This thesis deals with a variety of problems arising in credit risk modeling, defined as a possibility that a contractual counterparty does not meet its obligations stated in the contract with the result that the creditor suffers a financial loss. While the management of credit risk is a vast field, I focus on a specific class of problems: capturing dependence within portfolios of defaultable counterparties. The class of models used for this purpose can be roughly divided in two.

In the first class, we have factor models combined with a conditional independence assumption. The latent factor is interpreted as the state of the credit market. There are three main reasons for the usefulness of this class of models in credit risk applications. First, there is empirical evidence that rating transitions – and default probabilities in particular – vary over time and co-move with general economic conditions. As a result, capital buffers required for coping with credit losses need to vary over time as well. Second, financial markets have grown increasingly more liquid in the last decade. Subsequently, financial institutions have focused their risk management practices more on active management of credit portfolios. Finally, the Basel II Accord allows banks to perform many risk management tasks using internal models. Here dynamic models for default probabilities can provide a more efficient use of capital over stages of the business cycle. The main conclusion from the literature is that HMMs constitute a promising tool for dynamic credit risk modeling.

I contribute here by showing how the existing methodologies can be improved to provide more accurate forecasts of the default rates. Apart from the novelty in the theoretical approach, this result has two important practical aspects. First, I show that the credit market has its own dynamics, only mildly corresponding to those of the general state of the economy. Second, I extend the basic Hidden Markov Model by observable covariates to improve the fit to the data – this translates to more appropriate assessment of the capital buffer. Third, I present a case study on quantile forecasting, which is a prediction method encouraged by the current banking regulations.

In the second class of models, we can capture the risky assets behavior through their joint cumulative distribution functions (cdf), and more specifically - via copulas. Copulas allow us to separate the marginal behavior from the joint dependence structure of the random vector of interest. This allows for an accurate modeling of joint extreme co-movements of the assets, which are of primary importance in credit risk modeling. More specifically, we are interested in answering the following question: can it happen in a balanced
portfolio that a disproportional number of different companies default? If so, are we capable of making statistical judgements about those extreme events?

In this context I introduce a new flexible family of skewed-\(t\) distributions, which allows for capturing asymmetric behavior in the tails of the distributions. I derive analytical formulas for the tail dependence coefficients, which makes it possible to quantify the risk of extreme co-movements under such models. This distribution class is later applied to the problem of modeling Economic Capital for a portfolio of credit risky assets. The results of the conducted study highlight the importance of region and industry diversification of credit risk.

Apart from the time-invariant model discussed above, dynamical copula models have been increasingly popular in the field. We contribute to the existing literature in two ways. First, we propose a time-varying copula model that is parameter-driven. This approach is novel and has not been addressed in the existing literature. We also show how the parameters can be estimated using the familiar importance sampling methodology. Second, we provide an empirical example illustrating the time evolution of the lower tail dependence coefficient. This application is crucial from the credit risk management point of view and complements the existing research on stochastic correlations. Unlike the observation-driven models described before, our model can easily accommodate high-dimensional problems, as it does not make any special use of the bivariate structure.