Summary

Fire is one of the classical “elementary” phenomena in nature. It has a wide variety of applications such as in engines and as a heat source, but also appears naturally in less controlled situations such as forest fires and tunnel accidents. For combustion to occur one needs a fuel, a source of oxygen and an igniting “spark”. A combustion process can be regulated by controlling the amount of fuel (fed to the process), the amount of available oxygen or the heat source. The different physical and chemical aspects of combustion have been modelled extensively in the past. These models are often complicated and difficult to analyse mathematically.

In this thesis we study a combustion-radiation model that incorporates the relevant physical aspects, but is simple enough from a mathematical point of view to be investigated analytically and numerically. In particular we focus on the stability of travelling waves in a combustion-radiation free boundary model. The model describes premixed flames propagating in a gaseous mixture with inert dust. The model is based on the hypotheses of simple chemistry and high activation energy, and was suggested first by Buckmaster and Joulin.

Without radiative effects the model is known as the thermo-diffusive model for premixed flames, and consists of diffusion equations for the temperature and the fuel mass fraction. An important role is played by the Lewis number, the ratio between the diffusivities of the temperature and the fuel mass fraction. After ignition, the reaction is assumed to be confined to an infinitesimally narrow reaction zone (the flame front), which separates the fresh region in front of the flame from the burnt region behind the flame. At the flame front the mass flux going into the flame balances the heat flux coming out of the flame, with a temperature dependent Arrhenius type reaction rate. In this model, planar flames are one-dimensional travelling wave solutions. The natural control parameters for the travelling waves are the amount of fuel fed to the flame and the ambient temperature.

The radiative transfer of energy emitted and absorbed by dust particles is modelled using the Eddington approximation, which contains two radiative parameters,
the opacity of the medium and the Boltzmann number (measuring the relative importance of the radiative energy flux compared to the heat flux). Radiative transfer may significantly influence the flame speed and the temperature profile, depending on the opacity of the medium. An important feature of the radiative model is the Joulin effect: due to the radiative effects the flame temperature is higher than the adiabatic temperature (the temperature of the thermo-diffusive model), the flame propagates with a larger speed, and the ignition temperature is lowered.

Existence of travelling waves for the radiation-diffusion model, also referred to as the Eddington model, was proved by Brauner, Hulshof, and Ripoll using a Schauder type fixed point argument. The model features several physically interesting limit cases for the radiative parameters, including the adiabatic case. The limit case where the opacity of the medium goes to zero and the pre-heated zone becomes large, was investigated in detail by Baconneau et al. The amount of the fuel ahead of the flame (in the fresh region) was taken as a control parameter. There, the existence of travelling wave solutions curve with turning points was established. The analysis was restricted to the case that the Lewis number is equal to 1, because the bifurcation diagram (of the solutions) is independent of this number. Furthermore, the analysis of the stability of these waves suggested that stability was related mainly to changes of stability in the turning points.

A more difficult limit case, namely the transparent and weakly radiation dominated limit, in which the Joulin effect is most pronounced, was investigated by Van den Berg et al. More precisely, the law that describes the relation between the flame speed and the control parameters was determined. For this asymptotic regime the flame temperature reaches its upper bound.

In this thesis, we extend the result of the limit cases, and study the radiative combustion model for general Lewis number and general radiative parameters. We start with a linear version of the Eddington equation (replacing the nonlinear temperature dependence by a linear one) and investigate the bifurcation (solution) diagram. Moreover, we study extensively the stability both in the one-dimensional and two-dimensional setting. Finally, we examine the stability of the nonlinear version of the Eddington model numerically.

In Chapter 2, we study the travelling waves in the case that the Eddington equation is replaced by its linear version, but we keep the nonlinear reaction rate. We show that, dropping the reaction rate, there exists a travelling wave profile for every value of the flame speed. Taking the reaction rate into account, in the parameter plane where we use the burnt temperature on the horizontal axis and the speed of the flame on the vertical axis, we show that travelling waves are given by a single smooth curve which is parameterised by the flame temperature. This curve may
have turning points and exhibit $S$-shaped parts. The bifurcation diagram does not depend on the Lewis number. Finally, we determine the solution curve for physically interesting asymptotic regimes of the radiative parameters.

We continue the analysis of the linear version Eddington model and investigate the stability of the travelling waves in Chapter 3. We adopt the linearisation technique developed by Brauner et al, and linearise the free boundary model around the travelling wave solutions. The linearised problem is analysed by means of the Evans’ function. Eigenvalues of the linearised problem correspond to zeros of the Evans’ function. For stability of the travelling waves, it turns out that the Lewis number is of great importance. The other influential parameter is the derivative of the reaction rate with respect to the flame temperature. Changing the derivative of the reaction rate, while keeping the value of the reaction rate fixed, we change the tangent of the solution curve in the bifurcation diagram as well as the spectral stability of the travelling waves. Thus, using analytical and numerical methods we present the stability results in terms of the Lewis number and the derivative of the reaction rate.

We first consider the one-dimensional problem. We determine in detail the behaviour of the Evans’ function for small values of the eigenvalues. Because of the translation invariance, zero is always an eigenvalue. We show that the solution diagram having a vertical turning point is equivalent to the first derivative of the Evans’ function vanishing at zero. This condition is independent of the Lewis number. A triple zero of the Evans’ function in the origin occurs for a unique critical value of the Lewis number, from which a Hopf curve emanates. For some parameter values, the Hopf curve has a self-intersection, leading to a rich stability diagram. Only below the Hopf curve and below the turning point line spectral stability for the one-dimensional problem holds.

We conclude Chapter 3 with the stability analysis of the two-dimensional problem, where a lateral wave number appears. We first determine in detail the behaviour of the Evans’ function for small eigenvalues and small lateral wave numbers. We then describe how eigenvalues pass through zero on a “neutral dispersion curve” for each lateral wave number. Numerically we also examine the behaviour of the Hopf bifurcation curve as the lateral wave number is perturbed away from zero. It turns out that, for small values of the lateral wave number, Hopf curves move down and narrow the region of stable solutions. Thus, changes from spectral stability to spectral instability occur through small values of the lateral wave number, rather than for larger values.

Summing up, considering the plane with the Lewis number on the horizontal axis and the derivative of the reaction rate on the vertical axis, two-dimensional
spectral stability holds in the region lying below, firstly, all wave number dependent neutral dispersion curves, secondly, all wave number dependent Hopf curves, and thirdly, the turning point line corresponding to wave number zero (if relevant).

In Chapter 4, we continue with the analysis of the model with the nonlinear Eddington equation, and we turn from analytic considerations towards numerical computations. To study the stability problem, we linearise around numerically computed travelling waves. To construct the Evans’ function we extend the work by Allen and Bridges and others, and we use the wedges (exterior products) formulation to define two-dimensional spaces of bounded solutions on the left and on the right of the free boundary. Applying the jump conditions at the free boundary then leads to an (analytic) Evans’ function. Comparing the stability diagrams of the linear problem and the nonlinear problem, the differences are only quantitative in nature.