Dynamics of Reaching for Stationary and Moving Objects: Data and Model

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The nature of the interrelations among movement amplitude, movement time, and peak velocity was addressed in 2 experiments in which participants reached for stationary and moving objects. Movement time was found to scale with the distance between the hand and the object at the onset of movement but to be relatively independent of object speed. Peak velocity, however, was found to scale with both these variables. The origin of these interrelations cannot be understood within the framework of existing trajectory formation models that are based on optimization procedures. A dynamical perspective in which the movements are considered in an object-attached coordinate frame allows for their emergence. This is demonstrated by simulation of a nonlinear model, built up from Rayleigh and Duffing components, with the nonlinear dissipative parameter being associated with amplitude scaling.

Human prehension may be regarded as the coordinated unfolding of a reaching component and a grasping component. Whereas grasping involves preshaping of the hand in preparation for contact with the object, the task of the reaching component is to bring the hand to the right place in space and time (i.e., within adequate spatiotemporal vicinity of the target object). In focusing on the reaching component of prehension, as we do in the present article, we pursue two long-term goals: First, such a focus allows exploration of the extent to which reaching movements made in the context of prehension resemble reaching movements made in other contexts, with the latter having received much more experimental attention. Second, a thorough understanding of the reaching component is a sine qua non for the formulation of models of coordination in prehension (cf. Zaal, 1995; Zaal, Bootsma, & van Wieringen, 1998).

As is the case for reaching in general, a large number of the studies directed at an understanding of the mechanisms underlying the production of reaching movements in the context of prehension involved manipulation of specific task constraints such as movement amplitude (MA) and object size. These studies indicated first of all that the kinematics of reaching movements are highly stable over repetitions under the same experimental conditions. Furthermore, there are several consistent findings that involve changes in experimental conditions. (a) Longer acceleration and deceleration times (and thus longer overall movement times [MTs]) and higher peak velocities (PVs) were found when the distance between the initial hand position and the target object (defining MA under such conditions) was increased; these effects were found when movements were performed as quickly as possible (Bootsma, Marteniuk, MacKenzie, & Zaal, 1994; Jakobson & Goodale, 1991; Marteniuk, MacKenzie, Jeannerod, Athènes, & Dugas, 1987; Servos, Goodale, & Jakobson, 1992) as well as when they were performed at a preferred velocity (Gentilucci, Castiello, Corradini, Scarpa, Umlitâ, & Rizzolatti, 1991; Gentilucci, Chieffi, Scarpa, & Castiello, 1992; Jeannerod, 1981, 1984). (b) Shorter deceleration times (and thus shorter MTs) were found when object size was increased (Jakobson & Goodale, 1991; Marteniuk, Leavitt, MacKenzie, & Athènes, 1990; Marteniuk et al., 1987); Zaal and Bootsma (1993) showed that this latter effect was due to accuracy demands decreasing with increasing object size, and Bootsma et al. (1994) indicated that the effect of object size and MA had on MT combined in a way predicted by Fitts' law, allowing a generalization of the latter from aiming movements to reaching movements in the context of prehension. Also in line with research on reaching in general, sudden, discrete changes in task parameters such as target location (Gentilucci et al., 1992; Paulignan, MacKenzie, Marteniuk, & Jeannerod, 1991) and object size (Castiello, Bennett, & Stelmach, 1993; Paulignan, Jeannerod, MacKenzie, & Marteniuk, 1991) have been used to test the models of trajectory...
Movement Planning

In the cognitive movement-planning perspective, dependent on the model in question, specific aspects of the movement are planned prior to execution. Although adaptation of the ongoing movement on the basis of reafferent information is possible, such adaptations are supposed to be relatively small for feedback loops to remain stable (Hogan, Bizzi, Mussa-Ivaldi, & Flash, 1987). A well-known class of movement-planning models is formed by optimization models1 for a review, see Latash, 1993, in which the trajectory is determined through minimization of some cost function over MT. With respect to the reasons underlying the interrelations among MT, MA, and PV, a major problem with such models is that they do not account for the required a priori estimation of MT and its relation with task demands such as MA. Within the framework of cognitive movement-planning theory, Hoff (1994) is one of the few to have addressed this problem, proposing a two-stage cost function evaluation capable of generating MTs together with movement kinematics on the basis of specific boundary conditions. However, because in the second stage, MT sets the boundaries for the integral that is part of the cost function, information on MT prior to movement execution is still needed. If MT were really to emerge from the optimization process itself, the integral could not be involved in this very process.

An alternative approach is found in an encompassing model, called Knowledge II, presented recently by Rosenbaum, Loukopoulos, Meulenbroek, Vaughan, and Engelbrecht (1995) for the planning of reaching movements. In that model, stored representations of postures are assigned weights reflecting their judged effectiveness for reaching the specific target location, both in terms of spatial accuracy and travel costs. The target posture is derived from vectorial summation of the weighted stored postures, and a movement trajectory is achieved by progressively reducing the distance between the starting angle and the target angle of each joint. Unfortunately, although Knowledge II offers an original solution to the problem of selecting the final posture and determining movement direction for individual joints, the fact that the transition from initial to final posture is modeled as resulting from an optimization process reintroduces the problems signaled above.

Movement-planning models of trajectory formation have been demonstrated to be capable of dealing with sudden discrete changes in task parameters such as target location (Flash & Henis, 1991; Hoff, 1994). Basically, the procedure proposed consists of summation of the initial trajectory and a new trajectory derived on the basis of the new target location. Although such a solution seems quite acceptable for sudden changes in target location, it is evidently less suitable for conditions in which the target continuously changes location, because this would require ongoing planning and summation. In addressing such continuous target motion conditions, Rosenbaum et al. (1995) suggested that a future interception location needs to be anticipated, so that a reach directed to this location can be timed to end when the object arrives there. Movement-planning models thus require that both the future interception location and the time remaining until the object will arrive at this location need to be estimated prior to movement execution.

Emergent Movement

In the present study, in which we concentrate on reaching for stationary as well as continuously moving objects, an alternative approach is followed, one that draws its inspiration from dynamical systems theory (Beek, Peper, & Steegman, 1995; Jordan & Smith, 1987; Kelso, 1995). The latter approach has been most successful in accounting for cyclical movements (cf. Haken, Kelso, & Bunz, 1985), but recently it has also been applied to discrete movements (Saltzman & Kelso, 1987; Schöner, 1990). Inspired by this approach, we propose that interception of objects does not require anticipation of the interception location, but rather results from a pursuit-tracking-like strategy. Whereas the overwhelming emphasis in the literature on pursuit tracking is on its accuracy in terms of temporal and spatial error scores, Peper, Bootsma, Mestre, and Bakker (1994) formalized a procedure in which continuous tracking on the basis of a currently required velocity allowed interception of a moving object without the actor's knowing when and where contact would occur (for similar information-based movement regulation models, see also Bootsma, Fayt, Zaal, & Laurent, 1997; Bootsma & Peper, 1992; Michaels & Oudejans, 1992; and Todd, 1981).

In our approach to prehension, reaching to grasp a moving object is conceptualized as resulting from the interplay of attractor states in a dynamic landscape, reminiscent of what is proposed in equilibrium-point models of the control of reaching (Bizzi, Hogan, Mussa-Ivaldi, & Giszter, 1992; Feldman, 1986). With respect to the kinematics of human movement, two attractor types are relevant. The attractor could either be a point attractor or a limit cycle, where the former accounts for equifinality and the latter accounts for trajectory stability (Saltzman & Kelso, 1987). A succession of both attractor states would lead to the combination of trajectory stability and equifinality. Schöner (1990) suggested that discrete movements are the result of an interplay between fixed-point and limit-cycle regimes of a single

1 An example in the context of prehension is Hoff and Arbib's (1993) model of trajectory formation, which includes minimization functions for both the reach and the grasp components of prehension.
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Dynamical system. In this framework, both initial and final posture are modeled as stable fixed points; movement from initial posture toward posture at the target position is modeled as resulting from an intentionally stabilized limit cycle (by means of an intentional parameter resetting), followed by a destabilization of this limit cycle and subsequent relaxation toward the target posture. It is worth emphasizing that by scaling (in Schöner’s model, a discrete scaling) of a single parameter, the dynamics give rise to either stable posture or stable movement. Timing in this model is the result of the unfolding of the (intrinsic) dynamics. Experimentally observed relations among MA, PV, and MT can be understood as stemming from the underlying dynamical structure. In Schöner’s model, for instance, the increase in MT with increases in MA (as described by Fitts’ law) is an emergent property of the (adequately parameterized) dynamics. As a matter of fact, this model needs only one input (frequency) for both MA and PV to emerge. Of special interest for the present purposes is Mottet’s (1994; Mottet & Bootsma, 1995) attempt to give a dynamical account of Fitts’ law. By modifying Schöner’s model, Mottet succeeded in simulating not only the speed—accuracy trade-off but realistic velocity profiles as well. The dynamic proposed comprises a (velocity-driven) Rayleigh escapement component together with a quintic Duffing spring term. Although Mottet’s model was designed to accommodate cyclical movements, it can be modified to generate discrete movements as well.

To illustrate the dynamical account we propose for reaching in the context of prehension, we present generic phase plane representations of hand velocity during reaching as a function of hand position in Figure 1A. In these representations, semicircular reaching trajectories travel from the left—the point of movement initiation—to the right—the point of target acquisition. The phase plane trajectories are slightly skewed to the right, representing a relatively longer deceleration phase in the movement. As mentioned above, the relations among MA, MT (not directly represented in this phase plane representation), and PV are robust in human reaching. In the case of target motion, however, relations observed in reaching for stationary targets will not hold. Consider the situation in which the target moves away from the hand at constant velocity. In this situation, the motion of the target determines the relation between MA and MT to a large extent: Successfully acquiring the target implies that the hand meets the target at some interception location (defining MA), whereas the moment that the target arrives at this location is, of course, determined by the target’s motion characteristics. The target motion characteristics thus impose an upper limit on MT that can be shortened by delaying movement initiation but can in no way be lengthened. Because the resultant MA—MT relation typically differs from the naturally observed MA—MT relation in reaching for stationary targets, all details of the hand’s velocity profile, including PV, need to be different. The exemplary trajectory of a reach for a moving target in Figure 1A assumes the same MT as in the reach for the stationary target. The resulting differences in MA and PV are obvious in this example. If movement aspects like MA, MT, and PV are an explicit result of prior planning processes, this planning must deal with changing relations for every single case of target motion. In the example, target speed needs to be assessed, an interception location (or equivalently, an interception time) needs to be chosen, and the kinematic details appropriate for this situation need to be implemented.

We propose an alternative mechanism for target interception. The key idea is that hand movement is controlled with respect to the target, whether it is moving or not. A dynamic is set up in which the target acts as an attractor (e.g., Saltzman & Kelso, 1987). Kinematic details, such as MA, MT, and PV, are to a large extent a consequence of the unfolding of the same dynamic in different situations, such as those involving stationary or moving target objects.

From the assumption that a dynamic is set up in which the target acts as an attractor, specific predictions concerning the relations among MA, MT, and PV as a function of the task situation can be derived. These predictions are illustrated in Figure 1B. The same two reaches depicted in Figure 1A are now plotted in an object-attached frame of reference, a coordinate frame that travels with the target. This is the appropriate frame of reference for considering the movement details, because the dynamic is assumed to be set up relative to the (moving) target. In the case of a stationary target, the two representations in a world frame of reference in Figure 1A and in an object-attached frame of reference in Figure 1B, are identical. In contrast, trajectories of reaches for moving objects clearly diverge in the world frame of reference, whereas they almost fall on top of the reaches for stationary targets in Figure 1B (which, again, is the key idea of the proposal). Small differences are present, however. The trajectory of the reach for a moving target starts at a small negative velocity because the object has already started to move away from the hand at the moment of movement initiation.

Figure 1. Fictitious phase plane trajectories of movements toward a stationary object and a moving object in a world frame of reference (A) and an object-attached frame of reference (B).
In order to appreciate our first prediction, it is important to realize that our hypothesis is that movements toward stationary targets and movements toward moving targets result from the operation of the same underlying dynamic. Differences between the two conditions in the kinematics of hand movement thus result from differences in initial conditions. In the object-attached frame of reference, the initial distance between the location of the hand and the location of the target is equal to the distance between hand and target at movement initiation. Our first prediction is therefore that MT scales with initial hand-object distance (IHOD), rather than with MA—the distance between initial hand position and interception location, which is different from IHOD in the moving-target condition (see Figure 1A). Moreover, in the case of a moving target, the initial velocity is different from that in the stationary-target condition, because the target motion creates an initial negative hand velocity in an object-attached frame of reference. Thus, we expect the relation between IHOD and MT in the case of a moving target to be slightly different from that in the case of a stationary target, because some additional time is needed to neutralize the initial negative hand velocity.

The second prediction concerns the relation between object speed (OS) and hand velocity. Considered in an object-attached frame of reference, PV in reaching for moving objects is slightly larger than PV in reaching for a stationary object. Once again, the small difference arises from the initial negative hand velocity in this representation. Because the difference between the two representations—the world frame of reference in Figure 1A and the object-attached frame of reference in Figure 1B—is related to the object motion (the object-attached frame of reference moves with respect to the world frame of reference with a speed equal to the OS), PV in the world frame of reference equals PV in the object-attached frame of reference plus OS. Our second prediction therefore is that the empirically observed PV in the moving-object condition should equal PV in the stationary-object condition plus OS plus some additional value to compensate for the negative initial speed. For the latter reason, the slope of the regression line of PV onto OS should be somewhat larger than unity. Because, as for MT, PV is hypothesized to depend on initial conditions, PV should scale with IHOD in both stationary- and moving-target conditions.

To test the predictions formulated above, in two experiments we investigated the effects of OS on the kinematics of reaching. In the first experiment we contrasted reaching for stationary objects with reaching for moving objects. In the second experiment we studied the combined effects of OS and IHOD.

**Experiment 1**

**Method**

Four right-handed men and 6 right-handed women, 20 to 31 years in age, volunteered to participate in the experiment. Participants were seated at a table on which sat a drafting plotter (Roland DG DPX-2200); the plotter’s rectangular drawing board, the longer side of which was aligned with the participant’s sagittal plane, was situated 85 cm above floor level. A 5-cm-diameter cylindrical wooden disk with a height of 2.5 cm was placed on the plotter’s arm (5.7 cm above the drawing board), which could move parallel to the longer side of the plotter’s drawing board. The experiment had nine different experimental conditions. In six conditions, the participants were required to reach for and grasp the stationary disk at distances of 20, 25, 30, 35, 40, or 45 cm from the initial hand position; in these conditions, the plotter’s arm did not move during the trial. In the three remaining conditions, participants had to reach for and grasp the disk while the plotter arm, and thus the disk, moved away from its initial position (20 cm from the initial hand position) at a constant speed of 25, 35, or 45 cm/s. In the latter conditions, the object thus receded from the reaching hand along the sagittal plane.

Participants were informed about the upcoming condition. They were instructed to wait for a tone on a pair of headphones (presented in the moving-object conditions 100 ms after the plotter arm had started moving and the object had reached constant speed) and then to pick up the object with the thumb and index finger as accurately and as quickly as possible. The object was to be lifted a few centimeters. In a practice session preceding the experiment proper, each condition was presented five times, with conditions presented in random order. During the experimental session, 10 trials were presented under each condition, the order of presentation of the 90 trials being randomized.

Data were collected with a two-camera Selspot system. The cameras were situated above and to each side of the workspace. Three infrared emitting diodes (IREDs) were sampled with a frequency of 315 Hz. These IREDs were placed on the center of the target object, the upper medial corner of the thumbnail, and the upper lateral corner of the index fingernail. Hand position was operationalized as the point halfway between the two latter IREDs. We assessed the accuracy of the Selspot system using a method similar to that proposed by Haggard and Wing (1990). Three IREDs arranged in a triangular configuration were sampled under each experimental condition at the initial hand position and at the target position (note that in three conditions the target was moving). Whereas real distances between the IREDs were 8.8, 6.2, and 6.4 cm, measured distances (and standard deviations) were 8.51 (0.11), 6.05 (0.14), and 6.24 (0.15) cm, respectively.

High-frequency noise was removed from the recorded data with a second-order recursive Butterworth filter with a cut-off frequency of 8 Hz. Subsequently, we obtained three-dimensional real-world coordinates using the direct linear transformation method (Abdel-Aziz & Karara, 1971). The movement of the point halfway between the thumb and the index finger, calculated for each sample as the mean of the position data of the thumb and index finger projected onto the horizontal plane, was taken as representing the reaching movement of the hand. We calculated velocity and acceleration of hand position using local second-order polynomials. The onset of hand movement was defined as the moment at which velocity exceeded 5 cm/s; the moment of movement termination was defined as the moment that grip-closing velocity fell below 7.5

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2 Note that changes in object speed occurring during movement of the hand would be considered as constituting changes in initial conditions in such an approach.

3 For movement termination, a criterion based on grasp termination was chosen so as to allow a fair comparison between stationary and moving-object conditions. An alternative would have been a reaching component velocity threshold, but this threshold would have had to be defined relative to object speed. For this purpose, the IRED attached to the object would be needed, which would imply an extra noise source in the analysis.
cm/s. For technical reasons, 13 trials were removed from the data set, which left a total of 887 trials available for analysis.

**Results**

Velocity profiles of a single participant are presented in Figure 2 for each condition. For the stationary-object conditions, reaches show an increase in MT as well as an increase in PV with increasing MA. For the moving-object conditions, an increase of PV with increasing OS is observed, whereas MT varies only slightly. As depicted in Figure 3, these relations are clearly present in the MTs and PVs, both averaged over trials and participants, as well.

**Relation MA—MT.** A repeated measures analysis of variance (ANOVA) on the MTs in the stationary-object conditions revealed a significant effect of MA, $F(5, 45) = 243.46, p < .001$ (see also Table 1 and Figure 3A). Because Bootsma et al. (1994) suggested that prehensile reaching movements exhibit a speed—accuracy trade-off following the principles of Fitts’ law, we investigated the relation between MT and MA in the stationary-object conditions. First, a log-linear fit of MT onto MA, $F(1, 4) = 248.72, p < .001$, $R^2 = .984$, took the following form:

$$MT = 192.24 \ln (MA) - 216.49.$$  

(1)

$R^2$ ranged from .970 to .992 for individual participants. For reasons of comparison, both a linear relation, $F(1, 4) = 5.237.86, p < .001, R^2 = .999$, and a power relation,

$$MT = 6.26MA + 242.12,$$

(2)

$$MT = 98.08MA^{0.437}.$$  

(3)

Individual $R^2$ values ranged from .927 to .995 for the former relation and from .947 to .998 for the latter relation. Although the curve-fitting results on the averaged data suggest a better fit in the case of a linear relation between MA and MT, inspection of the individual $R^2$ values revealed that in 8 out of 10 participants the best fit was established using a power function; in 2 other cases the linear relation resulted in the highest $R^2$ values. The distance manipulation in this experiment apparently did not result in a range of MT data that speaks clearly in favor of any of the three explored relations. However that may be, the relation between MT

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4 Fitts’ law predicts a linear relation between movement duration (MT) and an index of difficulty, defined as $\log_2 (2MA/W)$, with movement amplitude $MA$ and width $W$. Bootsma et al. (1994) demonstrated that in prehension the available surface area for object-finger contact reflects the width parameter. Since we used cylindrical objects in the present experiment, the available contact area is not readily defined. This area, however, is constant over conditions. Since we did not know the available area for contact, we were not able to compute indices of difficulty. We proceeded by investigating a general log-linear relation between MT and MA. This relation reflects the effects of the amplitude manipulations. By conversion of the $e$-based logarithm to a 2-based logarithm and substitution of a width parameter, this relation can easily be transformed into a truly Fitts’ law based relation.
and MA reinforces the notion that in the context of prehension, the reaching movement is subject to constraints similar to those found in other contexts.

Although in the moving-object conditions, resultant MA increased significantly with increasing OS, $F(2, 18) = 683.81, p < .001$, MTs in those conditions were not significantly different, $F(2, 18) = 2.43, p = .117$. As a result of the instruction to wait for the tone, the object had moved several centimeters before participants initiated their movement. Averaged over participants, the momentary distances between the hand and the object at the moment of hand-movement initiation were 22, 23, and 25 cm in the conditions with OSs of 25, 35, and 45 cm/s, respectively. Inspection of Figure 3A suggests that MTs in the moving-object conditions were just slightly larger than the MTs realized in the stationary-object conditions, with amplitudes comparable to the distance between hand and object at the moment of hand-movement initiation.

Relation MA–PV. In the stationary-object conditions, PV increased with increasing MA (see Figure 3B), $F(5, 45) = 365.28, p < .001$. A linear regression of PV onto MA, $F(1, 4) = 297.26, p < .001, R^2 = .987$, resulted in

$$PV = 3.38MA + 30.06,$$

with $R^2$ values for individual participants ranging from .961 to .998. Phase portraits showed the velocity peaks appearing to the left of the origin (i.e., skewed to the right). The distance from the phase plane origin (defined halfway between the initial hand position and the object position; see Figure 1) to the location of the velocity peak divided by half of the MA served as an index of asymmetry. As can be seen in Table 1, this index was essentially constant over conditions. Although an ANOVA indicated the existence of a marginally significant effect of MA, no systematic relation was found, $F(5, 45) = 2.27, p = .063$.

PV in the moving-object conditions did not fit exactly the relation with MA found in the stationary-object conditions (see Figure 3B). Note that if the objects are stationary, MA equals the IHOD and is thus determined before the movement starts. In the case of moving objects, the participants may vary their MA by moving faster or slower. Because the IHOD was approximately the same in each moving-object condition (i.e., 20 cm) and MT was fairly constant, MA increased with increasing OS (see Figure 3B). Note that in all conditions, resultant MA was much smaller than the participant's maximal reaching distance (which is over 120 cm; Carello, Grososky, Reichel, Solomon, & Turvey, 1989), demonstrating that the object was grasped well before it reached the limits of the participants' workspace. PV in the moving-object conditions increased with increasing OS, $F(2, 18) = 113.00, p < .001$, and, thus, with MA. Combining the data of both the six stationary-object conditions and the three moving-object conditions, $F(2, 6) = 238.59, p < .001, R^2 = .988$ ($R^2$ for individual participants ranging from .936 to .992), yielded the following equation:

$$PV = 3.41IHOD + 1.78OS + 29.02,$$

with a standard error of 0.09 for the OS coefficient, which indicates that this parameter is clearly different from unity.

Discussion

By manipulating object width and MA in a prehension task involving stationary objects, Bootsma et al. (1994) demonstrated that these variables combined in a way predicted by Fitts' law, indicating that this law could be generalized from aiming movements to the reaching component of prehension movements. The present data on MT as a function of MA in the stationary-object conditions again fitted the log-linear relation predicted by Fitts' law quite well, although the present data could be at least equally well described by a linear function and a power function. Because MT is used as an input parameter in optimization models,

Table 1

<table>
<thead>
<tr>
<th>Condition</th>
<th>MA (cm)</th>
<th>MT (ms)</th>
<th>PV (cm/s)</th>
<th>AR</th>
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<tr>
<td></td>
<td>$M$</td>
<td>$SD$</td>
<td>$M$</td>
<td>$SD$</td>
</tr>
<tr>
<td>Stationary objects</td>
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<td></td>
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<td>462</td>
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</table>
these models are not suited for explaining differences in MT among experimental conditions.

From a dynamical perspective, two predictions concerning the relations among kinematic variables were formulated. First, we argued that MT in the moving-object conditions would be related to the IHOD rather than to resultant MA. The results obtained were in line with this hypothesis: MT in the moving-object conditions was not significantly affected by OS. Although nonsignificant, the data reported in Table 1 suggest a slight increase in MT with OS. Such a result also reinforces the explanation proposed, because this is exactly what was expected on the basis of the differences in initial conditions. Our results corroborated the second hypothesis as well: PV scaled with IHOD as well as with OS. Moreover, the OS coefficient in the regression equation of PV onto OS was expected to be somewhat larger than unity: The quality of the fits in the regression equation was satisfactory, and the OS coefficient was indeed larger than unity. We address the value of this coefficient in the General Discussion. In sum, the observed influence of OS on the kinematics was in line with the hypothesis that kinematic characteristics of movements directed toward stationary and moving target objects are similar if considered in an object-attached frame of reference.

Experiment 2

Whereas the conditions of the first experiment allowed us to investigate the influence of OS on MT and PV, the hypothesized relation between MT and IHOD could not be directly assessed, because the IHOD was always a constant 20 cm. To address the relations between IHOD and both MT and PV, we conducted a second experiment in which initial position of the object and its speed were varied in a factorial design. The design of the second experiment enabled us to test the hypotheses that (a) MT scales with IHOD and not with OS and (b) PV scales with both of these variables.

Method

Six right-handed volunteers (2 men and 4 women, aged 23 to 30 years) participated in the experiment, which involved the same experimental set-up and apparatus as in Experiment 1. Participants were required to pick up the disk, which moved away from them with a speed of either 15, 25, or 35 cm/s, starting from a distance of either 20, 25, or 30 cm from the initial hand position. Again, a practice session consisting of 5 trials in each condition was followed by an experimental session with 10 trials in each condition. The 90 experimental trials were presented in random order. New tests showed that the distances between IREDs were again reconstructed satisfactorily: Real distances of 11.7, 8.0, and 8.00 (0.03) cm, respectively. Data analyses were identical to those in Experiment 1. For technical reasons, 8 trials were removed from the data set, which left a total of 532 trials available for analysis.

Results and Discussion

Figure 4 and Table 2 present observed average MT and PV values as a function of the resultant movement amplitude in Experiment 2. Object speed conditions are represented by open squares, solid squares, and open circles for the 15, 25, and 35 cm/s conditions, respectively. Larger initial distances between hand and object corresponded with larger movement durations and larger peak velocities.

Table 2

Means and Intra-Individual Standard Deviations, Averaged Over Participants, for Movement Amplitude (MA), Movement Time (MT), and Peak Velocity (PV) in Experiment 2

<table>
<thead>
<tr>
<th>Condition</th>
<th>IHOD (cm)</th>
<th>OS (cm/s)</th>
<th>MA (cm)</th>
<th>MT (ms)</th>
<th>PV (cm/s)</th>
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<td>M</td>
<td>SD</td>
<td>M</td>
<td>SD</td>
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<tr>
<td>35</td>
<td>39.9 1.1</td>
<td>436 27</td>
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Note. IHOD = initial hand–object distance; OS = object speed.

Figure 4. Movement duration (A) and peak velocity (B) as a function of the resultant movement amplitude in Experiment 2. Object speed conditions are represented by open squares, solid squares, and open circles for the 15, 25, and 35 cm/s conditions, respectively. Larger initial distances between hand and object corresponded with larger movement durations and larger peak velocities.
of OS, $F(2, 10) = .91, p = .435$. The interaction between IHOD and OS did not reach significance either. MA was affected by both factors: IHOD, $F(2, 10) = 2.169.13, p < .001$; OS, $F(2, 10) = 406.38, p < .001$. However, the interaction was again not significant. The hypothesis concerning PV was confirmed by the data as well. Both IHOD, $F(2, 10) = 167.03, p < .001$, and OS, $F(2, 10) = 104.10, p < .001$, significantly affected PV, although the interaction between these factors did not reach significance. A multiple regression analysis, $F(2, 6) = 296.62, p < .001, R^2 = .990$, resulted in the following equation:

$$PV = 2.77\text{IHOD} + 1.80\text{OS} + 35.28,$$

with a standard error of 0.12 for the OS coefficient. $R^2$ ranged from .944 to .993 for the 6 participants. This experiment, indeed, supported both hypotheses formulated from a dynamical perspective. MT scaled only with IHOD, whereas PV was dependent on both IHOD and OS.

Whereas in Experiment 1 we addressed the influence of OS by contrasting reaches to pick up both stationary and moving objects, in Experiment 2 we investigated the influence of two parameters with respect to object movement—IHOD and OS—in more detail. The results showed that, at least in the case in which an object moves away from the participant at a constant speed, differences in kinematics can be understood if movements are considered in an object-attached coordinate frame.

**General Discussion**

The two experiments reported here demonstrate that reaching for stationary objects and reaching for moving objects can be understood within the same framework from the perspective of dynamical systems theory. The hand can then be best described as moving in an object-related frame of reference (i.e., as if the hand is attached to the object by a springlike device). In the moving-object conditions, the coordinate system moves along with the object, and the velocity relative to the object will develop along the same principles as in the stationary-object condition. This implies that MT scales with IHOD in both the stationary-object and moving-object conditions, irrespective of OS, as was indeed found in the present study. The only proviso is that in the moving-object conditions, the participants start with a negative velocity relative to the moving coordinate system. This accounts for minor MT differences in the moving-object conditions. The relation between PV on the one hand and IHOD and OS on the other hand can also be interpreted within a dynamical framework. In accordance with the predictions, OS coefficients in Equations 5 and 6 were found to be larger than unity. The interpretation that differences between reaches toward stationary objects and reaches toward moving objects can be accounted for by differences in IHOD in the object-attached frame of reference was therefore confirmed by the data.

The results of both experiments demonstrate a pattern in the kinematics compatible with the proposed dynamical account for trajectory formation. Compared with an account in which kinematic details are planned explicitly, the predictions based on the present model are considerably constrained. Whereas no principled predictions about the relations among MA, MT, and PV can be derived from existing planning models (i.e., no unified account of these kinematic relations in reaching for stationary objects exists in the literature, let alone of the kinematics of reaching for moving objects), our pattern of results conforms to the derived predictions. At this point it is, of course, premature to argue that no planning model with explicitly planned movement details could be constructed to account for the data. Such a model, however, would need to include relations between different kinematic landmarks for any conceivable situation involving target motion. In contrast, predictions from the dynamical model can easily be derived for any target motion, including target acceleration and deceleration. The key hypothesis remains that movement is fundamentally controlled with respect to instantaneous target location, such that hand movement in an object-attached frame of reference is fundamentally the same in any situation. Further experiments involving unpredictable object motion could provide more evidence with respect to the relative merits of our dynamical model vis à vis planning models.

Above, we argued that the relations among MA, MT, and PV can be understood if reaches are considered in an object-attached coordinate frame. From this perspective the discussed relations emerge from the dynamics. Here we develop a dynamical model capable of showing the observed behavior. In its most general form, the equation of motion for such a model reads as follows (Beek & Beek, 1988; Jordan & Smith, 1987):

$$\ddot{x} + g(x) + f(x, \dot{x})\dot{x} = 0,$$  \hspace{1cm} (7)

with a grouping of the conservative terms in the function $g(x)$ and a grouping of the dissipative terms in the function $f(x, \dot{x})$. The model we propose is built up from five essential aspects of the observed reaching movement. Following the lead of Schöner (1990), we consider discrete movement kinematics to emerge from the alternation of a fixed-point regime and a limit-cycle regime in one and the same dynamical system. For reasons of convenience, the origin of the phase plane is defined halfway between the starting position and the end position of the reach. The existence of fixed points at the latter two positions requires the stiffness function $g(x)$ to be nonlinear. For the case of two symmetrically fixed points at either side of the origin, a third-order polynomial in $x$ suffices. Thus, first,

$$g(x) = \omega^2x + \alpha x^3.$$  \hspace{1cm} (8)

As a second feature, stability of movement trajectories is included in the model through the ability to exhibit limit-cycle behavior. As a result, function $f(x, \dot{x})$ must be nonlinear in nature. A third consideration involves the choice of a specific nonlinear dissipative function; several options exist
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(e.g., see Beek & Beek, 1988). When scaled to amplitude, our data show negative asymmetry ratios, implying that the phase portraits of observed reaches are skewed to the right (see also Mottet & Bootsma, 1995, and Zaal & Bootsma, 1995). Thus, a Rayleigh function is appropriate for our modeling purposes,

\[ f(x, \dot{x}) = \beta + \gamma x^2, \]  

which leads to the following equation of motion:

\[ \ddot{x} + \omega^2 x + \alpha \dot{x} + \beta \dot{x} + \gamma x^3 = 0. \]  

Using the harmonic-balance technique (Jordan & Smith, 1987), one can assess the frequency, amplitude, and PV of the limit cycle, assuming that the system is weakly nonlinear. Accordingly, the approximate solution for Equation 10 yields

\[ x = A \cos (\Omega t), \]  

with amplitude \( A \) and movement frequency \( \Omega \). Substitution of this solution in the equation of motion, expansion of the third-order sine and cosine terms, and subsequent discarding of the \( \sin(3\Omega t) \) and \( \cos(3\Omega t) \) terms results in

\[ -A\Omega^2 \cos (\Omega t) + \omega^2 A \cos (\Omega t) + \gamma \alpha A^3 \cos (\Omega t) \]

\[ - \beta A \dot{\Omega} \sin (\Omega t) - \gamma \alpha^2 \Omega^2 \sin (\Omega t) = 0. \]

By matching the sine and cosine terms we arrive at

\[ -\Omega^2 + \omega^2 + \gamma \alpha A^2 = 0 \]  

Equations 13a and 13b state that PV (proportional to \( A\Omega \)) is a function of the dissipative terms, whereas movement frequency depends on the conservative terms. Equation 13a can be used to compare the model with the data collected in Experiment 1. Thus, the fourth aspect included in the model is the amplitude–frequency relation as observed in the stationary-object conditions of Experiment 1. Because the complete model postulates both relaxation and limit-cycle behavior to account for movement kinematics, the oscillator frequency cannot be directly inferred from movement duration. If the movement is close to harmonic, however, Approximation 11 holds, and oscillator frequency \( \Omega \) can be arrived at by dividing PV by amplitude. According to Equation 13a, squared amplitude and squared frequency are linearly related for (relatively) constant values of \( \omega^2 \) and \( \alpha \), as was indeed found to be the case (see Figure 5; linear regression: \( \Omega^2 = 92.8 - 0.0592 A^2 \); \( F(1, 4) = 351.02, R^2 = .989 \); note that the dynamically defined amplitude \( A \) equals \( \frac{1}{2} MA \). Thus, for the present experimental conditions, \( \omega^2 \) and \( \alpha \) can be considered to be constant over amplitude scaling in reaching (\( \omega^2 = 92.8 \text{ s}^{-2} \) and \( \alpha = \frac{1}{2}(0.0592) = -0.0789 \text{ cm}^{-2} \text{ s}^{-2} \)). This is clearly in contrast with most dynamical models presented to account for rhythmical behavior (e.g., Haken et al., 1985; Kay, Kelso, Saltzman, & Schöner, 1987) and the theoretical work on the dynamics for discrete movement (Schöner, 1990), in which \( \omega^2 \) is the controlled parameter. Recently, however, the tenability of fixed dissipative terms was challenged in a study of unimanual and bimanual rhythmical forearm movements (Beek, Rikkert, & van Wieringen, 1996) in which a frequency-dependent Rayleigh term was needed to account for the observed PV–frequency relations. Where frequency scaling in rhythmical unconstrained movement seems a logical mode of control, our data show that in goal-directed movement, the controlled variable is related to the dissipative terms in the equation of motion.

As revealed by Equation 13b, in our model PV scales with the ratio of \( \beta \) and \( \gamma \). The former parameter is related to the strength of attraction of the limit cycle and determines the extent of its asymmetry, which is the fifth aspect explicitly modeled. This is illustrated in Figure 6, in which the asymmetry ratio (the distance from the origin to the location at which PV is reached, relative to the amplitude) is depicted for a range of realistic values of \( \beta \) and \( \gamma \). Clearly, the linear dissipative parameter \( \beta \) is responsible for the extent of the asymmetry. Because our results show a rather constant asymmetry over the range of amplitudes in the experiment.

\[ \frac{\gamma}{\omega^2 s/cm^2} \]

Figure 5. Comparison of the relation between squared amplitude and squared frequency as observed in the stationary conditions of Experiment 1 with the theoretical relation of Equation 13a. Linear regression resulted in the following: \( \Omega^2 = 92.8 - 0.0592 A^2; R^2 = .989 \).

Figure 6. Asymmetry ratio as a function of \( \gamma \) for values of \( \beta = -5.0 \text{ s}^{-1} \) (triangles), \( -6.0 \text{ s}^{-1} \) (circles), and \( -7.0 \text{ s}^{-1} \) (squares). Other model parameters: \( \omega^2 = 92.8 \text{ s}^{-2}; \alpha = -0.0789 \text{ s}^{-2} \text{ cm}^{-2} \).
leads to 

\[ x + \omega^2 x + \beta x = 0. \]  

In the case of \( \omega^2 > 0 \) and \( \beta < 0 \), the specific parameter values determine whether the origin is a node or a spiral, but in all cases the origin is a repeller (Jordan & Smith, 1987). Linearizing around the fixed points at \( (x = 0; \ y = 0) \), the points refer to the origin of the phase plane and points at a distance of 34.30 cm on both sides of the origin, respectively. Beyond these outer fixed points the nonlinear spring becomes repellent, implying that reaches with an amplitude larger than 68.60 cm are not accounted for by the model. Therefore, the basin of attraction to the limit cycle is bounded\(^6\) (see Figure 7). Note that these considerations apply to the limit-cycle regime of the model. The stability of these fixed points within the limit-cycle regime can be assessed by considering the linearized system around these fixed points. For the fixed point at the origin, linear approximation results in

\[ \dot{x} + \omega^2 x + \beta x = 0. \]  

These fixed points are saddle points for \( \omega^2 > 0 \) and \( \beta < 0 \) (Jordan & Smith, 1987). In summary, for \( \omega^2 > 0, \alpha < 0, \beta < 0 \) and \( \gamma > 0 \), no stable fixed points exist and, within limits, trajectories are attracted toward a limit cycle.

Above, we concentrated on the limit-cycle regime of the model equations. Equation 15, however, also enables the assessment of the stability of the equilibrium points, which are located symmetrically around the origin, in the fixed-point regime. The condition that trajectories terminate at stable nodes implies that \( \omega^2 < 0 \) and \( \beta > 0 \) (Jordan & Smith, 1987). In order for the fixed point to exist at all, \( \omega^2 \) and \( \alpha \) must be of different signs, implying \( \alpha > 0 \). The transition from limit-cycle regime to fixed-point regime, thus, involves at least a change of sign of three parameters in this model. Whereas the limit-cycle properties of the model nicely represent the relations among amplitude, velocity, and frequency, and varying one specific variable results in the emergence of these observed kinematic relations, the transition toward a fixed-point regime seems to be more complicated. No efforts are made at this stage to further model this part.

We argued that modeling reaching from a dynamical systems perspective is appealing because one and the same framework provides an understanding of the kinematic characteristics of reaching to pick up stationary objects and reaching to pick up moving objects. The dependence of PV on OS is one of the predictions that emerges from this perspective. Indeed, our results show that the OS coefficients in regression Equations 5 and 6 are larger than unity. This is in line with the consideration that in the case of reaching toward a moving object, the trajectory in the object-attached reference frame starts with a negative velocity, such that this trajectory approaches the limit cycle from the outside. In the same vein, one should predict OS coefficients smaller than unity in the case of objects that come toward the participant. It is interesting that PV data reported by Chieffi, Fogassi, Gallese, and Gentilucci (1992) on reaching for and grasping objects approaching in the sagittal plane could be described by the equation \( PV = 2.27 \ \text{IHOD} + 0.83 \ \text{OS} + 31.77 \) (compare with Equation 5), thus confirming the prediction.

The proposed specific dynamical model enables the inspection of the PV–OS relation as well. A set of numerical simulations was performed in which, for a fixed IHOD (implying that all parameters were fixed in the model), initial

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\(^6\) This problem can be solved by adding a fifth-order polynomial term to the nonlinear stiffness function (Mottet, 1994; Mottet & Bootsma, 1995).
hand velocity relative to the object was varied (see Figure 8). In the case of a stationary object, the initial location in the phase plane would be on the negative side of the position axis, whereas in the case of a moving object, the initial location in the phase plane is situated vertically under the former (stationary) location (see Figure 8B). The difference emerging from the simulations between PV in the stationary-object situation and PV in the moving-object situation is rather small (see Figure 8A, no delay condition). Instead of a coefficient of 1.8 (Equations 5 and 6), the value of the OS coefficient would be about 1.1. The limit-cycle attraction is so strong that larger values of this coefficient are not to be expected if the trajectory starts with just a negative velocity relative to the situation with stationary objects. If, however, a delay of 200 ms—accounting for the time elapsing before the tone was presented plus a reaction time—is introduced in the simulations, the obtained order of magnitude of the coefficient is in accordance with the experimentally observed value (see Figure 8A, delay condition). Initial object location is shifted to the left relative to the zero delay situation (see Figure 8B). With the delay imposed, the simulations are in good agreement with the observed PV–OS relation (Equations 5 and 6).

Of course, the dynamical account proposed implies that information about object location is available during the movement. Although there is consensus that feedback gains need to be small in order for the system to remain stable, Bingham (1995) suggested that the need for small gains could well be less severe if feedback involves prospective information. We suggest that controlling hand movement relative to object motion is compatible with movement control on the basis of optical variables, such as (generalized) tau, which specifies a first-order time to contact (e.g., Bootsma & Oudejans, 1993; Lee, 1976; Zaal & Bootsma, 1995). In the case of reaching movement, the relevant tau variable is to be found in a combination of the relative rate of contraction of the optical gap between target and hand and the relative rate of contraction of the image of the hand (for details, see Bootsma et al., 1997; Bootsma & Oudejans, 1993; Zaal, 1995). Confronting participants with target motions that demand clear trajectory modifications could be fruitful in exploring the nature of the object-trajectory information used as well as its delays.

In conclusion, the dynamical systems perspective allows the interrelations among MA, MT, and PV that are to be observed under both stationary-object conditions and moving-object conditions to be accounted for in a principled and unified way, whereas existing movement-planning models (a) need an independent a priori specification of a number of these interrelations to deal with stationary-object conditions and (b) need additional a priori constraints (pertaining to the estimation of the future interception location) to deal with moving-object conditions. It is important to note that the predictions made concerning both the scaling of MT with IHOD rather than with MA and the scaling of PV with OS (with indication of the dependence of the scaling factor on the direction of object movement) were derived from a generic dynamical model and, therefore, do not depend on the specifics of the model developed. Nevertheless, the fact that a hybrid Duffing + Rayleigh form was derived from the present discrete movement data for reaching in the context of prehension as well as from continuous movement data for reaching in the context of aiming (Mottet, 1994; Mottet & Bootsma, 1995) suggests that the model proposed might be quite powerful.

References


binocular vision in prehension: A kinematic analysis. *Vision
Research, 32*, 1513–1521.
*Journal of Experimental Psychology: Human Perception and
Performance, 7*, 795–810.
account of reaching and grasping movements. Unpublished
doctoral dissertation, Free University, Amsterdam.
deceleration in prehension tasks. *Journal of Motor Behavior, 27*,
193–207.
Coordination in prehension: Information-based coupling of
reaching and grasping. *Experimental Brain Research, 119*,
427–435.

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