk of moral hazard: a rating agency may be subjective (biased upwards) in a bid to obtain purchases. For a more elaborate discussion, the reader is referred to Altman and Rijken [2005] and}

A second possible direction for the bank is to take the internal rating based (IRB) approach, which requires the bank to formulate its own rating system. The risk weight assigned to each commercial obligation is based on the rating assignment, so that higher rated obligations have lower risk weights. Under IRB, each bank is required to obtain estimates of probability of the default (PD) and EAD for each transaction. If the advanced IRB is in use, additional data requirements include Loss Given Default (LGD) and maturity (M). The expected loss is then calculated as a product of PD, EAD and LGD. Banks are encouraged to shift from foundation to advanced IRB: the main incentive comes from the bank’s ability to use actual LGD data instead of fixed assumptions. Also, the bank is allowed to use its own credit risk migration estimates to adjust PD, LGD and EAD. Moving to the advanced approach is expected to reduce the capital requirements of a bank.

Basel II introduces more sophistication in measuring credit risk as compared to Basel I, so there is definitely a step in the right direction. In addition, it moves the regulatory capital towards the economic capital. For that reason, it is widely regarded as one of the main reasons behind the rapid growth of credit risk models in the past years. Moreover, the current IRB approach does integrate elements of the portfolio perspective, which is an improvement compared to Basel I (see Hamerle et al. [2003] for a general introduction).

Around the time of the introduction of the regulations, the primary concern was increasing the flexibility of risk management practices of banks (in particular, lowering the overestimated capital buffers). Events of the second half of 2007 (widely referred to as the credit crunch or sub-prime crisis) show, that non-modeling issues can affect the perception of what an appropriate level of capital buffers is. Undoubtedly, once the proper assessment of losses suffered by banks in the subprime crisis has been conducted, this shall become a topic of further research. As of
1.4. WHY HMM

Dependence of defaults within a given portfolio is an important issue in credit risk management. Proper modeling requires capturing the timing, as well as the total number of defaults. Dynamic models of default rates have gained popularity in recent years. There are three main reasons for this phenomenon.

First, there is empirical evidence that rating transitions – and default probabilities in particular – vary over time and co-move with general economic conditions. Bangia et al. [2002] and Nickell et al. [2000], among others, show that downgrades are more likely to occur during recession than in expansion times. As a result, capital buffers required for coping with credit losses need to vary over time as well. Second, financial markets have grown increasingly more liquid in the last decade. Subsequently, financial institutions have focused their risk management practices more on active management of credit portfolios. Finally, as discussed in Section 1.3, the Basel II Accord allows banks to perform many risk management tasks using internal models. Here dynamic models for default probabilities can provide a more efficient use of capital over stages of the business cycle.

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Moody’s, S & P and Fitch dominate the market, which has an oligopoly structure due to high entry barriers and importance of reputation.
Figure 1.1: Default rate in the U.S. economy over the sample period.

Figure 1.1 shows the proportion of defaults among U.S. industrials recorded quarterly between 1981 and 2005. If we denote the number of “alive” companies during time period $t$ by $N_t$ and the number of defaulted ones by $D_t$, then the ratio $D_t/N_t$ shows us a default rate – as presented in the Figure. A visual inspection of the graph suggests, that the defaults are not independent over time – high and low default quarters appear to be clustered.

The model we propose in Chapter 2 captures these dynamics through a hidden Markov model (HMM), with the regime-switching probabilities depending on observed macro variables. This way, we are extending the model proposed in Diebold et al. [1994] to the credit risk context.

In the credit risk context latent state models have an advantage over those based solely on observables. Often there is little or no theory as to which factors would be optimal as proxies for systematic credit risk. This problem is avoided in the HMM setting. The (hidden) states of the Markov chain we propose correspond to the state of the credit market. We distinguish between normal and risky (“excited”) credit market

\[\text{We analyze the default rate instead of raw numbers as they come for two reasons: first, the portfolio sizes can vary significantly between applications. Second, since the beginning of the 1980s (that’s when properly gathered data sets tend to start) the size of the market has grown – so there are far more companies available in a portfolio. In case of the data set we analyze in Chapters 2 and 3, we are talking about a shift from 1300 to over five thousand.}\]
conditions. In risky credit market states, default probabilities increase. The transitions between the states are governed by probabilities that depend on selected macroeconomic variables.

The class of HMMs has been popularized by Hamilton [1989] and has proven useful for modeling and forecasting a variety of economic time series. Over the last years, several authors have implemented HMMs to model and forecast credit risk dynamics. Crowder et al. [2005] uses the familiar model of Hamilton [1989] with two regimes: a high and low default rate regime. In the model of Crowder, the hidden Markov layer is observed via a binomial layer, where the binomial draw is the number of defaults. Banachewicz et al. [2006] extend the model of Crowder by making the transition probabilities in the MC dependent on macro-variables.

A different approach is taken by Wendin and McNeil [2005] and Koopman and Lucas [2005]. They distinguish a continuum of possible states, where the state follows a simple time series process such as a low order autoregression. As in Crowder et al. [2005], the observed time series of default counts is modeled as a binomial process, where the success probability depends directly on the hidden Markov layer. Koopman et al. [2006] further extend this approach to a continuous time setting, where the hidden layer drives the intensity of a point process.

The main drawback of the HMMs is that they are more cumbersome to estimate. Instead of straightforward maximum likelihood, one has to resort to simulated maximum likelihood, EM, or Bayesian methods. This is why some previous authors have used observable state variables instead of a hidden layer. For example, Nickell et al. [2000] use GDP growth rates, while Bangia et al. [2002] use NBER business cycle classifications to distinguish between high and low default rate regimes. Koopman et al. [2006] and Duffie et al. [2007], however, show that even if one includes observable macro variables, a hidden layer is still needed to capture default rate dynamics correctly. Moreover, Lucas and Klaassen [2006] show that default regimes based on observables like GDP growth and NBER business cycle classifications result in a substantial underestimation of the annual default rate volatility. This is in line with Dacco and Satchell [1999], who show that a slight mis-classification of regimes can lead to a substantial deterioration in forecasting performance of regime switching models. We address this issue in Chapter 3.

The main conclusion from the literature is that HMMs constitute a promising tool for dynamic credit risk modeling. So far, however, no systematic study has appeared that compares the adequacy of the proposed competing HMM specifications for forecasting. The models from the literature differ in the number of regimes they distinguish,
in the hidden layer dynamics, and in whether they have a discrete or continuous state space.

Based on the model formulated in Chapter 2 we address a number of issues arising in the portfolio credit risk context. First, we investigate how much information on credit markets is contained in macro variables like GDP, interest rates and financial markets returns. Intuitively, in an expanding economy we should observe a decrease in default risk. We verify this by estimating the model in two versions. One model has a simple latent risk state as in Crowder et al. [2005]. The other model has an observable part (macro factors) and a hidden part (interpreted as the condition of the credit market). In addition we analyze whether the industry sector influences the impact of economic conditions on default risk. The current approach with a latent component is less prone to the choice of incorrect macro proxies, as signaled in Lucas and Klaassen [2006]. Because the credit state is a latent component, we can still have a systematic credit risk factor at the portfolio level even though all macro variables are incorrect. The model will then collapse to the basic HMM of Crowder et al. [2005].

Based on the HMM for defaults, we can construct an early warning mechanism for high default probability regimes. The importance of such regimes for setting capital buffers was clearly illustrated in Bangia et al. [2002] in a switching model context. We confront our predictions with the NBER classification of business cycle states. In this way we can analyze the dependence between the business cycle (behavior of the general economic variables) and the credit cycle (fluctuations of the recovered hidden credit risk state process), both in particular sectors and in the economy as a whole. We only find a mild correspondence between the two, indicating that credit and business cycles can have their own separate dynamics. Our current model classifies credit market conditions into a finite number of different levels for default intensities. This makes the model easy to estimate using standard methods like the EM algorithm. Our approach complements related papers that either use observed rather than hidden regimes, such as Nickell et al. [2000], or a continuous number of states for economy-wide default intensities, see Koopman et al. [2005].

Next, we address the issue of diagnostics for the HMM of the type discussed in Chapter 2. The focus in Chapter 3 is on the effect of miscalibration in the hidden layer’s state space dimension and dynamics on the quality of quantile forecasts. This appears particularly important given the limited number of time series observations typically available for default rate modeling: annual, quarterly, or monthly time series since the 1980s. For such data, it is far from trivial to reliably determine the
appropriate dimension of the state space or the dynamics in the hidden Markov layer from the data.

We focus on forecasting quantiles rather than expected values. This is in line with the predominant use of quantiles rather than expected values for risk management purposes in the financial industry. Following the Basel II Accord, quantiles can be used directly by financial institutions to determine the size of required capital buffers. It will turn out that the quality of quantile forecasts may be affected in a substantially different way by mis-specification than forecasts of means.

We contribute to the existing literature in three ways. First, we provide a systematic comparison of the forecast accuracy of different non-Gaussian state space models for credit risk. In particular, we are the first to compare both discrete-state and continuous-state HMM specifications as they have been put forward in the recent literature. Second, we concentrate on the effect of mis-specification in hidden layer dynamics on quantile forecasts. And finally, we apply the various methodologies to an empirical example.

We conduct a controlled simulation experiment, where we vary the number of regimes. We systematically study the effect this has on quantile forecasts. We find that underestimating the number of regimes has a significant impact on forecast quality. For the high quantiles typically used in practice, the differences appear economically significant. Overestimating the number of regimes appears to have less effect. Surprisingly, however, even correctly specified models have difficulty in adequately estimating the high quantiles of the true distribution of future default rates.

The continuous state models behave substantially differently from the discrete state models at high quantiles. Typically, continuous state models result in much higher quantiles. If the true data generating process (DGP) has a discrete state space, the continuous state model vastly overestimates the high quantiles. Conversely, if the true DGP has a continuous state space, the discrete state models substantially underestimate high quantile default rates.

We also apply the different methods to an empirical data set of U.S. corporate default rates, obtained from Standard and Poor’s. We find that a discrete state model with three states provides a good empirical description of the data. In terms of forecasting performance, the continuous state model picks up the dynamics of the realized default rates better than the discrete state model. As usual, however, the forecasts lag the realizations if we forecast further out of sample. The main problem with the continuous-state specification is that its high-quantile forecasts are very large. Due to the lagging behavior of the forecast with respect
to the realization in the more than one step ahead context, these large
predicted quantiles might result in overly conservative default scenarios
and, thus, in overly conservative capital requirements.

1.5. WHY COPULAS

One of the main interests of the credit risk manager is the impact
of extreme events on the portfolio of the financial institution. More
specifically, we are interested in answering the following question: can
it happen in a balanced portfolio that a disproportional number of dif-
ferent companies default? If so, are we capable of making statistical
judgements about those extreme events? It is well known, that such phe-
omena are not well modeled using multivariate normal distributions -
see Frey and McNeil [2003]. Introducing copulas into the mathematical
setting of the problem is an attempt to overcome this difficulty. The
theory of copulas is well established in statistics. A copula is basically
a function describing the relations between the multivariate probability
distribution of a random vector and the individual univariate (marginal)
distributions.

In recent years, this theory has been widely recognized for its use-
fulness by the financial practitioners and academics – resulting in a
multitude of papers, both from applied and theoretic perspective.

The model we propose in Chapter 4 is something of a compromise
between the approaches mentioned above. On the one hand, it allows for
asymmetry and extreme dependence, thus overcoming the well-known
fallacies of the multivariate normal distribution. It can be viewed as
an extension of the grouped-\(t\) copula proposed by Daul et al. [2003] so
that asymmetry can be introduced. On the other, it retains analytical
tractability due to the fact, that is based on a mixture type of
construction.

After formulating the model and analyzing its extremal properties,
we turn to an application. In credit risk, concentrated exposure to a
certain factor is considered one of the most dangerous problems and
there is an ongoing discussion about the relative importance of country–
and industry-related effects. This problem can be analyzed on many
levels.

First, one can be interested in the impact of diversification on ex-
pected returns. This problem has been addressed in the literature by
a number of authors. Wang et al. [2003] find, using a factor model,
that industry effects have dominated country related ones for index re-
turns. Equity returns have been analyzed by Ammer and Wongswan
[2004], who analyze both the cash flow and discount rate components of equity returns. An interesting view on the country effects in the real estate context is provided by Glascock and Kelly [2007] – the authors analyze the property type vs country-based diversification and find the latter to be more effective for risk management purposes. Nijman et al. [2004] address the problem of investigating the momentum effect in excess stock returns and the determinants of this phenomenon. Their regression model allows one to distinguish between individual stock, industry and country effects by taking those effects into account simultaneously.

As far as the impact of economic sectors and company domiciles on correlation is considered, Serra [2000] focuses on emerging markets. She shows, that emerging markets returns are mainly driven by country factors and that cross-market correlation is not affected by the industrial composition of the indices. However, ignoring the industrial mix deteriorates the diversification effect. A similar study, although with different conclusions, is Sener and Salavitabar [2004]. Analyzing Pearson correlation coefficients, they find that industry effects dominate over country effects.

The problem of concentration effects and related exposure risk in the credit context has been addressed in e.g. Dullmann and Masschelein [2006], who show how the exposure to a country or industry factor can affect the Economic Capital. Jarrow and van Deventer [2005] provide an excellent review of the analysis of the diversification problem in terms of its importance for systematic risks in defaultable portfolios.

As can be seen from this brief review, country and industry effects have been analyzed from different perspectives. However, the same type of problems is present in the context of extreme dependence and, to the best knowledge of the authors, has not been investigated in such detail. This is the gap we intend to fill.

There is a number of practical reasons behind focusing on this particular problem. First, it is the joint extreme (downward) movements of the assets that pose the biggest threat of generating an unexpected loss. Second, a proper modeling of the tail dependence translates directly into more accurate estimates of the Economic Capital. Mis-specification can result in capital buffers being either too conservative, or worse, insufficient in case of an emergency. The sub-prime mortgage crisis of 2007 is an example of financial institutions not being sufficiently prepared for such a fatality.

Factor models have been employed in credit models before, see Schönbucher [2001] for a review. Their applicability is widely recognized, as they combine dimension reduction and intuitive interpretability. We
take the factor model approach and merge it with the copula approach of Li [2000].

A dimension reduction procedure has to be introduced in a rigorous manner. We proceed in two steps. First, each counterparty in a portfolio is mapped to a performance index, that is country and industry specific. A second step involves modeling the dependence structure of the explanatory indices. For that purpose, we propose a general mixing construction – a so called meta-\textit{t} distribution, first proposed by Daul et al. [2003]. This flexible copula-based method allows for different modifications and is particularly suited for our credit risk application later on.

Our approach is a counterpart to the research of Nijman et al. [2004]. The authors of that study focus on expected returns and correlations. Our research addresses a similar problem in the extreme regions of the joint return distribution. By conducting a regression type of analysis, we can determine whether region- or industry-related effects are more relevant for inducing extreme dependence.

Our model is applied to the analysis of Economic Capital, which is a crucial quantity in credit risk management. As documented in a number of studies, credit risk concentration has been a reason behind several bank failures in recent years. Heitfield et al. [2006] identify sector concentration as the main contributor to Economic Capital for portfolios of varying sizes.

Chapter 5 contributes to the existing literature in two ways. First, we address the issue of clustering in the context of extreme dependence. For that purpose, we model the data with the grouped-\textit{t} copula. We use a method of moments approach to test whether industry or country effects are more important determinants of the dependence.

Second, we apply the model to the problem of Economic Capital (EC) in a controlled simulation environment. We analyze the impact of different dependence structures on the portfolio distribution quantiles. We find dramatic increases in the EC as the popular Gaussian model is replaced with different members of the meta-\textit{t} family. Even more interestingly, allowing for heterogeneity in tail dependence in either the country or region dimension, or in both, substantially increases the Economic Capital beyond that based on the simple Student-\textit{t} copula.

The two Chapters discussed above introduce a time-invariant model and apply it to analyze the unconditional distribution of assets in a credit risky portfolio. However, time-varying models have also been very popular in the field. As mentioned in 1.4, temporal dynamics of the assets dependence is widely recognized as one of the main factors governing the behavior of the capital buffers. Time series models based on copulas
have grown increasingly popular in recent years due to their flexibility in capturing extreme dependence of random variables combined with dynamical properties. This allows for a more focused analysis.

The core reference in this field are the seminal results obtained in Patton [2002] and extended further in Patton [2006], who proposed a way to adapt the theory of unconditional copulas to the conditional case. This way, time-varying dependence structure could be modeled in a direct manner. The methodology is applied to modeling dependence between Deutsche Mark and yen exchange rates against a U.S. dollar. This line of research has been followed by e.g. Dias and Embrechts [2004] for examining structural changes in foreign exchanges data and Palaro and Hotta [2004], who investigated risk management issues.

Another approach to time varying copulas is Bielecki et al. [2006] who propose a Markov copula: the changes in the parameters are governed by an underlying Markov Chain. This model is further applied to pricing credit index derivatives. Chollete and Heinen [2006] uses a similar HMM-based setting to investigate high- and low-correlation periods for equity indices in international markets. Dueker [1997] uses stock market volatility forecasts as predictions of option-implied volatilities with the same apparatus. An interesting study on contagion in international financial markets, which combines the two approaches (GARCH-and HMM-based), is Rodriguez [2007].

The conditional copula approach of Patton [2006] is observation driven. The transformed residuals are the driving force behind fluctuations in the extreme dependence structure. By contrast, the HMM of Bielecki et al. [2006] is parameter driven, but only distinguishes two different copula states. The problem of constructing parameter-driven models with continuous states has not been addressed in the copula environment. This is the gap we intend to fill.

The best way to understand the importance of the parameter-driven setting is an analogy with classical results in time series models for volatility. First, we have the class of GARCH models, introduced by Engle [1982] and then generalized by Bollerslev [1986]. These models have become an industry standard. The idea to link the observable volatility clustering to the actual series has attracted considerable interest and spawned numerous extensions over the last two decades. This class of models is entirely observation-driven.

Another way to account for shifts in volatility regime is to postulate a Hidden Markov type of regime governing the shifts in volatility. This approach allows for a greater flexibility in modeling the time-dependent variance. The main conceptual difference between this model class and
the class of GARCH models is the presence of an underlying Markov Chain which is the driving force behind the dynamics.

Finally, there is a vast class of stochastic volatility models. An excellent introduction to the topic is provided in Ghysels et al. [1996]. The main difference from the dynamics discussed above is that we have two random processes driving the evolution of the volatility – see Kim et al. [2003] for a more elaborate discussion of the topic. This allows for more accurate fit and prediction, at the price of more complexity in the estimation process. Due to a lack of closed expressions, computational techniques such as importance sampling have to be called upon to make maximum likelihood methods numerically feasible.

In Chapter 6 we propose a parameter-driven stochastic copula model aimed at accurate modeling of extreme dependence in the lower tail of portfolio asset distributions. This type of co-monotonicity is of particular importance for diversification purposes in the credit risk management context.

We contribute to the existing literature in two ways. First, we propose a time-varying copula model that is parameter-driven. This approach is novel and has not been addressed in the existing literature. We also show how the parameters can be estimated using the familiar importance sampling methodology. Second, we provide an empirical example illustrating the time evolution of the lower tail dependence coefficient. This application is crucial from the credit risk management point of view and complements the existing research on stochastic correlations. Unlike the observation-driven models described before, our model can easily accommodate high-dimensional problems, as it does not make any special use of the bivariate structure.