INFORMATION EFFECTS IN TRANSPORT WITH STOCHASTIC CAPACITY AND UNCERTAINTY COSTS*

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In this paper the impact of information provision to travellers using a model based on a stochastic equilibrium concept is analysed. A new analytical framework is proposed by including a specific term related to the costs of travel-time uncertainty, per se, into the generalised cost function. Different driver information models are analysed to assess the efficiency-improving features of driver information systems compared with the (optimal) first-best policy of road pricing. The results reveal that the potential of driver information systems as an efficiency-improving policy instrument are underestimated if costs of uncertainty are ignored.

1. INTRODUCTION

The congestion-relieving properties of providing drivers with traffic information have recently gained much interest, both in the public and academic domain. In the U.S.A., the E.U. and Japan, special research programmes are dedicated to investigating the impact of so-called telematics, while at the same time private firms are investing a significant amount of money to develop the necessary tools for implementing these technologies. As the feasibility of adding capacity to the existing infrastructure by means of building new roads is regarded to be rather low (due to environmental and financial concerns), emphasis has moved to using the current infrastructure as efficiently as possible (Boyce 1988; Emmerink et al., 1994).

Besides the provision of traffic information, road pricing is one of the other available instruments to improve the efficiency of existing infrastructure. In fact, a fluctuating road-pricing scheme (fluctuating with the level of congestion) can be shown to be the so-called first-best instrument in theory, since it is able to give drivers the financial incentive to behave (system) optimally. However, the political and social feasibility of road pricing has often been questioned (Emmerink et al., 1995a; Johansson and Mattsson 1995). On the other hand, the provision of traffic information is a second-best instrument. It cannot be proven that information

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provision will always lead to system-optimal road usage. In contrast, due to the existence of transport externalities, such as congestion, pollution, and safety, this will generally not be the case. However, owing to its user friendliness, public and political support for introducing information technologies in transport networks exceeds the backing of road pricing by far.

From an economic perspective, a thorough analysis of the discrepancy in efficiency between these first- and second-best instruments is essential. In this paper we will do so by focusing on two points. In Section 2, the welfare economic properties of information provision are analysed by answering the question: Does information provision lead to a strict Pareto improvement? Secondly, in Section 3, and related to the first point, attention is paid to the size of potential efficiency gains due to the provision of traffic information. This should lead to a better understanding of the key factors that determine the efficiency impact of driver information. Finally, Section 4 concludes.

This paper complements the work by Emmerink et al. (1996a, 1996b, 1996 (forthcoming)) by including a term related to the costs of travel-time uncertainty into the generalised cost function. In so doing, the model presented hereafter gains much in realism, as the importance of uncertainty costs has frequently been stressed in the literature (Arnott et al., 1990, Hendrickson and Kocur 1981, Noland et al., 1994). Furthermore, the work in this paper advances work by Arnott et al. (1991, 1996) in two respects. First, besides public information we also consider the case of club information. Club information deals with the situation in which only a specific group of travellers is provided with information. Second, attention will be paid to endogenous information provision, implying that travellers will acquire information only if the private benefits of the information exceed the private costs. Arnott et al. (1991, 1996) assumed that information is available for free for the whole population.

The analysis in this paper is applied to information in transport networks. Clearly, the analysis can be used for any congestible facility. One could, for instance, think of when and where to go on a potentially crowded beach; telephone calls in a congested communication network; and information on congested computer networks such as the Internet.

2. STOCHASTIC NETWORK EQUILIBRIUM MODELS WITH INFORMATION AND COSTS OF UNCERTAINTY

In this section, four models for road use in which drivers take costs of uncertainty into account are presented. Previous work (see Arnott et al., 1996, and Emmerink et al., 1996a) considered the case in which travel costs were solely determined by stochastic costs related to travel time. Costs resulting from uncertainty or risk were not explicitly considered. Here, it is assumed that travellers base their behaviour on the following generalised cost function:

\[ E(\text{travel costs}) = \alpha \cdot E(\text{travel time}) + \beta \cdot Sd(\text{travel time}) \]

where \( Sd \) indicates the standard deviation of the stochastic variable \textit{travel time}, and \( E \) denotes the expectation operator. The parameters \( \alpha \) and \( \beta \) can be interpreted as
the monetary value-of-time and value-of-uncertainty (or value-of-risk), respectively. This type of generalised cost function has previously been considered in the transportation literature; see for example, Arnott et al., (1990), Hendrickson and Kocur (1981), Noland and Small (1995), and Small (1982). These authors stressed the importance of including costs related to travel-time uncertainty in a generalised travel cost function. Empirical work revealed the important role played by uncertainty in travel behaviour. More precisely, the term $\beta \cdot \text{Sd(travel time)}$ in equation (1) refers to planning costs that uncertainty in travel time imposes on individuals. For instance, one could think of costs associated with disruption of meetings with coworkers, stress, and so forth (Noland et al., 1994). Such costs make individuals effectively risk-averse with respect to travel time. In economic terms, the model assumes that travellers have rational expectations of the first and second moments of the travel-time distribution. In other words, drivers perceive the correct mean and variance of the travel-time distribution. For peak-hour travellers it seems reasonable to assume that drivers have no misperceptions of either expected travel time or its variance.

In order to confine the analysis to relevant situations the parameter $\beta$ is restricted to values within the interval $[0, \alpha]$. First, negative $\beta$-values would imply risk-loving travelling behaviour; something that is rather unrealistic in the context of travel-time valuation. Second, $\beta$-values exceeding $\alpha$ would imply that reducing the uncertainty by a number of minutes is worth more than a reduction in travel time by the same amount. In the sequel, values of $\beta$ greater than $\alpha$ will only be used for purely illustrative purposes. In addition, the size of the $\beta$-parameter is also restricted by the observation that it is rather unrealistic to assume that the expected travel costs under uncertainty exceed the highest possible travel costs without uncertainty.

Four models will be described, representing four different starting points concerning the availability of information. In each of these, travellers are seeking to optimise the generalised costs of travelling. As we have imposed the assumption of rational expectations in terms of first and second moments of the travel-time distribution, the potential adverse effects of information provision due to over-reaction (Ben-Akiva et al., 1991) are ruled out. In contrast with the present study, in simulation studies drivers’ behaviour has generally been modelled using simple behavioural rules. In those models, overreaction might play an important role as the rational expectations assumption is not necessarily implied by those rules (Emmerink et al., 1995, Mahmassani and Jayakrishman 1991, Noland 1995).

In the first model, indicated as model $N$, there is no information available to the road users, and travellers base their trip-making decisions on expected travel costs. In the second model, model $I$, information on the actual traffic costs is available to all road users. Consequently, these will consider actual rather than expected costs in their trip-making decision. In the third model, denoted as model $P$, information is

\footnote{In fact, travellers with extremely large penalty costs for lateness might have a value of $\beta$ that exceeds $\alpha$. However, it seems rather unlikely that the whole population of potential travellers exhibits such extreme risk-averse behaviour.}
available to an exogenously determined fraction of the road users. Finally, in model E, information is modelled endogenously. In this model, the traveller's choice of being informed depends on the private benefits and private costs associated with the information. Drivers will then acquire traffic information when the internal private benefits derived from the information exceed the costs of information.

In the past, several studies have introduced stochasticity in the context of road transportation. For stochasticity in travel demand see, for example, d'Ouville and McDonald (1990), and Kay (1979). Stochasticity in capacity has been considered by Arnott et al. (1991, 1996), De Vany and Saving (1977), Emmerink et al. (1996a, 1996b, forthcoming), and Verhoef et al., 1996. We follow this trend and apply a recently developed methodology that explicitly treats travel time as random variables (Emmerink et al., 1995b). Uncertainty originates from stochastic shocks in the cost function. Information, then, provides travellers with realisations of these random variables. In order to keep the model tractable, it is assumed that the stochastic travel time functions follow a Bernoulli distribution. With probability \((1 - p)\) the travel time function is given by \(C^0(N)\), while with probability \(p\) travel time is given by \(C^1(N)\), where \(N\) denotes the level of road usage. As both \(C^0(N)\) and \(C^1(N)\) reflect travel time, these are increasing functions in \(N\). The two realisations of the stochastic travel time variables are referred to as state 0 (\(C^0(N)\)) and state 1 (\(C^1(N)\)). State 1 reflects the situation with low capacity (e.g., due to traffic accidents, road works, unpredicted lane closures); in state 0 capacity is relatively high. Research by De Rose (1964), Giuliano (1989), Golob et al. (1987), Hall (1993) and Lindley (1987) has emphasized the need to model stochastic, unpredictable congestion explicitly.\(^3\)

The relation between the travel time in state 0 (high capacity) and state 1 (low capacity) can be written as:

\[
(2) \quad C^0(N) < C^1(N) \quad \text{and} \quad \frac{\partial C^0(N)}{\partial N} < \frac{\partial C^1(N)}{\partial N} \quad \text{for all } N > 0
\]

Hence, evaluated at a given number of road users, both the travel time itself and the rate at which average travel time increases with an additional road user is higher under state 1.

Before presenting the four models in detail, a final remark concerns the elasticity of travel demand. In the models it is assumed that demand for using the transport network is elastic; this in contrast with most of the well-known work that makes use of the bottleneck model, where demand is assumed to be fixed (Al-Deek and Kanafani 1993, Arnott et al., 1990, 1991, 1992, 1994).\(^4\) In our model, the situation of fixed demand is just the limiting case where the elasticity of the demand function approaches zero. The inverse demand function is given by \(D(N)\), that is, at a cost level \(\kappa\), we postulate that \(D^{-1}(\kappa)\) road users will use the network.

\(^3\) In the transportation literature, this type of congestion is known as nonrecurrent congestion.

\(^4\) A few researchers have paid some attention to the case of elastic demand. See, for example, Arnott et al. (1993), and Ben-Akiva et al. (1986).
2.1. **Model N: No Information Available.** In order to implement the model based on individual optimising behaviour following equation [1], an expression for the standard deviation of the stochastic travel times is needed. By defining $\bar{C} = (1 - p)C^0 + pC^1$, it can easily be shown that:

\[
\text{Var(travel time)} = (1 - p) \cdot (C^0 - \bar{C})^2 + p \cdot (C^1 - \bar{C})^2
\]

\[
= p \cdot (1 - p) \cdot (C^1 - C^0)^2
\]

where $\text{Var}$ indicates the variance operator. It follows that:

\[
\text{Sd(travel time)} = \sqrt{p \cdot (1 - p) \cdot (C^1 - C^0)^2}
\]

because, by construction, $C^1 > C^0$, see expression [2].

The equilibrium condition for model $N$, where no information is available, is then given by:

\[
\alpha \cdot ((1 - p) \cdot C^0(N_N) - p \cdot C^1(N_N))
\]

\[
+ \beta \cdot \sqrt{p \cdot (1 - p) \cdot (C^1(N_N) - C^0(N_N))} = D(N_N)
\]

where $N_N$ denotes the equilibrium level of road usage of model $N$. The subscript is used to distinguish the equilibrium levels of road usage from this model with the ones to be presented hereafter. The left-hand side of expression [5] gives the expected travel costs, while the right-hand side denotes the willingness-to-pay for using the transport network. Clearly, for the marginal network user $N_N$, expected private costs should equal private benefits: he or she is indifferent between using the network or not.

Using the following explicit linear functions for $C^j(N)$ and $D(N)$:

\[
C^j(N) = k^j + b^jN \quad (j = 0, 1)
\]

\[
D(N) = d - aN,
\]

the equilibrium level of road usage $N_N$ is given by:

\[
N_N = \frac{d - \alpha \cdot k - \beta \cdot \sqrt{p \cdot (1 - p) \cdot (k^1 - k^0)}}{a \cdot b + \beta \cdot \sqrt{p \cdot (1 - p) \cdot (b^1 - b^0)} + a}
\]

where a bar indicates an expected value. As expected, an increase in the value-of-time ($\alpha$) and the value-of-uncertainty ($\beta$) leads to a decrease of $N_N$.

2.2. **Model I: Information Available for All Travellers.** In model $I$ it is assumed that all potential road users are perfectly informed on the actual traffic
situation. This implies that the travellers base their trip-making decision on actual rather than expected costs (as opposed to model $N$), and do not face any costs related to travel time uncertainty. The two equilibrium conditions that describe the model are given by:

\begin{align}
\alpha \cdot C^0(N^0) &= D(N^0_I) \\
\alpha \cdot C^1(N^1_I) &= D(N^1_I)
\end{align}

These conditions reveal that for the marginal network users $N^0_I$ (when state 0 occurs) and $N^1_I$ (when state 1 occurs) private costs are equal to private benefits in both states. Marginal user $N^1_I$ is indifferent between using the network in both states or just in state 0, while marginal user $N^0_I$ is indifferent between using the network in state 0 or not using it at all. The parameter $\beta$ plays no role in these expressions, since informed drivers do not face uncertainty costs. Clearly, there is no interaction between $N^0_I$ and $N^1_I$ as the road users are perfectly aware of the actual traffic conditions in model $I$.

When comparing models $N$ and $I$, it can be shown that $N_N < N^0_I$. Regarding $N^1_I$, two regimes can be distinguished:

\begin{align}
(1) \quad N^1_I < N_N < N^0_I &: \text{for relatively small } \beta; \\
(2) \quad N_N < N^1_I < N^0_I &: \text{for relatively large } \beta.
\end{align}

An analysis of the properties of model $N$ and model $I$, under the condition of linear demand and travel time functions, leads to Proposition 1, containing the most important results of a welfare theoretical comparison of the two models.

**PROPOSITION 1.** In a one-link network assuming linear demand and cost functions, and assuming that expression \[2\] holds, then due to information to all travellers: (1) expected road usage increases; (2) expected network travel costs decrease; (3) none of the road users is worse off and consequently, the system welfare in model $I$ does not fall short of system welfare in model $N$.

**PROOF.** Available upon request from the authors.

Some implications of Proposition 1 are illustrated in Figure 1 and Figure 2. On the $x$-axis, the travellers are ranked according to decreasing willingness-to-pay, while on the $y$-axis the additional expected net private benefits (owing to information provision) are shown. The additional expected net private benefits are defined as the expected net private benefits for model $I$ minus the expected net private benefits for model $N$. Using the assumptions of linear demand and cost functions, it can be shown that the additional expected net private benefits look either like
Figure 1 or like Figure 2.\(^5\) The kinks in these figures occur at the levels of road usage that are given by the solutions of the equations \([5]\), \([7]\), and \([8]\), respectively. At these ‘solution levels’ of road usage regime switches are taking place. For instance, travellers on the left-hand side of \(N_N\) will use the network in model \(N\), while travellers on the right-hand side of \(N_N\) do not use the network in model \(N\). Similarly, travellers on the left-hand side of \(N_I^1\) use the network in both states in model \(I\); travellers in the interval \(\langle N_I^1, N_I^0 \rangle\) use the network only in state 0 in model \(I\), while travellers on the right-hand side of \(N_I^0\) do not use the network in model \(I\).

Some observations are in order. First, depending on whether \(N_I^1\) is smaller or larger than \(N_N\), most benefits from information provision accrue to either travellers in segment \(II_1\) and some of those in \(III_1\) (in Figure 1), or those in segment \(I_2\) (in Figure 2). Second, the benefits depicted in Figure 1 and Figure 2 are the sum of the so-called internal and external individual benefits. External benefits are beneficial effects to a particular traveller due to changes in trip-making decisions by other travellers. For example, following Proposition 1, travellers who—dependent of the state of the network—always use the transport network in both model \(N\) and model \(I\) gain from a decrease in expected network travel costs. This is an example of an external benefit. However, travellers who have different trip-making decisions in model \(N\) and model \(I\) (for example, those in the interval

\(^5\) The ordinate values on the y-axis of Figure 1 and Figure 2 can be easily derived. For instance, the additional expected net private benefits of model \(I\) minus model \(N\) for traveller \(N_N\) follow from the following calculation:

- Expected private costs in model \(N\) for traveller \(N_N\) are given by \(D(N_N)\), see equation \([5]\).
- Expected private costs in model \(I\) for traveller \(N_N\) are given by \((1 - p)D(N_I^0) + pD(N_N)\), see equations \([7]\) and \([8]\). \((pD(N_N)\) in state 1 as in state 1 traveller \(N_N\) will not use the network.)

Hence, the decrease in expected travel costs between model \(I\) and model \(N\) is given by:

\[D(N_N) - ((1 - p)D(N_I^0) + pD(N_N)) = (1 - p)(D(N_N) - D(N_I^0)).\]
\( (N^I_l, N^I_0) \) in Figure 1 benefit from \textit{internal} decision-making benefits. This means that due to the information provided these travellers respond to the different levels of congestion (state 0 and state 1) by means of altering their trip-making decision. The distinction between internal and external benefits is important as it reveals that (rational) travellers are not willing to pay the whole amount of their additional expected net private benefits to become informed. For example, drivers in segment \( I_1 \) are not necessarily prepared to pay the whole amount of their information benefits, that is, \( D(N_N) - D(E(N^I_l)) \), to acquire information. Part of their information benefits is external in nature and is caused by changes in behaviour by other road users. In fact, drivers in segment \( I_1 \) are willing to pay a maximum of

\[
\beta \cdot \sqrt{p \cdot (1-p) \cdot \left( C^I(N^I_l) - C^0(N^I_0) \right)}
\]

to become informed. This amount represents the monetary costs of uncertainty that they are faced with. Furthermore, as theoretically shown in Emmerink et al. (1996a) and Haltiwanger and Waldman (1985), the marginal effect of information decreases with the fraction of users who are informed. Simulation experiments by Mahmassani and Jayakrishnan (1991) led to even stronger results. They showed that information benefits might actually become negative as the fraction of informed road users exceeded a particular threshold value. The theoretical model presented here, however, shows that the system welfare (measured as the sum of the individual benefits minus the sum of the individual costs) in model \( I \) does not fall short of the system welfare in model \( N \) (see Proposition 1). In other words, information increases network efficiency.

Figure 1 was also discussed in Emmerink, Verhoef, Nijkamp and Rietveld (1996a), where no costs of uncertainty were considered \((\beta = 0)\). When including
monetary costs of uncertainty ($\beta > 0$) however, the pattern of benefit distribution may change to the one shown in Figure 2 for sufficiently high values of $\beta$.

2.3. Model P: Information Available for an Exogenously Determined Group of Travellers. Thus far, it was assumed that either none (model $N$) or all (model $I$) potential road users are supplied with information on the actual traffic situation. Next, in model $P$, it is assumed that information is provided to an exogenously determined group of potential road users denoted by subscript $i$ (referring to informed), while uninformed travellers are denoted by subscript $u$. Respective inverse demand functions for these two groups of potential road users are given by $D_i(N_{P,i})$ and $D_u(N_{P,u})$, where as before a capital $N$ indicates the level of road usage and the subscript $P$ refers to the model under consideration. From these two groups, informed road users are perfectly aware of the prevailing (actual) travel costs and consequently do not face any uncertainty regarding the prevailing travel time; the cost component with the standard deviation is equal to zero. On the other hand, uninformed potential road users are not aware of any day-specific travel costs and base their behaviour on expected rather than actual costs. The three equilibrium conditions that fully describe model $P$ are given in expressions [10] to [12].

\[
\begin{align*}
(10) & \quad \alpha \cdot C^0(N_{P,i}^0 + N_{P,u}^0) = D_i(N_{P,i}^0) \\
(11) & \quad \alpha \cdot C^1(N_{P,i}^1 + N_{P,u}^1) = D_i(N_{P,i}^1) \\
(12) & \quad \alpha \cdot ((1 - p) \cdot C^0(N_{P,i}^0 + N_{P,u}^0) + p \cdot C^1(N_{P,i}^1 + N_{P,u}^1)) \\
& \quad \quad + \beta \cdot \sqrt{p \cdot (1 - p)} \cdot (C^1(N_{P,i}^1 + N_{P,u}^1) - C^0(N_{P,i}^0 + N_{P,u}^0)) = D_u(N_{P,u}^1)
\end{align*}
\]

Equilibrium conditions [10] and [11] state the equality between private costs and private benefits for the informed marginal network user $N_{P,i}^0$ (when state 0 occurs) and $N_{P,i}^1$ (when state 1 occurs). The third equilibrium condition reflects the equality between expected private costs and private benefits for the uninformed marginal network user $N_{P,u}^1$; network user $N_{P,u}^0$ is indifferent between using the network or not.

In order to assess the welfare economic properties of model $P$, its features are compared with model $N$. To do so, model $N$ is first presented for two groups of drivers. The equilibrium conditions of this model are straightforward:

\[
\begin{align*}
(13) & \quad \alpha \cdot ((1 - p) \cdot C^0(N_{N,i}^0 + N_{N,u}^0) + p \cdot C^1(N_{N,i}^1 + N_{N,u}^1)) \\
& \quad \quad + \beta \cdot \sqrt{p \cdot (1 - p)} \cdot (C^1(N_{N,i}^1 + N_{N,u}^1) - C^0(N_{N,i}^0 + N_{N,u}^0)) = D_i(N_{N,i}^1) \\
(14) & \quad \alpha \cdot ((1 - p) \cdot C^0(N_{N,i}^0 + N_{N,u}^0) + p \cdot C^1(N_{N,i}^1 + N_{N,u}^1)) \\
& \quad \quad + \beta \cdot \sqrt{p \cdot (1 - p)} \cdot (C^1(N_{N,i}^1 + N_{N,u}^1) - C^0(N_{N,i}^0 + N_{N,u}^0)) = D_u(N_{N,u}^1)
\end{align*}
\]
Clearly, the former group of ‘informed’ potential road users in model $P$ is here also basing its behaviour on expected costs; see expression [13]. Using the linear demand and stochastic link travel time functions, the properties of model $P$ and model $N$ are given by Proposition 2.

**Proposition 2.** In a one-link network assuming linear demand and cost functions, and assuming that expression [2] holds, then as a result of information provision to an exogenously determined group of travellers: (1) total expected road usage increases; (2) expected road usage of the group of informed travellers increases; (3) expected network travel costs of the group of informed travellers decreases; (4) none of the travellers of the informed group is worse off and consequently, welfare for the informed travellers does not decrease.

**Proof.** Available upon request from the authors.

There are many similarities between Proposition 1 and Proposition 2. However, there is one important difference. Proposition 2 does not address the welfare economic effects of the information provision to the uninformed travellers. In fact, in model $P$ it is impossible to prove that welfare of uninformed travellers will not decrease. The illustration in Figure 3, which gives the results of a simulation experiment with model $N$ and model $P$, shows that under certain conditions welfare of uninformed drivers decreases; in particular, if the value of uncertainty $\beta$ is sufficiently high.

![Figure 3](image-url)
In Figure 3, the expected network travel costs are depicted as a function of the value-of-uncertainty parameter $\beta$. As proven in Proposition 2, informed road users are better off with information. However, this is not necessarily true for the uninformed ones. As can be seen in Figure 3, with the given parameter values, when $\beta$ gets beyond $\beta = 1.8$ (which exceeds $\alpha$), the uninformed road users are worse off with information provision. This can be explained as follows. First, notice that the value-of-time and value-of-uncertainty parameters are assumed to be identical for all potential road users, both informed and uninformed. Clearly, as the informed drivers do not face travel time uncertainty, the uncertainty component becomes zero in their expected travel cost function; see expressions [10] and [11]. This, in turn, will lead to a larger number of informed travellers using the network. The increase will be larger with high values of $\beta$. As a consequence, network travel times will increase, which leads to larger expected travel costs for the uninformed road users.

On the left-hand side of Figure 3 at $\beta = 0$, uninformed drivers are absolutely certain to benefit from the information provision to informed drivers, results which have been derived in Emmerink et al., (1996a).

In the literature, several authors have called attention to the potential negative effects of information provision for uninformed drivers; see for example Bonsall and Parry (1990). However, those assertions were often based on qualitative analyses. Our formal analysis indicates that under the provision of perfect information, some of the uninformed road users might be worse off when the costs of uncertainty ($\beta$) are very high. With a more realistic value-of-uncertainty parameter, however, this is unlikely to take place.

2.4. Model E: Information Available for an Endogenously Determined Group of Travellers. In model E it is assumed that the choice of being informed depends on the internal private benefits derived from the information and the private costs of being informed. In contrast to the previous two models, it is now assumed that information is a commodity that can be purchased for a fixed price $\pi$, where the price $\pi$ refers to the costs of the necessary information equipment and the costs of the subscription to information services. In this model, potential road users are faced with two decisions: either to use the network or not, and either to buy the information or not. Clearly, these decisions are interdependent.

Due to the static equilibrium nature of the models discussed in this paper, the price of information $\pi$ to be considered here lacks any time dimension. In other words, whereas one would intuitively think of $\pi$ as an individual investment, the internal benefits of which were to be reaped during a subsequent (large) number of travel decisions, such reasoning is not in the spirit of static equilibrium analysis. Hence, for the translation of the model into more practical terms, one should interpret $\pi$ as the daily equivalent of some purchase price $\Pi$, where $\pi$ reflects daily interest and depreciation.

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6 Based on previous work (Emmerink et al., 1996a), the following parameters were used to produce Figure 3: $d_i = d_o = 50, a_i = a_o = 0.03, p = 0.25, k^b = k^l = 20, b^0 = 0.015, b^l = 0.04, \alpha = 1$.

7 Without including costs of uncertainty, endogenous provision of information was dealt with in Emmerink et al., (1996b).
Before deriving the equilibrium conditions of model $E$, a final remark is worth making. Without loss of generality, only parameter values leading to at least a few travellers acquiring information are considered. This assumption allows us to write the equilibrium conditions of the model in terms of equalities. If none of the travellers is willing to buy the information (either because the price $\pi$ is too high or the stochasticity in the network is too low, i.e., $p$-values close to zero or one) then the model simply collapses to model $N$ (see Section 2.1).

It is not straightforward to derive the equilibrium conditions for this model. Let us first split up the group of potential road users into four parts, as in Figure 1. Road users in segment $I_1$ will always use the network, independent of the state of the network. For these drivers, information reduces their uncertainty costs to zero. Since the expected costs of the uncertainty are identical for these drivers, either all or none of the road users in segment $I_1$ will acquire information depending on the price of the information.

Next, consider the road users in segment $II_1$: Without being provided with information, they will always use the network, and therefore face travel costs related to both travel time and uncertainty. However, when provided with information, they will only use the network in state 0; their private benefits in state 1 fall short of their private costs in this state. Therefore, information will allow these drivers to reduce the costs of uncertainty to zero, and in addition, will provide them with decision-making benefits in state 1. In fact, the benefits in state 1 stem from the possibility of avoiding network use when capacity is low. A comparison of the information benefits of drivers from segment $I_1$ and $II_1$ shows that segment $II_1$ drivers benefit more than drivers from segment $I_1$ in Figure 1.

Now, we turn to the road users in segment $III_1$. When they base their trip-making behaviour on expected costs, these will exceed the private benefits of using the network. Therefore, they will not use the network when not provided with information. This also implies that drivers in segment $III_1$ do not face any costs related to uncertainty. When road users from segment $III_1$ are informed on the actual network conditions, they will decide to use the network only when state 0 prevails. These drivers will thus obtain individual benefits from improved decision-making. Finally, the hypothetical road users of segment $IV_1$ will never use the network, nor will they therefore buy the information.

To derive the equilibrium conditions of model $E$, we have to distinguish between two different situations. First, let us assume that for drivers in segment $I_1$ and $I_2$ it is not beneficial to be equipped with the information device. This implies that:

$$\beta \cdot \sqrt{p \cdot (1 - p)} \cdot \left( C^1(N^0_E) - C^0(N^0_E) \right) < \pi$$

The above inequality shows that the benefits of being informed do not offset the costs of acquiring the information for drivers who will always use the network, and therefore, it ensures that drivers in segment $I_1$ and $I_2$ will not buy the information. The equilibrium conditions of model $E$ under the assumption that inequality [15] holds can now be given:

$$\left( 1 - p \right) \cdot \left( D(N^0_E) - \alpha \cdot C^0(N^0_E) \right) = \pi$$
The term in the big parentheses in expression [16] denotes the net internal private benefits to driver $N^0_E$ from segment $III_1$ in state 0. Multiplied by the probability of occurrence, $1 - p$, the left-hand side of expression [16] gives the net private benefits of informed road usage experienced by the informed driver $N^0_E$. To ensure that driver $N^0_E$ is the marginal informed driver, the net benefits should equal the costs of acquiring the information $\pi$.

The first term in the big parentheses of expression [17] denotes the net private benefits to informed drivers in segment $II_1$, accruing from the fact that they are not using the network in state 1 when provided with information. Remember that without being provided with information, drivers from segment $II_1$ would always decide to use the network. The expected value of these benefits is obtained by multiplying this term with the probability of occurrence of state 1, $p$. The second term of expression [17] gives the benefits of avoiding the costs of uncertainty. Without being provided with information, these drivers would always use the network and therefore suffer from uncertainty costs. To ensure that driver $N^1_E$ is the marginal informed driver of segment $II_1$, the internal private benefits from being provided with information should equal the costs of information $\pi$. This then leaves us with expression [17]. Notice that in expression [17] the equilibrium levels of road usage $N^0_E$ and $N^1_E$ interact in the term that relates to the costs of uncertainty.

Now, consider the case that inequality [15] does not hold, and hence:

$\beta \cdot \sqrt{p \cdot (1 - p) \cdot (C^1(N^1_E) - C^0(N^0_E))} > \pi$

This implies that all drivers in segment $I_1$ and $I_2$ will acquire information, since the benefits owing to the information exceed the costs of buying the information. Under this assumption, the marginal informed driver $N^1_E$ in segment $I_2$ is now marginal in the sense that he or she is indifferent between using the network in both states or just in state 0 when provided with information. Remember that for inequality [15] to hold, the marginal informed driver $N^1_E$ is indifferent between either using the network in both states and not buying the information or using the network in state 0 and buying the information. Under inequality [18], however, marginal driver $N^1_E$ is not indifferent between buying information or not. He or she will buy information as the benefits of avoiding the travel time uncertainty already offset the costs of acquiring the information; see inequality [18]. The two equilibrium conditions that fully determine $N^0_E$ and $N^1_E$ are now given by expressions [19] and [20]:

$\beta \cdot \sqrt{p \cdot (1 - p) \cdot (C^1(N^1_E) - C^0(N^0_E))} = \pi$

Expression [19] is similar to expression [16]. Expression [20], however, is different from expression [17]. Equilibrium condition [20] ensures that the informed traveller...
\(N^1_E\) is indifferent between either using the network in both states or in state 0 solely: the left-hand side represents the costs of using the network in state 1, while the right-hand side denotes the costs of not using the network in state 1.\(^8\)

A comparison of the properties of model \(N\) and model \(E\), under the assumption of linear demand and stochastic travel time functions, leads to Proposition 3:

**Proposition 3.** In a one-link network assuming linear demand and cost functions, and assuming that expression [2] holds, then as a result of endogenous provision of information: (1) expected road usage increases; (2) for all road users expected network travel costs do not increase; (3) none of the travellers is worse off and consequently, the system welfare in model \(E\) does not fall short of system welfare in model \(N\).

**Proof.** Available upon request from the authors.

With respect to the third point of Proposition 3, it can be shown that the additional net expected private benefits resulting from the endogenous provision of information (net expected private benefits of model \(E\) minus net expected private benefits of model \(N\)) have a similar pattern as the curves in Figure 1 and Figure 2. Of course, subscript \(I\) has to be replaced by subscript \(E\).

It is important to note that the negative external effects to uninformed road users are now not taking place. Even for very large values of \(\beta\), uninformed road users are benefiting from the endogenous provision of information. An obvious explanation for this difference between model \(P\) and model \(E\) is that the negatively affected uninformed road users of model \(P\) would buy the information in model \(E\) to ensure that they will also benefit from the information.

To conclude, in the previous sections various stochastic network equilibrium models were analysed. It was proven that information leads to a Pareto improvement for the informed drivers, so none of the informed drivers is worse off due to information. In model \(P\), however, uninformed travellers might be negatively affected by information, but this is unlikely to happen with realistic parameter values. Finally, in model \(E\), where the choice of being informed is endogenous, it was shown that travellers who decide not to acquire information will always be better off. Therefore, a preliminary conclusion of the analysis is that information will lead to a strict Pareto improvement.

It should be stressed, however, that the results obtained are to some extent dependent on the assumed functional forms of demand and cost functions. For example, Verhoef et al., (1996) have shown that information provision may induce welfare losses when a kinked demand curve is used. Furthermore, concerning the supply-side of the system, it can be shown that even with linear cost functions welfare losses due to information provision occur under the unrealistic assumption.

\(^8\) Notice that equation [17] collapses to equation [20] if

\[\beta \cdot \sqrt{p(1-p)} \cdot \left(C^1(N^1_E) - C^3(N^0_E)\right) = \pi\]

Hence, in this situation the two cases discussed are identical.
that the slope of the cost function in state 0 is larger than the slope of the cost function in state 1.

3. INFORMATION PROVISION AND SYSTEM OPTIMAL BEHAVIOUR

Information provision is a second-best policy; provision of information does not necessarily direct the network performance to system optimal (first-best) levels. In the present section, the analysis is focused on the efficiency of endogenous information provision relative to the optimal policy. The level of welfare generated by endogenous information provision is compared with welfare of the optimal policy and welfare of the nonintervention policy, respectively. The nonintervention policy considered is given by model $N$. The optimal (welfare-maximising) policy is presented in Section 3.1. The welfare effects of endogenous information provision are then investigated in Section 3.2.

3.1. System Optimum (First-best) Policy. The theoretical concept of road pricing, stemming from Pigou (1920) and Knight (1924), and further explored by Walters (1961), Smeed (1964), Sharp (1966), and Vickrey (1969, 1971), is based on levying a tax equal to the difference of the marginal social costs and marginal private costs (Pigouvian tax). In doing so, the negative external effects on other road users when joining a road are accounted for in the user's decision-making process: the system optimum will coincide with the user equilibrium (Wardrop 1952). In economic terms, the road price ensures that only economically efficient trips are undertaken.\(^9\)

In this section, the theory of road pricing is extended to situations in which road users suffer from costs related to uncertainty. A system optimal policy can be implemented by means of a fluctuating (depending on the state) pricing scheme; fluctuating in the sense that the road price is equal to $f^0$ ($f^1$) when state 0 (state 1) prevails.\(^{10}\) To analytically present the welfare-maximising model—where welfare is measured as the sum of the individuals' private benefits minus the sum of their private costs—we have to consider three situations:

1. some of the travellers acquire information;
2. all travellers acquire information;
3. no information is available.

3.1.1. System optimum, situation 1: some travellers acquire information. First, let us assume that inequality [15] holds true. This implies that travellers in the interval $[0, N^a_E]$ are not acquiring information. Only, for drivers in segment $[N^a_E, N^b_E]$ it is beneficial to buy information. In order to realise the welfare-maximising traffic

\(^9\)Due to limited political feasibility, there are only a few sites where a road pricing scheme has been implemented. For arguments pro and contra road pricing, we refer to the publications by Borins (1988), Emmerink et al., (1995a), Evans (1992), Giuliano (1992), Goodwin (1989), Grieco and Jones (1994), Hills and Evans (1993), Johansson and Mattsson (1995), Jones (1991), Small (1992).

\(^{10}\)Of course, the static model with fluctuating fees depending on the actual state is an abstraction from reality. A dynamic model with time-dependent fluctuating fees would be more realistic.
scheme, the regulator is faced with the following maximisation problem:

\[
\max_{f^0, f^1} (1 - p) \cdot \left( \int_0^{N_E^0} D(x) \, dx - \left( \alpha \cdot C^0(N_E^0) \cdot N_E^0 \right) \right) \\
+ p \cdot \left( \int_0^{N_E^1} D(x) \, dx - \left( \alpha \cdot C^1(N_E^1) \cdot N_E^1 \right) \right) \\
- \beta \sqrt{p(1-p)} \cdot \left( C^1(N_E^1) - C^0(N_E^0) \right) \cdot N_E^1 - \pi \cdot (N_E^0 - N_E^1)
\]

subject to

\[
(1 - p) \cdot (D(N_E^0) - \alpha \cdot C^0(N_E^0) - f^0) = \pi \\
p \cdot (\alpha \cdot C^1(N_E^1) + f^1 - D(N_E^1)) + \beta \sqrt{p(1-p)} \cdot (C^1(N_E^1) - C^0(N_E^0)) = \pi
\]

The integral term provides the area under the inverse demand curve. The term \(\pi(N_E^0 - N_E^1)\) in the objective function gives the costs of information, while the uncertainty cost component is multiplied by the number of road users faced with uncertainty: \(N_E^1\). The two restrictions ensure that information is allocated in an individually rational manner. Apart from the incorporation of the fluctuating road price, the restrictions are similar to expressions [16] and [17]. The second restriction implies that driver \(N_E^1\) is indifferent between either always using the network or buying information and using the network in state 0 only; the first restriction implies that driver \(N_E^0\) is indifferent between either never using the network or buying information and using the network in state 0 only. The optimal fluctuating fees \(f^0\) and \(f^1\) can be solved for by using the technique of Lagrange, yielding:

\[
f^0 = \alpha \cdot C^0(N_E^0) \cdot N_E^0 - \beta \sqrt{\frac{p}{1-p}} \cdot C^0(N_E^0) \cdot N_E^1
\]

\[
f^1 = \alpha \cdot C^1(N_E^1) \cdot N_E^1 + \beta \sqrt{\frac{1-p}{p}} \cdot C^1(N_E^1) \cdot N_E^1
\]

The first terms on the right-hand side of [22] reflect the traditional external congestion costs of road traffic. The second term then represents the external costs of uncertainty imposed on the road users in the interval \([0, N_E^1]\). This additional term decreases the value of \(f^0\) and increases the value of \(f^1\) in order to diminish the fluctuations in travel time, and hence the external costs of uncertainty.

3.1.2. System optimum: situation 2: all travellers acquire information. Next, assume that inequality [18] holds true, implying that all users of the network are
acquiring information. The regulator then faces maximisation problem [23]:

\[
\max_{f^0, f^1} \left( (1 - p) \cdot \left( \int_0^{N^0_E} D(x) \, dx - (N^0_E \cdot N^0_E) \right) + p \cdot \left( \int_0^{N^1_E} D(x) \, dx - (N^1_E \cdot N^1_E) \right) - \pi \cdot N^0_E \right)
\]

subject to

\[
(1 - p) \cdot (D(N^0_E) - \alpha \cdot C^0(N^0_E) - f^0) = \pi
\]
\[
\alpha \cdot C^1(N^1_E) + f^1 = D(N^1_E)
\]

The term \(\pi N^0_E\) in the objective function reflects the costs of information, while the restrictions ensure individually rational behaviour. Maximisation problem [23] can be solved using the technique of Lagrange. The optimal fluctuating fees are:

\[
f^0 = \alpha \cdot C^0(N^0_E) \cdot N^0_E
\]
\[
f^1 = \alpha \cdot C^1(N^1_E) \cdot N^1_E
\]

which is the traditional expression for the optimal (first-best) congestion prices. Clearly, uncertainty does not play a role in these expressions, since none of the drivers in the network faces uncertainty costs as everyone is acquiring information.

3.1.3. System optimum; situation 3: no information available. Finally, let us assume that the costs of information (\(\pi\)) are relatively high. Then it might be optimal from a system point of view to supply none of the drivers with information. In such a situation, the network regulator faces the problem of finding the system optimum of model \(N\) (see also Section 2.1):

\[
\max_f \int_0^{N_N} D(x) \, dx - (1 - p) \cdot \alpha \cdot C^0(N_N) \cdot N_N - p \cdot \alpha \cdot C^1(N_N) \cdot N_N
\]

subject to

\[
\alpha \cdot ((1 - p) \cdot C^0(N_N) + p \cdot C^1(N_N))
\]
\[
+ \beta \sqrt{p(1 - p)} \cdot (C^1(N_N) - C^0(N_N)) + f = D(N_N)
\]

Using the technique of Lagrange, the optimal flat fee \(f\) has the following form:

\[
f = \alpha \cdot ((1 - p) \cdot C^0(N_N) + p \cdot C^1(N_N)) \cdot N_N
\]
\[
+ \beta \sqrt{p(1 - p)} \cdot (C^1(N_N) - C^0(N_N)) \cdot N_N
\]
The first term on the right-hand side of [26] denotes the expected external congestion costs; the second term reflects the external uncertainty costs. Depending both on the functional forms of the demand and travel time functions, and on the values of the parameters, one of the three situations discussed above yields the system optimum for the transport network. In the next section, the welfare-improving properties of information provision are compared with welfare levels under system optimum (first-best) behaviour.

3.2. Relative Welfare Effects of Endogenous Information Provision. The impact of information on the relative efficiency of road usage will be measured using the performance measure $\omega$, defined as (Arnott et al., 1991; Verhoef et al., 1995):

$$
\omega = \frac{\text{Welfare(Endogenous Information)} - \text{Welfare(Nonintervention)}}{\text{Welfare(System Optimum)} - \text{Welfare(Nonintervention)}}
$$

$\text{Welfare(Endogenous Information)}$ denotes the amount of welfare generated by model $E$ (Section 2.4), while $\text{Welfare(Nonintervention)}$ represents welfare under model $N$ (no information available, Section 2.1). $\omega$ then provides the achievable welfare gains as a proportion of the theoretically possible welfare gains. Clearly, $\omega$ cannot exceed the value of one. In addition, $\omega$ cannot be smaller than zero, since it was shown in Proposition 1 that endogenous provision of information leads to a strict Pareto improvement, implying that the numerator of $\omega$ cannot take on negative values.

Various experiments have been conducted to assess the impact of key parameters of the model on the relative efficiency of information provision. Similar to the specifications underlying the propositions in Section 2, the functional form of the demand function used in the experiments is given by $D(N) = d - aN$; the link travel time functions are specified as $C_i(N) = k_i + b_iN$ ($j = 0, 1$).

In previous papers, costs related to uncertainty were not included in the generalised cost function, that is, $\beta$ was assumed to be equal to zero. Figure 4, however, shows that the inclusion of uncertainty costs strongly influences the relative efficiency of information provision. The more important the uncertainty costs component, the higher the efficiency of information provision. Hence, the results obtained in previous work provide a lower bound for the potential efficiency gains of information; the positive impacts of information are underestimated when costs of uncertainty are ignored.

4. CONCLUSIONS

In this paper, the impacts of information provision to travellers were analysed using a stochastic equilibrium concept. Previous work was advanced by including an additional term related to the costs of travel time uncertainty into the generalised

11 The following parameters were used to produce Figure 4: $d = 50, a = 0.015, p = 0.25, k^0 = k^1 = 20, b^0 = 0.015, b^1 = 0.04, a = 1, \pi = 2$. 
cost function. Costs of uncertainty have frequently been mentioned as being important in the travel behaviour literature, particularly with respect to driver information systems.

Four models were discussed. It was proven that for realistic values of the value-of-uncertainty parameter ($\beta$), information is beneficial to both the informed and uninformed drivers, that is, information will lead to a strict Pareto improvement. For exceptionally large $\beta$-values however, exogenous provision of information (model $P$) might negatively affect the uninformed drivers.

The relative efficiency of endogenous information provision (model $E$) was analysed, using the system optimal welfare level as a benchmark. System optimal welfare can be achieved by means of a fluctuating (depending on the level of congestion) road pricing scheme. With the inclusion of costs related to uncertainty, the optimal fluctuating road price depends on both the external congestion costs and the external uncertainty costs. Experiments revealed that the larger the costs of uncertainty, the more attractive information is as a policy option to optimise traffic flows.

There are plenty of promising future research directions; we will mention four of these. First, it would be interesting to consider a heterogenous group of travellers, that is, with value-of-time and value-of-uncertainty parameters differing throughout the population. Second, the application of the stochastic equilibrium principle is limited to a simple transport network; the model would considerably gain in realism if more complex networks were dealt with. Third, the present analysis has ignored
the additional beneficial effect that information provision might reduce drivers’ perceptual errors regarding traffic conditions. In our model we have assumed that potential road users are perfectly aware of average network conditions. It is not directly obvious what the consequences of relaxing this assumption are. Finally, the models discussed above are static in nature, and it is a major future challenge to apply this stochastic equilibrium concept to a dynamic model as well. This would require both advanced mathematical techniques and enormous computing power.

REFERENCES


