Summary

In state space models, indirectly observed dynamic phenomena are modelled using a mathematical description of the dynamics and observation mechanisms. Initially such models were applied in the aerospace industry to estimate the location of objects such as rockets from indirect and not completely reliable measurements. Due to the generality of the models and the algorithms to estimate the indirectly observed state variables, state space models were increasingly used in other disciplines. Generally a state space model can be described as a model of a system or phenomenon which changes through time and which is not observed directly. Clearly, many problems from economics and finance fit within this framework. Important examples are estimation of business cycles, seasonal influences on economic activity, general price indices, the volatility of prices and dynamic model parameters (which are assumed to be constant in classical regression analysis).

The key algorithm in state space analysis is known as the Kalman filter, which is a set of equations to calculate the optimal estimates of the state variables from the observations. The Kalman filter is efficient due to its recursive nature: new observations are combined with the current best estimate to produce new estimates. Within the class of linear state space models with Normally distributed variables, the Kalman filter produces the minimum mean square error estimates of the state variables. When the dynamics or the observations in state space models are non-linear, the optimal estimates are difficult to calculate. The exact solution consist of recursive integral equations, often of high dimensions without close form solutions. In practical applications, generally some type of approximation is required.

The introducing chapter of this dissertation describes estimation of the general non-linear state space model. The mathematical formulation and the exact optimal estimators can be found in this chapter. The following two chapters discusses various approximation methods.

Non-linear estimation techniques can generally be divided in simulation, or Monte-Carlo based algorithms, and non-simulation based algorithms. In chapter 2, we examine two non-simulation based algorithms in detail, namely the extended and the unscented Kalman filter. The extended Kalman filter is the oldest and most established non-
linear state space filtering algorithm. It is derived as the application of the standard Kalman filter to a first order approximation of the non-linear model. In this chapter, the extended filter is also provided for models with diffuse initial conditions.

The unscented Kalman filter is a more recent non-linear filter which is gaining acceptance in engineering disciplines. Using an innovative transformation, it utilises a higher order Taylor approximation without the need to calculate higher order derivatives.

Chapter 3 describes Monte Carlo estimation. Although naive Monte Carlo estimators are readily derived from the Bayesian recursive solution to the filtering problem, any practical application requires some variance reduction techniques. This chapter concentrates on importance sampling, both in a global form, where an approximating model is built for the entire state process before the simulation commences, as well as in a sequential form in which approximating model are obtained using simulations in each time step. The latter approach is usually termed particle filtering.

The next four chapters demonstrate economic applications of general state space models. The first three are extensions of the well-known unobserved components (UC) models, in which time series are decomposed in stochastic processes which represent trends, cycles, seasonal effects, noise and possibly other sources of variation.

In chapter 4 logistic and smooth spline functions are used to permit variation in the evolution of the parameters of the cycle component. This is strictly speaking a linear state space model in the sense that the transition and observation processes are both linear. The parameters are transformed as a non-linear deterministic functions of time, which allows more flexibility compared to the basic specification. The specification appears to be suitable for U.S. macro economic times series, and is relatively easy to estimate. However, this comes at the cost of having to use many parameters.

Chapter 5 develops a non-linear cycle specification in which the length of the period or the frequency depends on the steepness of the cycle. This can capture asymmetry in the cycle, a phenomenon often observed in macro-economic time series, in which expansions and contractions occur at different velocities. The asymmetry parameter is estimated using an importance sampling based likelihood. In the U.S. macro-economic time series we estimate that GDP and investment tend to fall significantly faster than that the rise, while unemployment increases faster than it decreases.

In chapter 6, the seasonal component in a UC model is linked to both the trend and the cycle, such that the degree of seasonal variation may depend on the other components. We show that this specification can be used to model increasing seasonal variation in cases where a logarithmic transformation is not appropriate, and that it captures cycle dependent seasonality. In U.S unemployment it is found that seasonal effects are more pronounced during expansions than during recessions, and that in dwellings productions seasonality is attenuated in the cycle and has decreased overall
together with the rise in the trend.

Finally, in chapter 7 two importance sampling based estimators are compared in their effectiveness to estimate stochastic volatility models. The performance between the methods which arrive at model approximations from rather different paths are comparable, and both methods can be used effectively to estimate stochastic volatility model with Normal and Student t disturbances.