Essays on Simultaneous Search Equilibrium

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Essays on Simultaneous Search Equilibrium

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door
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My first encounter with simultaneous search was in - of all places – the real world. Writing my master's thesis at CPB convinced me that I wanted to continue doing research by pursuing a doctorate in economics. Despite being unfamiliar with the literature on search, it was not hard to understand that sending several applications to PhD programs was a safer strategy than sending just one. And so, I looked up some interesting projects and sent my CV and application letter to every one of them. Soon after, I was confronted by some of the concepts for which I would learn the terminology only a few months later: coordination frictions, information asymmetries and limited recall.

The way I met Pieter Gautier comes pretty close to random matching: he did not know me and I initially did not know that he had a vacancy. But once these frictions were resolved, his explanation of the similarities between an unemployed factory worker and Angelina Jolie fascinated me and the choice became easy. And it was a good choice; I could not have wished for a better supervisor. We met almost every day to discuss the progress of our papers and, without exception, each of these meetings was both very productive and very pleasant. Pieter, I fear that we have to check the proofs in our papers once again: it seems that in real life random matching can, in fact, very well generate efficient outcomes.

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Chicago, September 2008

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1 Introduction

1.1 Background

The start of modern economic modeling goes back to Cournot (1838), who pioneered the use of calculus to analyze economic markets, like monopoly, duopoly and competitive markets. His book, which also introduced the concept of a demand function, later inspired Marshall (1890), to implement an upward sloping supply function and a downward sloping demand function as tools for finding the market price. This model of supply and demand became the fundamental building block for the next generations of economists.

However, over time economists started to recognize that this simple model has some important shortcomings. First of all, it is does not explain how supply and demand meet and how the equilibrium price is determined. Walras (1874) introduced the concept of an auctioneer who suggests a price, registers demand and supply, and lowers / raises the price in case of excess supply / demand, until the market-clearing price is reached. But, in most markets there is no central market maker that can play the role of the Walrasian auctioneer. Market participants are often not aware of the actions of other market participants and coordination is often impossible due to the large size of the market. A second shortcoming of the simple, competitive model is that it implies the *law of one price*: a homogeneous product is always sold at the same, unique equilibrium price. This also contradicts what one typically observes in reality: different firms often charge different prices for the same product and similar workers can earn very different wages.

In the early sixties, several authors tried to improve on these two shortcomings, by creating models in which buyers/workers sample a number of prices/wages from a fixed exogenous distribution. In a paper by Karlin (1962), sellers sequentially receive price offers and have to decide whether to accept the current offer or to reject it and hope for a better offer in the next period. Around the same time, Stigler (1961, 1962) analyzed models in which buyers/workers sample multiple times simultaneously and accept the most favourable draw. He wrote:

> "Prices change with varying frequency in all markets, and, unless a market is completely centralized, no one will know all the prices which various sellers (or buyers) quote at any time. A buyer (or seller) who wishes to ascertain the most favorable price must canvass various sellers (or buyers) - a phenomenon I shall term *search*." (Stigler, 1961)

Since then, a large search literature has been developed with models that are more explicit about the meeting process and price/wage dispersion. In the next section, I briefly summarize the most important contributions.[1] The aim of this thesis is to contribute to the literature by analyzing models in which workers can send multiple applications simultaneously. A more detailed summary of each of the three essays in this thesis is given in section 1.3.

1.2 Search Literature

The first contributions to the search literature were directly based on the pioneering work by Karlin (1962) and Stigler (1961, 1962). Several authors, for example McCall (1965, 1970) and Nelson (1970), argued that the simultaneous sampling design used by the workers in Stigler's models was not optimal.[2] Workers would prefer a sequential approach with a reservation wage, as in Mortensen (1970). A second, and perhaps more fundamental,

[1] Rogerson et al. (2005) provide a more thorough review of the theoretical search literature. For an overview of the empirical literature, see Eckstein and van den Berg (2007).

[2] Ever since Stigler's papers, some authors have written models for buyers, sellers, and prices in a goods market, while others have used a labor market setting with workers, firms and wages. Typically, there is no fundamental difference between both paradigms. For reasons of consistency, and in line with the topic of the thesis, I will often stick to labor market terminology when discussing consumer search models.

criticism was put forward by Rothschild (1973). He pointed out that the models described a *partial partial-equilibrium* theory, i.e. a theory of one side of the market only. The models explained the behavior of workers when confronted with wage dispersion, but not why there was dispersion in the first place. In fact, if firms could optimize their wage setting in these models, dispersion would disappear.

Two years earlier, Diamond (1971) had already derived a similar result in a slightly different setting. He showed that search costs among workers lead to a unique equilibrium in which all firms offer the reservation wage. This result became known as the Diamond paradox, because even with a large number of firms in the market and arbitrarily small (but positive) search costs, search frictions leave the workers with no surplus, while the competitive wage as in Bertrand (1883) would give them the entire surplus in this setup.

In the following years many authors tried to overcome this result by relaxing one of the assumptions of the simple model. A popular choice, exploited by e.g. Salop and Stiglitz (1977), Reinganum (1979), Varian (1980), and Albrecht and Axell (1984), was to relax the assumption of homogeneity of the market participants. Generating wage dispersion in a market with fully homogeneous agents turned out to be more difficult. Butters (1977) and Burdett and Judd (1983) found a way by reintroducing simultaneity. The idea being the following: if some, but not all, workers observe several employment opportunities simultaneously, then firms face a trade-off between the wage offer and the hiring probability. Firms that offer low wages will only hire workers that do not have better offers, but make a high profit per hired worker. On the other hand, firms offering high wages attract workers more easily, but make a lower profit per hired worker. In equilibrium the effects offset, such that all firms make the same profit. The resulting wage distribution has a continuous support and its density is strictly increasing.

Burdett and Mortensen (1989, 1998) translated the simultaneity idea of Burdett and Judd (1983) into a sequential setting with wage posting. In the continuous time model workers encounter one job opportunity at a time, but they continue to search for better offers while employed. Hence, workers can compare any new wage offer to their current wage. This again generates a trade-off for firms: higher wages give a higher hiring probability but a lower profit per hired worker than low wages. Also this model has been extended in many ways. Van den Berg and Ridder (1998), Bontemps et al. (1999), and Bontemps et al. (2000) introduced worker and/or firm heterogeneity in the model, mainly to obtain a better fit of the data. Postel-Vinay and Robin (2002a,b) did the same, but also allowed the incumbent and the poaching firm to participate in Bertrand competition for the worker's services. Other recent extensions include Burdett and Coles (2003, 2007), who allow firms to post wage-tenure contracts, and Burdett et al. (2008), who analyze human capital accumulation by the workers.

A new class of models (Diamond, 1982; Mortensen, 1982a,b; Pissarides, 1984, 1985) was developed from the early eighties onwards. These models dropped the assumption that firms make take-it-or-leave it offers and instead assumed that wages are the result of Nash bargaining between firms and workers. A second novelty in these articles was the introduction of an exogenous matching function, which determined the number of matches per period as a function of unemployment and the number of vacancies. Mortensen and Pissarides (1994) extended the model by allowing for endogenous job destruction.[3] Although this model does not generate wage dispersion in markets with homogeneous agents, its analytical tractability and the ease with which it can be extended have made it a popular tool for analyzing a wide variety of policy questions, such as unemployment benefits, minimum wages, taxes, and business cycle fluctuations.[4] Currently there is a debate in macroeconomics whether the model is able to explain the cyclical behavior of vacancies and unemployment (see Shimer, 2005b).

In these models, workers and firms meet randomly. Consequently, as was shown by Hosios (1990), the resulting equilibria are in general not constrained efficient, implying that government intervention could improve welfare. A relatively new branch of literature, the so called *directed search* or *competitive search* models, relaxes the assumption of a random meeting process. In this setup, workers observe all wages posted by the firms before they decide where to apply. Early specifications of such models include Peters (1984, 1991) and Montgomery (1991). Typically these models find that the market equilibrium is

[3] See Pissarides (2000) for other extensions.

[4] See Mortensen and Pissarides (1999a,b) for an overview.

efficient (see e.g. Moen, 1997) . Since then a large number of extensions and applications has emerged (see e.g. Acemoglu and Shimer, 1999a,b; Shi, 2001, 2002; Mortensen and Wright, 2002; Coles and Eeckhout, 2003; Shimer, 2005a).

The focus on simultaneous search has returned in recent literature, for example in Shimer (2004b) and Chade and Smith (2006). In the *competing auction* model of Julien et al. (2000, 2006) workers do not search themselves but are contacted randomly by firms. A worker who is contacted by exactly one firm gets her reservation wage. However, if more than one firm contacts a specific worker, the firms in question bid up to the productivity level, thereby generating wage dispersion. Albrecht et al. (2003, 2004) extend the urn-ball matching function model of Butters (1977) by allowing for multiple applications by the workers. Gautier and Moraga-González (2005) use this matching function and show that even when all workers apply to multiple jobs, the matching technology generates variation in the number of job offers workers get, implying that wage dispersion as in Burdett and Judd (1983) continues to be an equilibrium outcome. Kaas (2007) introduces a slightly different specification of the urn-ball matching function in which not only the number of job offers but also the number of applications sent by workers is stochastic, which improves the analytical tractability of the model.

The directed search literature is also increasingly witnessing a surge in studies of the effect of multiple applications. Albrecht et al. (2006) allow firms offering the job to the same worker to increase their initial bid, while Galenianos and Kircher (2008) and Kircher (2007) study price posting. The first two papers find that simultaneous search destroys the efficiency of the equilibrium. The latter paper argues that efficiency reemerges if firms are allowed to contact all of their applicants, rather than one. Delacroix and Shi (2006) use simultaneity in the Burdett and Mortensen (1998) sense. In their model with wage posting, workers send one application at a time, but they can search on-the-job, implying that they can compare their wage offer to their current wage. All these papers find wage dispersion with a discrete support.

The first chapter of this thesis contributes to the literature by allowing for firm heterogeneity in a similar framework as in Albrecht et al. (2006). The second and third chapter consider a random search framework with a matching function as in Albrecht et al. (2003) and/or Kaas (2007). These chapters respectively highlight the choice of search intensity and on-the-job search. A more detailed summary of each chapter is given in the following section.

1.3 Structure of the Thesis

This thesis contains three essays on simultaneous search in the labor market. Each essay relaxes the assumptions of the standard model in a different way. The first and the third essay are mainly theoretical contributions to the literature, while the second essay also presents an important methodological and empirical part.

The first essay studies the effect of heterogeneity in firm productivity in a simultaneous search setting. In a competitive environment more productive firms will eventually hire all workers and the less productive firms are driven out of the market. In a world with frictions, high and low productivity firms can coexist. Then the question arises whether in equilibrium the allocation of workers over both types of firms is optimal. The essay analyzes this for an economy with two types of firms (high and low productivity firms), in which workers can send two applications. Firms that compete for the same candidate can increase their wage offers as often as they like. It is shown that there is a unique symmetric equilibrium where workers mix between sending both applications to the high and sending both to the low productivity sector. However, social welfare would be maximized if they diversify their applications over the sectors, because a higher matching rate in the high-productivity sector can then be realized with fewer applications (and consequently fewer coordination frictions). But, in the market the workers' payoff is determined by how much the firm with the second highest productivity is willing to bid, which prevents a worker from applying to both sectors. The essay shows that the equilibrium outcomes are the same under directed and random search for many values of the exogenous parameters. This implies that the inefficiency result is not driven by the random search assumption, as in many other papers. Further, an extended version of the model in which there is free entry of vacancies is discussed. It is shown that this creates a second source of inefficiency. Finally, the chapter discusses the effects of increasing the number of applications and shows that the results can easily be generalized to a larger number of firm types.

The second essay takes the sector in which workers search for a job as given, but focuses on the question whether workers send the socially desired number of applications. In most countries there exist active labor market programs that try to increase the search intensity of unemployed workers. Workers who send more applications typically benefit in the sense that they find a job more quickly. However, their change in behavior imposes various externalities on the other agents in the market. The increase in search intensity makes it harder for other workers to get a job offer, makes it easier for firms to find a worker, and will affect the wage offer distribution. For a proper evaluation of these programs it is necessary to have a framework that incorporates all these effects. This essay presents such a framework, in which the wage distribution, job search intensities, and firm entry are simultaneously determined in market equilibrium, taking the search costs of workers as exogenously given primitive variables. Using wage data the search cost distribution, the implied matching probabilities, the productivity of a match, and the flow value of non-labor market time are structurally estimated; these estimates are then used to derive the socially optimal firm entry rates and distribution of job search intensities. Three inefficiencies are found. It turns out that, from a social point of view, (i) too few workers participate in the labor market, (ii) some unemployed search too much and (iii) for given search intensities, entry is excessive. The low participation rate reflects a standard hold-up problem and the excess number of applications result is due to coordination frictions and rent seeking behavior. Sizable welfare gains (about 12%) can be realized by correcting these three inefficiencies. It is argued that a modest binding minimum wage or conditioning unemployment benefits on applying for at least one job per period is welfare increasing, because it stimulates participation without rewarding excessive search.

The last essay in this thesis studies an equilibrium search model of the labor market in which workers can search for a better match while employed. An important novelty compared to existing on-the-job search models is that workers can communicate their current wage to the firm offering them a new job. Based on this information, firms make a wage offer to which they commit. By using a simultaneous search framework wage dispersion remains to be an equilibrium outcome, because firms take into account potential competition from other firms for the same job candidate. The essay shows that the equilibrium wage density has continuous support and a unique interior mode, even in a market with identical workers and firms. High wage levels cannot be reached directly from unemployment, but require several job-to-job transitions. The speed with which workers climb the wage ladder is stochastic. By calibrating the model it is shown that changes in the level of unemployment benefits have a larger effect on the job offer arrival rate of the unemployed than of the employed. The opposite holds when considering changes in the job destruction rate.

2 Simultaneous Search and Heterogeneous Firms

2.1 Introduction

In most labor markets, heterogeneous firms compete for workers. In a competitive environment more productive firms can always bid more than less productive firms so they will hire all workers and ultimately the unproductive firms are driven out of the market. In a world with frictions, high and low productivity firms can coexist but the question whether the allocation of workers over both types of firms is optimal given the frictions remains. In this essay we show that under arguably small coordination frictions the optimal allocation breaks down. We consider the following deviations from the competitive model: (i) workers do not know to which firms other workers apply to, (ii) firms do not know which candidates receive offers, (iii) applications are costly and firms can consider only a fraction of their candidates. While keeping our model as simple as possible we want to capture a number of factors that we feel are important in real world labor markets like heterogeneity, the possibility of simultaneous search and ex post competition for workers with multiple offers. At the same time we want to rigorously model the matching process, the strategic interactions between workers with each other and with the firms.

Specifically, we study a portfolio problem where identical unemployed workers must decide in which sector(s) to search; the high and or the low productivity sector. Within a sector, all firms are identical. Workers can send 0, 1 or 2 applications at a cost $k > 0$ for each application. Each vacancy that receives one or more candidates randomly picks

This chapter is based on Gautier and Wolthoff (2006).

a candidate and offers the job to him. The other applications are rejected. In the simplest version of the model, workers know the productivity in each sector but only learn about the wage at a specific firm after applying there. We then show that our results still hold in the much more complicated case where search is fully directed: i.e. firms can ex ante post a wage which is observed by all workers before they decide where to send their applications. Firms that compete for the same candidate can increase their offers as often as they like, so we do not restrict the firm's strategy space in this dimension. We are interested in symmetric pure strategy equilibria (in terms of the number of applications) and their efficiency properties.

Interestingly, in the simplest version of our model it cannot be an equilibrium for workers to send just one application because then firms have no incentives to offer a positive wage. This is basically the Diamond (1971) paradox. Therefore, if k is sufficiently low, workers always send two applications, hoping to get a positive payoff by receiving two offers. But this in turn implies that workers will never apply to both sectors (HL) because this strategy is strictly dominated by sending both applications to the low productivity sector (LL). The intuition behind this result is that in any equilibrium where workers are willing to apply to the low productivity sector, the expected number of applications must be lower there. However, the expected payoffs of receiving an offer from a high and a low productivity firm is the same as receiving offers from two low productivity firms because a high productivity firm that (Bertrand) competes with a low productivity firm for the same candidate will win and pay the productivity level of the worker at the low productivity firm. So, the worker's payoffs conditional on getting two offers are the same for a worker who sends both applications to the low productivity sector (LL) and a worker who plays HL, but the probability of receiving two offers is higher for the first worker. We then show that there is a unique mixed strategy equilibrium where workers send both applications with probability q^*_{HH} to the high productivity sector and with probability $1 - q^*_{HH}$ to the low productivity sector where q^*_{HH} depends on the relative productivity and the relative supply of vacancies in each of the sectors. As in Albrecht et al. (2006) there are two coordination problems in the matching process: (1) workers do not know where other workers apply to and (2) firms do not know which candidate other firms consider.

By allowing workers to apply to different sectors, the degree of coordination frictions becomes partly endogenous, even for a given number of applications per worker. However, workers do not internalize the effects of their portfolio choice on the employment opportunities of other workers. They just want to maximize the productivity-weighted probability to receive multiple offers. We show that the resulting equilibrium is not efficient and unemployment is too high. An important reason for the inefficiency is that a social planner would like some or all workers to apply to both sectors in order to reduce the coordination problems in the matching process. More H matches can be realized by letting workers accept the job in the most productive sector in case of multiple offers. In the market, workers never play HL because the expected payoffs of this strategy are too low, since high productivity firms would either pay the monopsony wage or the productivity level of a low productivity firm in case the worker has two offers. Since the expected payoff of playing HL is independent of high productivity output, workers incentives are distorted. Another source of inefficiency is that because of the coordination frictions, the matching function is non-monotonic in the number of applications. When there are relatively few vacancies, the second coordination problem is severe and the matching rate is decreasing in the number of applications. The planner internalizes this while individual workers diversify too little and apply too often to the high productivity sector. A similar problem arises in the academic job market or the market for Ph.D. candidates where the top universities typically receive too many applicants.[1]

If the number of firms in the market or the difference in productivity between both sectors is not too large, the equilibrium outcomes under random search are the same as in the directed search equilibrium where firms can post a wage ex ante and workers observe all wages.[2] The reason for this is the same as the one in Albrecht et al. (2006) where posted wages are zero. They consider the case where all workers and firms are identical and show

[1]In small labor markets, more matches are realized if all workers play HL than if 50% plays LL and 50% plays HH. However, in large labor markets there is no difference between these two cases.

[2]Usually, the equilibrium in directed search models is constrained efficient, e.g. Burdett et al. (2001), Moen (1997), Montgomery (1991), Peters (1991).

that the existence of ex post competition makes it still attractive for workers to apply to firms who offer the monopsony wage. Offering a higher wage than the monopsony wage only marginally increases the number of applicants in expectation, because workers mainly care about the probability to get multiple offers, while the expected firm payoffs in case of a match drop linearly. This implies that our results are not driven by the fact that search is random because for a fixed supply of vacancies and applications, the Albrecht et al. (2006) model is constraint efficient while the directed search version of our model is not.

In section 2.4 we also allow for free entry of vacancies. We do this by allowing the output of both sectors to be traded in a competitive goods market where consumers with love-for-variety demand both types of goods.[3] Now, not only the workers' incentives are distorted, but also firms' incentives are distorted. Vacancy supply in each sector can both be too high or too low while typically, the market assigns too few workers to the high productivity sector. Even if we restrict the planner to playing only HH and LL, the inefficiency remains.

There are a couple of other papers related to what we do. First, Shimer (2005a) and Shi (2002) consider a directed search model with two-sided heterogeneity where workers can only apply to one job and ex post competition is irrelevant. They find that the decentralized market outcome is constrained efficient. We show that this result may break down if workers can simultaneously apply to multiple jobs and there is ex post competition for their services. In Gautier and Moraga-González (2005) workers and firms are also identical and workers only learn about the wage after a firm is contacted. There, wages and the number of applications are determined in a simultaneous move game and the worker's portfolio problem is trivial: each application should go to a random vacancy. Chade and Smith (2006) and Galenianos and Kircher (2008) also consider portfolio problems of workers who can apply to multiple jobs. In the latter paper, all jobs have the same productivity but because firms must commit to their posted wages they respond to the worker's

[3] The fixed vacancy supply case can be considered to be a special case with Leontief demand. Further, if output in both sectors are perfect substitutes, only one good will be produced namely the one where the expected value of a vacancy is highest.

desire to diversify. This desire to diversify is driven by the fact that the expected payoff is equal to the maximum wage offer of a worker and not to the average one. This also creates non-trivial portfolio problems. Interestingly, because of the ex ante wage commitment of firms, workers diversify as much as possible over the different wages that are offered by the firms. Chade and Smith (2006) is not an equilibrium model but it considers a general class of portfolio problems in the absence of ex post competition. Finally, Davis (2001) analyzes a model in which workers and firms can decide to invest in human capital and job quality respectively. Because they cannot capture the full increase of the match surplus generated by these investments, both firms and workers tend to underinvest. In equilibrium there is excessive supply of inferior jobs and inferior workers.

The chapter is organized as follows. Section 2.2 describes the basic version of the model in which the number of vacancies is assumed to be exogenously given. We derive the equilibrium and determine whether it is efficient. In section 2.3 we check whether our conclusions are sensitive to the simplifying assumptions we make. In section 2.4 we extend the model by allowing for free entry. Finally, section 2.5 concludes.

2.2 Basic Model

2.2.1 Labor Market

Consider a labor market with u risk neutral workers and v risk neutral firms. All workers are identical, but the firms are divided into two different types. There are v_H high-productivity firms and v_L low-productivity firms, with $v = v_H + v_L$. We refer to those firms as *highs* and *lows*. Each firm has exactly one vacancy.

Workers can send zero, one, or two applications at costs $k > 0$. Those applications can be directed to a specific type of vacancy, but workers do not observe ex ante the wage that a particular firm offers. If a worker receives multiple job offers, there is Bertrand competition for his services. Basically, workers must decide whether they want to send both applications to high type vacancies, both applications to low type vacancies, or one application to a high type and one to a low type vacancy. In section 2.2.5 we show that if there are not too many firms in the market and if the productivity of the low type firms is not too small, our results carry over to a directed search setting, where workers observe ex ante the wage offered by each individual firm.

We make three important further assumptions. First, we assume that the labor market is large, i.e. $u \to \infty$ and $v \to \infty$, keeping $\theta_i \equiv v_i/u$ fixed $\forall i \in \{H, L\}$. For the moment, we assume that θ_H and θ_L are exogenously given. We relax this assumption in section 2.4. Second, we focus on symmetric equilibria, which means that identical agents must have identical strategies. Third, we assume that the labor market is anonymous: firms must treat identical workers identically and vice versa. So, a worker's strategy may only be conditioned on the type (H or L) of the firm. This excludes equilibria that require a lot of coordination amongst workers, something that seems hard to imagine in a large labor market.

2.2.2 Setting of the Game

The model that is closest related to ours is the one used in Albrecht et al. (2006). There are two important differences: (i) we allow for heterogeneity amongst firms and (ii) search is not fully directed (in section 2.2.5 we discuss the directed search equilibrium). The setting of the game is as follows:

1. Each vacancy posts a wage mechanism.

2. Workers observe all vacancy types (but not the wage mechanism) and send $a \in \{0, 1, 2\}$ applications. In section 2.2.5 we allow workers to also ex ante observe the wage mechanism.

3. Each vacancy that receives at least one application, randomly selects a candidate. Applications that are not selected are returned as rejections.

4. A vacancy with a processed application offers the applicant the job. If the applicant receives more than one offer, the firms in question can increase their bids as often as they like.

5. A worker that receives one job offer will accept that offer as long as the offered wage is non-negative. A worker with two offers will accept the one that gives him the highest wage, or will select a job randomly if the offered wages are equal.

If a type i firm matches with a worker, it produces y_i units of output. Without loss of generality we assume that $y_L < y_H = 1$. The payoff of a firm that matches with a worker equals $y_i - w$, where w denotes the wage that the firm pays. A worker hired at wage w receives a payoff that is equal to that wage. Workers and firms that fail to match receive payoffs of zero.

2.2.3 Decentralized Market

We start the analysis of the decentralized market by showing that no firm posts a positive wage. This is basically the Diamond (1971) paradox.

Lemma 2.1 *In equilibrium all firms post a wage equal to zero.*

Proof. Note that workers can direct their applications to a specific kind of vacancy, but not to a particular firm. So, posting a higher wage (or more general: a more generous wage mechanism) does not attract more applicants and does not affect the matching probability. This implies that there is no incentive for a firm to offer the worker more than zero.[4] ■

A direct result of this lemma is that workers never send only one application.

Corollary 2.1 *No equilibrium exists in which there are workers that only send one application.*

Proof. Note that if a worker sends one application, there will never be ex post competition for his services. Firms offer a wage equal to zero, so the worker's payoff always equals $-k$. Hence, applying to one job is strictly dominated by not applying at all and therefore never part of an equilibrium strategy. ■

[4] Note that this argument implies that posting a wage equal to zero does not only dominate posting a strictly positive wage, but also all other feasible wage mechanisms.

Whether a worker applies twice or not at all depends on the cost k of sending an application. For example if $k > 0.5$, each worker will decide not to apply, because applying twice costs more than the competitive wage ($2k > 1 = y_H$). On the other hand, all workers apply to two jobs if k is sufficiently small, because this gives a strictly positive expected payoff, while not applying results in a payoff of zero. Here we restrict ourselves to the situation in which k is small enough to guarantee that $a = 2$ with probability 1.[5] In this respect our model differs from Shimer (2005a) and Shi (2002) where $a = 1$.

Three different strategies are possible: a worker can either apply to two high type vacancies, two low type vacancies, or one high type and one low type of vacancy. Denote the respective probabilities by q_{HH}, q_{LL}, and q_{HL}, where $q_{HH} + q_{LL} + q_{HL} = 1$. Using the fact that each worker uses the same strategies, this implies that the total number of applications to firms of type i is equal to $(2q_{ii} + q_{HL})\,u$. The expected number of applications a specific vacancy receives, is therefore given by

$$\phi_i\left(q_{ii}, q_{HL}, \theta_i\right) = \frac{2q_{ii} + q_{HL}}{\theta_i}. \tag{2.1}$$

Since our labor market is large, the actual number of applications to a specific vacancy follows a Poisson distribution with mean ϕ_i.[6] Next, consider an individual who applies to a type i firm. The number of competitors for the job at that firm also follows a Poisson distribution with mean ϕ_i, because there is an infinite number of workers. In case of n other applicants, the probability that the individual in question will get the job equals $\frac{1}{n+1}$. Therefore, the probability that an application to a type i firm results in a job offer equals

$$\psi_i = \sum_{n=0}^{\infty} \frac{1}{n+1} \frac{e^{-\phi_i}\phi_i^n}{n!} \tag{2.2}$$

$$= \frac{1}{\phi_i}\left(1 - e^{-\phi_i}\right). \tag{2.3}$$

[5] An explicit expression for the upper bound K on k in that case is derived below.

[6] For ease of exposition we omit the arguments of functions whenever this does not lead to confusion.

Note that this expression is not well defined for $\phi_i = 0$. For convenience we define $\psi_i(0) = \lim_{\phi_i \to 0} \psi_i(\phi_i) = 1$.

Whether a worker's second application results in an offer does not depend on whether the first application was successful or not. A worker who plays ij (i.e. applies to a type i firm and a type j firm) with $i, j \in \{H, L\}$ therefore has a probability $\psi_i \psi_j$ of getting two job offers and a probability $\psi_i \left(1 - \psi_j\right) + \psi_j \left(1 - \psi_i\right)$ of getting one job offer. The matching probability of such a worker equals one minus the probability that he does not get a job offer and is therefore equal to $1 - \left(1 - \psi_i\right)\left(1 - \psi_j\right)$ (see Albrecht et al., 2006 for a proof in the case with homogeneous firms). This matching probability is obviously strictly increasing in both ψ_i and ψ_j and depends on the worker's portfolio choice.

If a worker receives two high job offers, Bertrand competition between the two firms results in a wage equal to $y_H = 1$. In case of two low offers, the firms increase their bids until the worker's wage equals y_L. A combination of one high and one low offer also implies a wage of y_L, because at that wage level the low type firm is no longer willing to increase its bid. This is the standard result from Bertrand competition. As shown above, a worker who receives only one job offer gets a wage equal to zero.

Next, we prove that workers never send one application to a high and one to a low productivity firm:

Lemma 2.2 *Workers never play HL, since this strategy is strictly dominated.*

Proof. The expected payoff for a worker who plays HL is $\psi_H \psi_L y_L - 2k$, i.e. the probability that he receives two job offers times the productivity of the low type firm minus the application cost. Likewise, the expected payoffs of playing HH and LL are $\psi_H^2 y_H - 2k$ and $\psi_L^2 y_L - 2k$ respectively. Suppose that $\psi_H \geq \psi_L$. In that case all workers play HH, since that strategy gives a strictly higher payoff than HL and LL. This however implies that $\phi_L = 0$ and thus that $\psi_L = 1$, which contradicts $\psi_H \geq \psi_L$. Hence, in equilibrium it must be the case that $\psi_L > \psi_H$. Then, playing LL gives a strictly higher payoff than HL. So, HL is strictly dominated. ■

In the following proposition we show that the model has a unique equilibrium for all parameter values.

Proposition 2.1 *A unique equilibrium exists for any $\theta_H > 0$, $\theta_L > 0$, and $y_L \in (0,1)$. The equilibrium is a pure strategy equilibrium if and only if the following condition holds:*

$$\frac{\theta_H^2}{4}\left(1-\exp\left(-\frac{2}{\theta_H}\right)\right)^2 \geq y_L. \tag{2.4}$$

Otherwise, the equilibrium is a mixed strategy equilibrium, which can be characterized by the value q_{HH}^* that solves the equality $\psi_H^2 = \psi_L^2 y_L$.

Proof. We can rule out the possibility that workers play HL because of lemma 2.2. First, note that an equilibrium in which $q_{LL} = 1$ does not exist, since a deviant that applies twice to a high firm gets a higher payoff (y_H) than the equilibrium payoff $\psi_L^2 y_L < y_L$.[7]

On the other hand, $q_{HH} = 1$ can be an equilibrium if y_L is low enough. The equilibrium payoff in this case equals $\psi_H^2 = \frac{\theta_H^2}{4}\left(1-\exp\left(-\frac{2}{\theta_H}\right)\right)^2$. Deviating to LL gives a wage y_L for sure. So, $q_{HH}^* = 1$ is an equilibrium if condition (2.4) holds.

If condition (2.4) does not hold, only a mixed strategy equilibrium can exist, in which the workers are indifferent between playing HH and LL, i.e. where $\psi_H^2 = \psi_L^2 y_L$. If we substitute $q_{LL} = 1 - q_{HH}$, the only free parameter in this condition is q_{HH}. To see that a unique equilibrium value q_{HH}^* exists, note that the left hand side of the condition is continuous and strictly decreasing in q_{HH}, while the right hand side is continuous and strictly increasing in q_{HH} (see figure 2.1). Furthermore, we have

$$\lim_{q_{HH}\to 0}\psi_H^2 = 1 > \frac{\theta_L^2}{4}\left(1-\exp\left(-\frac{2}{\theta_L}\right)\right)^2 y_L = \lim_{q_{HH}\to 0}\psi_L^2 y_L$$

and

$$\lim_{q_{HH}\to 1}\psi_H^2 = \frac{\theta_H^2}{4}\left(1-\exp\left(-\frac{2}{\theta_H}\right)\right)^2 < y_L = \lim_{q_{HH}\to 1}\psi_L^2 y_L.$$

[7] Since we only consider strategies in which workers apply twice, we can safely ignore the application cost k in this proof. This parameter only plays a role in comparing the payoffs of strategies that differ in the number of applications sent.

Applying the Intermediate Value Theorem now shows that there exists a unique value $0 < q^*_{HH} < 1$ such that $\psi_H^2 = \psi_L^2 y_L$ holds. ■

Hence, we have a pure strategy equilibrium in which all firms post a wage equal to zero and all workers apply twice to high type vacancies if condition (2.4) holds. This condition imposes very low upper bounds on y_L for any reasonable value of θ_H (e.g. $\theta_H = 0.5$ implies $y_L < 0.06$). The case in which the condition does not hold is therefore more interesting. Unfortunately, we are not able to derive an explicit expression for q^*_{HH}. Figure 2.1 shows the equilibrium as the intersection point of the ψ_H^2-curve and the $\psi_L^2 y_L$-curve for $\theta_H = \theta_L = 0.5$ and $y_L = 0.5$. For those values 63% of the workers plays HH, while 37% plays LL. Figure 2.2 shows the effect of the low productivity y_L on the equilibrium value q^*_{HH}. A change in this productivity level affects the payoff in case a worker receives two job offers from low type firms. A higher productivity of these firms is therefore associated with more applications to this sector.

In equilibrium the expected payoff for a worker equals $\psi_H^2 - 2k = \psi_L^2 y_L - 2k$. The requirement that this value should be larger than the payoff of not applying at all, i.e. zero, implies that k should be smaller than $\frac{1}{2}\psi_H^2 = \frac{1}{2}\psi_L^2 y_L$. This assumption seems reasonable. It is hard to imagine that the cost of a particular application exceeds half the expected wage of a job.

2.2.4 Efficiency

In the mixed strategy equilibrium that we derived in the previous subsection, a fraction q^*_{HH} of the workers matches with probability $1-(1-\psi_H^*)^2$ to a high firm and produce output $y_H = 1$. The remaining workers match with probability $1-(1-\psi_L^*)^2$ to a low firm and produce output y_L. The total output Y^* per worker in this equilibrium is therefore given by

$$Y^* = q^*_{HH}\left(1-(1-\psi_H^*)^2\right) + (1-q^*_{HH})\left(1-(1-\psi_L^*)^2\right) y_L.$$

The main question of this essay is whether the equilibrium value q^*_{HH} is constrained

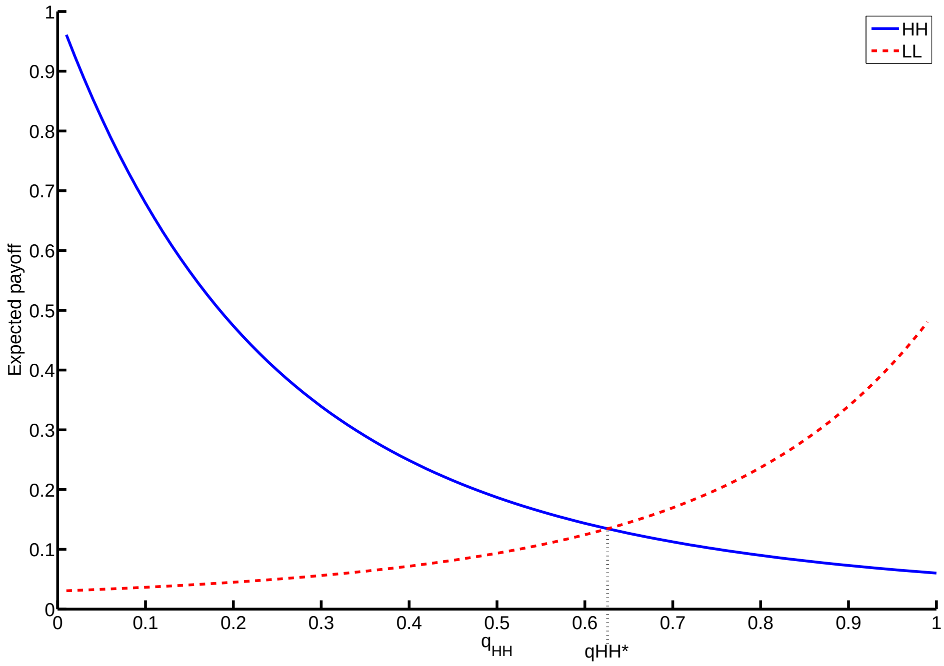


Figure 2.1: Expected payoff of playing HH and LL for $\theta_H = \theta_L = 1$ and $y_L = 0.5$.

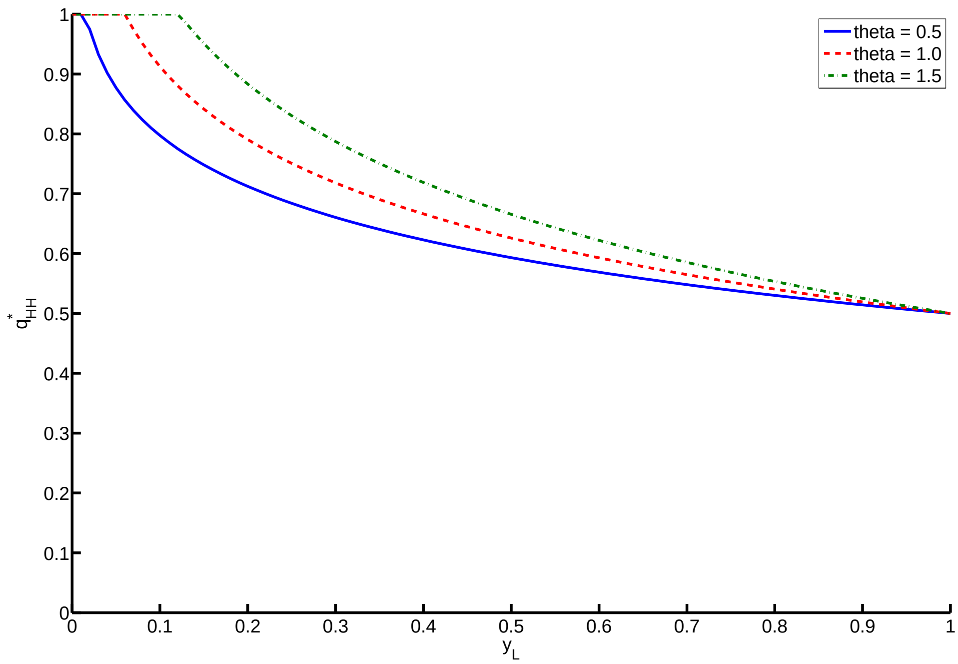


Figure 2.2: q^*_{HH} as a function of y_L for several values of $\theta_H = \theta_L = \frac{1}{2}\theta$.

efficient. In order to answer this question we consider a social planner who maximizes total output in the economy. The planner cannot eliminate the coordination frictions, but he can decide to which firms the workers apply. In other words, he can control q_{HH}, q_{LL}, and q_{HL}. In section 2.4 we allow for free entry of vacancies and let the planner also determine θ_H and θ_L. We assume that the social planner can also decide which job a worker will take if he receives both a high and a low job offer. Suppose that he sends a fraction α of those workers to the high type firm and a fraction $1-\alpha$ to the low type firm. Then we can derive χ_{ij}^{k}, $i,j,k \in \{H,L\}$, which represents the probability that playing ij results in a match with a type k firm. These probabilities are functions of α, ψ_H, and ψ_L:

$$\chi_{HH}^{H} = 1-(1-\psi_H)^2 \tag{2.5a}$$

$$\chi_{HL}^{H} = \alpha\psi_H\psi_L + \psi_H(1-\psi_L) \tag{2.5b}$$

$$\chi_{LL}^{L} = 1-(1-\psi_L)^2 \tag{2.5c}$$

$$\chi_{HL}^{L} = (1-\alpha)\psi_H\psi_L + \psi_L(1-\psi_H). \tag{2.5d}$$

The remaining probabilities, like χ_{HH}^{L}, are equal to zero. Using this notation, we can write the per-worker output created by the high and the low type firms as respectively:

$$Y_H = q_{HH}\chi_{HH}^{H} + q_{HL}\chi_{HL}^{H} \tag{2.6}$$

and

$$Y_L = \left(q_{LL}\chi_{LL}^{L} + q_{HL}\chi_{HL}^{L}\right) y_L. \tag{2.7}$$

This implies that the social planner's problem is:

$$\max_{q_{HH},q_{LL},q_{HL},\alpha} Y = \max_{q_{HH},q_{LL},q_{HL},\alpha} q_{HH}\chi_{HH}^{H} + q_{HL}\chi_{HL}^{H} + \left(q_{LL}\chi_{LL}^{L} + q_{HL}\chi_{HL}^{L}\right) y_L, \tag{2.8}$$

subject to $q_{HH}+q_{LL}+q_{HL}=1$.

Solving this maximization problem gives us the optimal values q_{ij}^{**} and α^{**}, which can be used to calculate Y^{**}, the level of output. First note that $\alpha^{**}=1$, i.e. when a worker gets a job offer from both a high type and a low type firm, the planner wants him to take the high type job. The intuition for this result is clear. If a worker receives a job offer from both a high and a low firm, he must always take the job at the high type firm because his marginal productivity is higher there. Next, we can formally prove that the mixed strategy market equilibrium is inefficient: the social planner creates a higher output.

Proposition 2.2 *The equilibrium described in proposition 2.1 is not constrained efficient if* $y_L > \exp\left(-\frac{2}{\theta_H}\right)$.

Proof. Note that $\exp\left(-\frac{2}{\theta_H}\right) < \frac{\theta_H^2}{4}\left(1-\exp\left(-\frac{2}{\theta_H}\right)\right)^2 \forall \theta_H$. First, consider the pure strategy equilibrium in which $q_{HH}^* = 1$. Let the planner instead impose $\alpha = 1$, $q_{HH} = q_{HH}^* - q_{HL}$, q_{HL}, and $q_{LL} = 0$. This generates output equal to

$$Y = (1-q_{HL})\left(1-(1-\psi_H)^2\right) + q_{HL}\left(\psi_H + \psi_L(1-\psi_H)y_L\right)$$

Taking the derivative with respect to q_{HL} and evaluating the result in $q_{HL} = 0$ gives

$$\left.\frac{\partial Y}{\partial q_{HL}}\right|_{q_{HL}=0} = (1-\psi_H)\left(y_L - \exp\left(-\frac{2}{\theta_H}\right)\right),$$

which is positive if $y_L > \exp\left(-\frac{2}{\theta_H}\right)$. Hence, the pure strategy equilibrium is not constraint efficient if this condition holds.

Second, consider the case in which $y_L > \frac{\theta_H^2}{4}\left(1-\exp\left(-\frac{2}{\theta_H}\right)\right)^2$. This implies a mixed strategy market equilibrium in which a strictly positive fraction of the workers sends two applications to the high sector and another strictly positive fraction sends two applications to low type firms. Next, consider a social planner who faces this equilibrium. One way in which he can increase output is by selecting a worker that plays HH and a worker that plays LL and by letting them both diversify their applications amongst the sectors. By matching HH-workers and LL-workers in this way, the total number of vacancies in each sector remains constant, implying that the matching probabilities ψ_H^* and ψ_L^* do not change. So, let the planner impose $\alpha = 1$, $q_{HH} = q_{HH}^* - \frac{1}{2}q_{HL}$, q_{HL}, and $q_{LL} = q_{LL}^* - \frac{1}{2}q_{HL} = 1 - q_{HH}^* - \frac{1}{2}q_{HL}$, where the market equilibrium corresponds to $q_{HL} = 0$. The output Y in that case equals

$$\begin{aligned} Y &= \left(q^*_{HH} - \frac{1}{2}q_{HL}\right)\left(1-(1-\psi^*_H)^2\right) + q_{HL}\left(\psi^*_H + \psi^*_L(1-\psi^*_H)\,y_L\right) \\ &\quad + \left(q^*_{LL} - \frac{1}{2}q_{HL}\right)\left(1-(1-\psi^*_L)^2\right) y_L. \end{aligned}$$

Taking the derivative with respect to q_{HL} gives

$$\begin{aligned} \frac{\partial Y}{\partial q_{HL}} &= \frac{1}{2}\left(\psi^{*2}_H - 2\psi^*_H\psi^*_L y_L + \psi^{*2}_L y_L\right) \\ &> \frac{1}{2}\left(\psi^*_H - \psi^*_L\sqrt{y_L}\right)^2 \geq 0. \end{aligned}$$

This expression is strictly positive for all q_{HL}. Hence, the mixed strategy market equilibrium is not constrained efficient. ■

From this proof it is immediately clear that q_{HH} and q_{LL} cannot both be strictly larger than zero in the planner's solution. The planner can match HH-workers and LL-workers and thereby increase output until one of both groups is completely exhausted. Note that although the resulting situation generates a higher social welfare than the market equilibrium, there is no reason to believe that it is the optimum. Other strategies might increase welfare even more. Unfortunately, an explicit expression for the planner's solution cannot be derived, because of the non-invertibility of ψ_i and χ^i_{ij}. Therefore, we maximize equation (2.8) numerically.[8]

We find that for many values of $\{\theta_H, \theta_L, y_L\}$ the planner lets all workers play HL. This is for example the case for $\theta_H = \theta_L \leq 0.5$ and $y_L \in \left(\frac{\theta_H^2}{4}\left(1-\exp\left(-\frac{2}{\theta_H}\right)\right)^2, 1\right)$. As mentioned above, this contrasts with the decentralized market where nobody plays HL. Workers do not play HL because they are only interested in getting two job offers in the same sector. However, from the planner's point of view two job offers to the same worker is always inefficient, because in that case one firm remains unmatched, while it could have matched with a worker without any job offers. Hence, all workers ideally receive

[8] The numerical results in this essay are obtained using Ox version 3.40 (see Doornik, 2002).

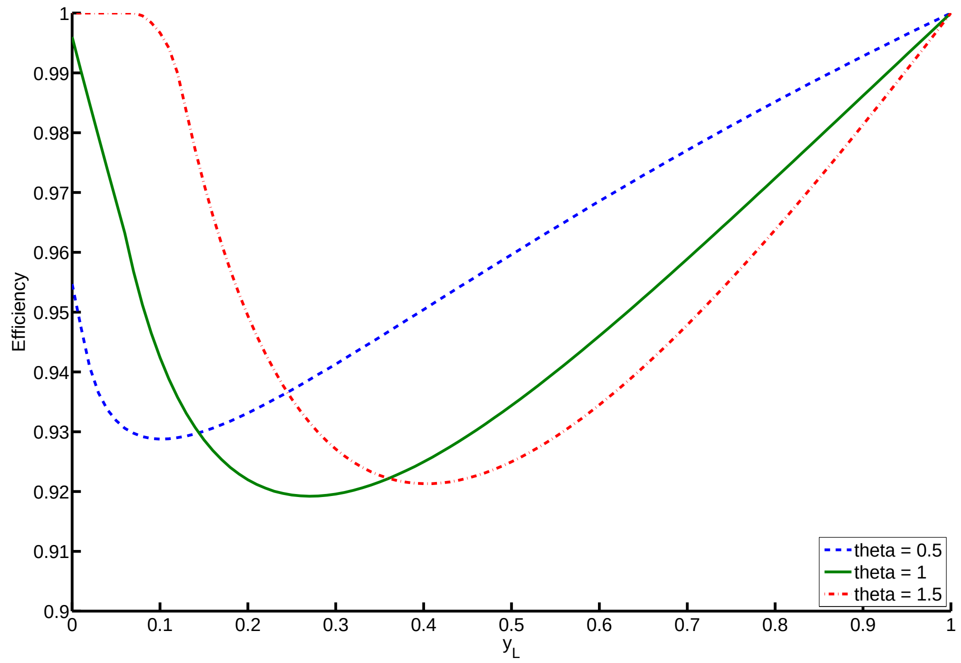


Figure 2.3: Efficiency of the decentralized equilibrium (Y^*/Y^{**}) as a function of y_L for several values of $\theta_H = \theta_L = \frac{1}{2}\theta$.

only one job offer. The planner can however not coordinate the job offers, so the only way in which he can reduce the coordination problem is by spreading the applications as much as possible, i.e. by playing HL. The planner only considers HH or LL if (i) the productivity of the L-types firms is very low, (ii) the number of firms in the market is very large, or (iii) there is a large difference between the number of high type firms and the number of low type firms.

Next, we consider the ratio $\frac{Y^*}{Y^{**}}$, i.e. the ratio between the total output in the decentralized equilibrium and the output level created by the social planner. This ratio is displayed in figure 2.3. This figure confirms that the decentralized equilibrium is in general not efficient. The output in the mixed strategy equilibrium is only equal to the optimal level for $y_L = 1$ because then there is essentially no difference between high and low firms. For $y_L = 0$, the market equilibrium is not efficient for $\theta = \theta_H + \theta_L = \frac{1}{2}$ or 1 because the optimal number of applications per worker to the H-sector is smaller than 2 for those values of θ. The planner can use the L-sector as "garbage can" to reduce the number of applications to the H-sector which reduces the probability that two firms consider the same candidate. For $\theta = \frac{3}{2}$, the optimal number of applications is equal to 2 and the market equilibrium is constrained efficient. We also see that for low values of y_L, the equilibria with high θ perform relatively well as compared to the planner's choice, while for high values of y_L, the equilibria with low θ are closer to the constrained optimum. In the first case, almost all workers play HH, which makes the second coordination friction large (many H-firms loose their candidate to a rival firm). When θ is large, this second coordination friction is less severe. For larger values of y_L, it is less desirable to play HH because L-firm matches become more valuable but for high θ, $\frac{\partial q_{HH}}{\partial y_L}$ is smaller (see figure 2.2), so q^*_{HH} adjusts too slow and therefore the low-θ equilibria are closer to the planner's solution.[9]

The model has two important characteristics that could both potentially cause the inefficiency: (i) the fact that workers in the decentralized market never play HL, while the social planner does and (ii) the fact that workers can not direct their applications to specific firms. Below we prove that our results are not driven by (ii). First, we show that (i) is neither solely responsible for the inefficiency: when we do not allow the social planner to let workers play HL, then he still does better than the market.

Proposition 2.3 *A social planner who cannot impose HL, but only HH and LL, generates a higher output than the decentralized market.*

Proof. The only decision variable for a planner who cannot impose HL is q_{HH}, the fraction of workers sending two applications to the high type sector. The remaining workers, a fraction $q_{LL} = 1 - q_{HH}$, applies twice to low type firms. Hence, output in this case equals

$$Y = q_{HH}\left(1-(1-\psi_H)^2\right) + (1-q_{HH})\left(1-(1-\psi_L)^2\right)y_L. \tag{2.9}$$

The derivative of output with respect to q_{HH} is equal to

$$\frac{\partial Y}{\partial q_{HH}} = \psi_H^2 + 2e^{-\phi_H}(1-\psi_H) - \psi_L^2 y_L - 2e^{-\phi_L}(1-\psi_L)y_L. \tag{2.10}$$

[9] Note that we do not say that the low θ equilibria are more desirable. Decreasing θ lowers output, but the planner's output decreases as well.

Evaluating this expression in the market equilibrium gives

$$\left.\frac{\partial Y}{\partial q_{HH}}\right|_{q^*_{HH}} = 2e^{-\phi^*_H}(1-\psi^*_H) - 2e^{-\phi^*_L}(1-\psi^*_L)\,y_L, \tag{2.11}$$

which typically is not equal to zero. ■

This result implies that the inefficiency does not only depend on the fact that workers fail to diversify their applications over the sectors. Part of the inefficiency arises due to the non-optimal way in which workers choose between HH and LL. For example, for $\theta_H = \theta = 0.75$ it turns out that workers play LL too often as compared to HH for most values of y_L. However, it is good to note that, although strictly positive, the level of inefficiency is relatively small. For the considered values of θ_H and θ_L, it never exceeds 1%.

2.2.5 Directed Search Equilibrium

In this subsection, we investigate to what extent the inefficiency in our model depends on the assumption of random search. In other words, we check whether efficiency would be restored if we allow workers to direct their applications to specific wages. We find that this is not the case. General expressions for an equilibrium in a directed search framework are hard to derive, but the equilibrium outcomes of our model coincide with the equilibrium outcomes of a directed search model for many values of θ_H, θ_L, and y_L.

Compared to the setup described in the previous section there is one important difference: workers can observe the wages posted by the firm before they send out their applications. This allows the worker to choose not only the sectors but also the wages to which he wants to apply.[10] Let $\psi_i(w)$ denote the probability that an application send to a firm in sector i offering wage w results in a match. Then, the worker's problem is to

[10]Note that given the anonymity assumption, a worker randomizes over all firms in a specific sector that are offering the same wage.

choose sectors i and j and wages w_1 and w_2 that maximize his expected payoff:

$$\psi_i(w_1)\left(1-\psi_j(w_2)\right)w_1+\psi_j(w_2)\left(1-\psi_i(w_1)\right)w_2+\psi_i(w_1)\,\psi_j(w_2)\min\left\{y_i,y_j\right\}.$$

Firms take this into account when they decide which wages to post. This provides firms with an incentive to consider positive wages because a higher wage leads to a larger arrival rate of applicants. Nevertheless, for many parameter configurations, all firms post wages equal to zero, as we state in the following proposition.

Proposition 2.4 *Assume that k small enough to guarantee that all workers send two applications.*[11] *Then, for θ_H and θ_L sufficiently small or for y_L sufficiently large, the equilibrium outcomes described in section 2.2.3 are the same as in the directed search version of our model where workers observe all wages before they apply.*

Proof. See appendix. ■

Figure 2.4 shows for which values of $\theta_H = \theta_L = \frac{1}{2}\theta$ and y_L the random search equilibrium values are the same as the directed search equilibrium values. The intuition is the same as in Albrecht et al. (2006). First, posted wages are lower if workers apply to multiple jobs than if they apply to one job because Bertrand competition makes it very valuable to have multiple offers. So workers place a relatively larger weight on short expected queue length than on posted wages. The reason that wages go down all the way to zero is that the benefits of a downward deviation are constant but the cost of a downward deviation (in terms of less applications) are decreasing in the wage. As we prove in the appendix, only for the low type sector there exist configurations for which there is a profitable deviation from the candidate equilibrium where all firms post $w_L = 0$. For example, if there are many firms relative to workers or if the low type firms have a low productivity, which makes it unattractive for the workers to apply there, $w_L > 0$ and the standard positive

[11] Under directed search we can have an equilibrium with $a = 1$ for some values of k. Since this is a special case of the model described in Shimer (2005a), we focus on sufficiently low values of k such that $a = 2$.

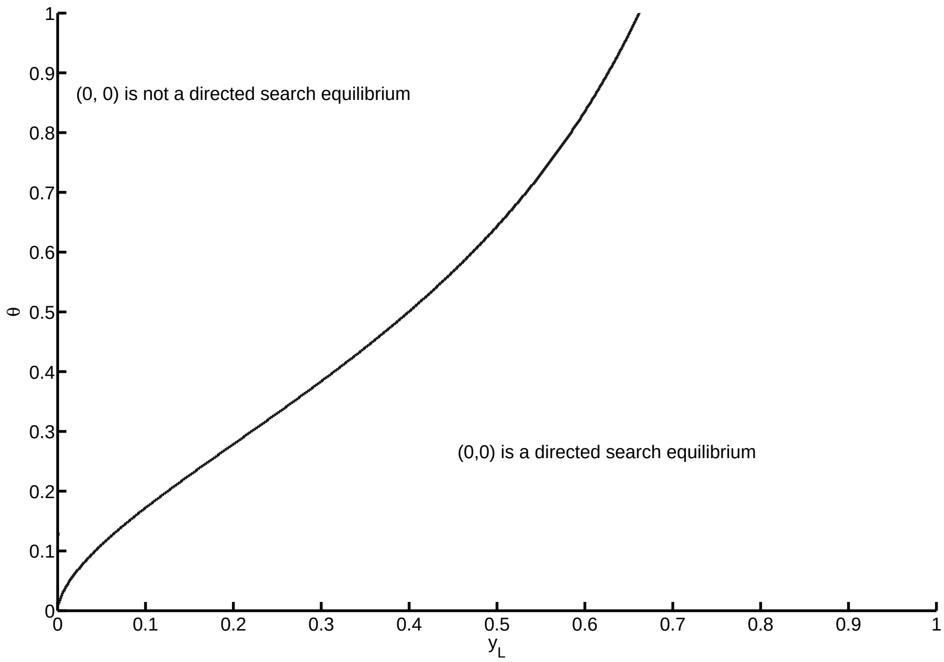


Figure 2.4: Combinations of y_L and $\theta_H = \theta_L = \frac{1}{2}\theta$ for which $\{w_H = 0, w_L = 0\}$ is a directed search equilibrium.

relation between posted wages and productivity can break down. In Postel-Vinay and Robin (2002b) this happens for similar reasons. In their model, workers agree to accept a lower initial wage at high productivity firms because of future possibilities of wage increases through Bertrand competition with rival firms. In the directed search version of our model, high productivity firms always get away with posting the reservation wage while low productivity firms do not because the payoff of receiving multiple offers from high productivity firms is more attractive than from low productivity firms.

The fact that the equilibrium values under random search and directed search can coincide implies that the inefficiency of the decentralized equilibrium can not be eliminated by making search fully directed. This result is contradictory to for example Burdett et al. (2001) and Moen (1997), who found that the equilibrium in their directed search models was constrained efficient.

To sum up, for a fixed supply of vacancies the market equilibrium is inefficient predominantly owing to workers never playing HL. Playing HL has the advantage that more H-

matches can be realized by setting $\alpha = 1$ (in case of two offers, always take the H-offer). Therefore, the coordination frictions are larger than necessary. Interestingly, Galenianos and Kircher (2008) also find that worker's market portfolios of applications are socially inefficient. They only have ex ante competition for workers and show that even if workers and firms are homogeneous, workers have a desire to diversify and firms respond to this desire by offering different wages. In their model, workers choose to apply both to the high and the low wage firms but with a higher probability to the high wage firms whereas it would be socially efficient if workers apply to each firm with equal probability. Finally, note that in Albrecht et al. (2006) the portfolio inefficiency is absent because they consider both identical workers plus jobs and allow for ex post competition. They show that entry is excessive when workers apply to multiple jobs. In this section we fixed θ_i, so their inefficiency does not arise here. In section 2.4 we relax this assumption to see whether the entry decision is also distorted in our model.

2.3 Robustness

In this section we discuss to what extent our results are sensitive to the following three simplifying assumptions we made: (i) there are only two firm types, (ii) a worker cannot send more than two applications, and (iii) if a firm fails to hire its candidate it cannot make an offer to the next candidate.

More than two firm types

Suppose there are N rankable firm types where $y_{n+1} > y_n$. Then it is straightforward to show that workers never diversify because the application-portfolio strategy, $(n+i,n)$, is dominated by (n,n). The only way for workers to receive a positive payoff is by getting two job offers. For both portfolios, Bertrand competition leads to a wage of y_n but because the expected queue length is shorter in the least productive sector, the probability of receiving two offers is larger for the (n,n) than for the $(n+i,n)$ portfolio. One can easily generalize proposition 2.2 to show that also in this case the market outcome is inefficient.

Therefore, considering only two firm types is not restrictive.

More than two applications

The second simplifying assumption is that a worker cannot send more than two applications. Allowing workers to apply to more than two jobs makes the analysis more difficult but does not change the nature of the portfolio problem. Still workers are only interested in the productivity-weighted probability to get more than one job offer, while the social planner wants to spread applications in order to reduce the coordination frictions. So, the fact that we restrict the workers to at most two applications is not driving our main result. If we allow workers to send three applications, (HHL) can be a symmetric equilibrium portfolio for very large θ_L and θ_H and y_L. The L-application is used to increase the probability of two offers. θ_L must be sufficiently large to make this effect large enough, y_L must be sufficiently large to make the payoffs of HL-offers close to the payoffs of HH-offers and θ_H should be sufficiently large that it is not profitable to play (HHH). If workers apply to four jobs there exist more equilibria with diversification. Suppose $\theta_L \to \infty$, then for y_L sufficiently high, workers will send two applications to the L-sector which will result in two offers with a probability close to one. The marginal contribution of sending the remaining two applications to the L-sector are close to zero so they can best be sent to the H-sector. For five and more applications we cannot rule out regions where workers send three applications to the L-sector and the rest to the H-sector. This only happens for θ_L sufficiently large but smaller than one. The L-applications are used to secure a job while the H-applications are used to get a large payoff. We do know for sure that workers never send just one application to the H-sector $\forall a$ because the resulting wage in case of HL-offers equals the wage in case of LL-offers but the probability of occurrence is higher for the LL-portfolio.

The desire to diversify in our model is less than in Chade and Smith (2006) or Galenianos and Kircher (2008) who only have ex ante competition but no ex post competition for workers. This is caused by the fact that in our model the wage is not determined by the productivity at the most productive firm but by the productivity of the second-highest-productivity firm that makes an offer. In the portfolio problems that they consider, the firms commit ex ante to a wage. Under ex post competition, workers have incentives to generate similar offers. Allowing workers to send more than two applications will not restore efficiency because the planner will reduce coordination frictions by letting workers diversify as much as possible between sectors while workers have strong incentives to send applications to the same sector.

Finally, note that in our setting the marginal improvement algorithm (MIA) of Chade and Smith (2006) does not work. This algorithm first picks the application with the highest expected payoff, the next application is sent to the location with the highest marginal improvement and so on and so forth. If the marginal contribution of an application is negative then the previous one is the final application. In our setting, the first application has a negative marginal payoff. Moreover, if an agent has played LL, an additional H-application always has a smaller marginal contribution to the portfolio than a single L-application but as we argued before, for some configurations, the $LLHH$-portfolio dominates the $LLLL$-portfolio. This makes it computationally hard to find the optimal portfolio for the case with many firm types and many applications.[12]

Multiple job offers

The third important assumption is that firms can offer the job to one worker only. This can be restrictive even if we assume that the marginal productivity of a second worker is zero. For example, it can be profitable for a firm to increase its matching probability by offering the same job to more applicants. The drawback of this strategy is that the firm then runs the risk that more than one worker accepts the offer. In that case, the firm has to pay a wage to all the workers it hires, while only one of them can be used in producing output.

Deriving the optimal strategy in such a model is not straightforward. First, timing matters. Suppose that a firm gives two job offers. Initially, it offers a wage equal to zero to both applicants. If one of the candidates has also received another offer, the firm must

[12] There may exist algorithms where the marginal contribution of pairs or triples of applications can be used rather than comparing complete portfolios with each other but we have not been able to prove this.

decide whether it will compete for this worker. The strategy of the firm depends on the result of the second job offer it has made. Therefore, one must make assumptions about the exact moment at which the firm learns the result of each job offer.

One way to solve the timing problem is by assuming that if their candidate has multiple offers, the firms participate in a second-price sealed bid auction, rather than Bertrand competition.[13] In that case all firms submit one bid w_i and the bids are revealed simultaneously. The winning firm hires the worker and pays a wage equal to the bid of the competing firm (and zero if there was no competing firm). If firms can make only one job offer, it is optimal for them to bid the productivity level, $w_i = y_i$. Hence, in that case the payoffs are identical to the payoffs described in the previous sections, i.e. under the assumption of Bertrand competition.

If firms can however make more than one job offer, deriving the optimal wage offer remains difficult. First, it is relevant whether the other offer of the firm's candidate is at a firm with multiple candidates or not. If it is not, the other firm will bid more aggressively. Second, there is no pure strategy equilibrium because each candidate equilibrium wage pair is dominated by either offering one of the candidates a zero wage or offering them ε more. This is essentially the well known Burdett and Judd (1983) argument. An alternative is the shortlisting assumption of Albrecht et al. (2006) where firms pick a first candidate and a second candidate to whom they offer the job (if she is still available) in case they fail to hire their first candidate. At each of the firms they apply to, workers can be in three possible states: first candidate, second candidate or neither. This makes the algebra tedious but the bottom line is that none of the coordination frictions is eliminated . Even if a firm makes b job offers, it is still possible that it remains unmatched, because all the workers accepted offers from other firms. Moreover, workers still only care about receiving two offers while the planner wants to maximize the output-weighted number of matches. Finally, Gautier et al. (2005) and Kircher (2007) consider the case where firms can consider as many applicants as they like. Kircher (2007) shows that if firms commit to their posted wage, the directed search equilibrium is efficient. If firms can increase their

[13] See Julien et al. (2000), Kultti (1999) and Shimer (1999).

initial bids, in case their (final) candidate has multiple offers, the remaining equilibrium remains inefficient.

2.4 Goods Market and Free Entry

2.4.1 Setting of the Game

The aim of this section is to investigate whether heterogeneity distorts entry decisions under multiple applications. Therefore, we extend the basic model by introducing free entry of firms. Before creating a job opening, firms need to buy one unit of installment capital which costs c_H for high type firms and c_L for low type firms. If a firm matches with a worker, then it can use the value of the output to cover these costs. Otherwise, it incurs a loss. Risk-neutral firms enter until the point where expected benefits are zero.

Before we continue, it is good to note that the model discussed in the previous section implicitly assumed that the output created in the low type sector and in the high type sector were perfect substitutes to each other. Free entry is not so interesting in that case because usually a corner solution is obtained where either it is more profitable to create a low type vacancy or it is more profitable to create a high type vacancy. Therefore, we focus on a specification in which the goods are imperfect substitutes. Then we get an internal solution where both the L- and the H-commodity are produced.

Both types of firms now produce the same amount of output in case of a match ($y_H = y_L = 1$), but the value of these outputs on the goods market may differ. Those values are denoted by $p_H = 1$ (after normalization) and p_L respectively.[14] The demand on the goods market is determined by the workers who receive utility from consuming the high and the low commodity according to the following Cobb-Douglas utility function with the

[14]The assumption $y_L = 1$ is without loss of generality, since only the total value of the output, i.e. $y_L p_L$, is relevant in our analysis. Fixing y_L to a value different from 1 therefore only implies a rescaling of p_L.

exogenously given constant $0.5 < \lambda < 1$:[15]

$$u(x_H, x_L) = x_H^{\lambda} x_L^{1-\lambda}, \tag{2.12}$$

where x_i represents the consumption of commodity i. Consumers maximize this utility function under the budget constraint

$$x_H + p_L x_L \leq w, \tag{2.13}$$

where w denotes the wage of the worker. Basically, output from both sectors is traded in a competitive goods market where λ reflects the relative preference for the H-good. Individuals have love for variety and therefore strictly positive quantities of both goods are consumed. The other characteristics of the model remain the same. Workers still send two applications and firms can increase their initial bid in case their candidate receives multiple offers.

2.4.2 Decentralized Market

Several of the results derived for the basic model carry over to this more extended version. For example, it remains optimal for all firms to initially post a wage equal to zero. Again, if a worker receives two job offers, the firms will increase their bids and Bertrand competition pushes the wages to the marginal product. Therefore, the expected wage of a worker who applies twice to a type i firm is equal to $\psi_i^2 p_i$, the probability of receiving two job offers multiplied by the value of the output of a type i firm.

The main difference with the model of the previous section is that workers playing HL and receiving two job offers can now be hired by either the high or the low type firm. Which firm hires depends on the value of p_L, which now is an endogenous variable. As long as $p_L < 1$, the high type firm wins the Bertrand game and hires the workers at a

[15]Note that the labels *high* and *low* no longer refer to the productivity of a firm, since the productivity is assumed to be the same for both types. We nevertheless stick to these labels in order to keep notation consistent. Instead, one can interpret the labels in the following way: high type firms create a commodity that has a larger weight $\left(\lambda > \frac{1}{2}\right)$ in (2.12) than the commodity created by the low type firms $\left(1 - \lambda < \frac{1}{2}\right)$.

wage p_L. On the other hand, if $p_L > 1$ the worker matches with the low type firm at a wage equal to 1. In the case that $p_L = 1$ both firms employ the worker with probability $\frac{1}{2}$. Hence, the expected wage of a worker who plays HL is $\psi_H \psi_L \min\{1, p_L\}$.

However, again one can show that HL is dominated by either HH or LL. The proof is similar to the one in lemma 2.2. Only if $p_L = 1$ and $\psi_H = \psi_L$, workers are indifferent between playing HH, LL, and HL, but this is only because in that case all jobs are identical. In all other cases, workers will only consider playing HH and LL.

A firm of type i has a positive revenue if it attracts at least one applicant and if the worker to which it offers the job, does not receive a second job offer. The first event happens with probability $\left(1 - e^{-\phi_i}\right)$, while the probability of the latter equals $(1 - \psi_i)$.[16] Therefore, the expected profit of such a firm equals

$$\pi_i = \left(1 - e^{-\phi_i}\right)(1 - \psi_i)\, p_i - c_i, \tag{2.14}$$

which under free entry is equal to zero in equilibrium. From this, one can see that an equilibrium in which HL is not strictly dominated, i.e. with $p_L = 1$ and $\psi_H = \psi_L$, can only arise if $c_H = c_L$.

In equilibrium, the ratio of the prices of the commodities must equal the (absolute value of the) marginal rate of substitution (MRS):

$$\frac{p_L}{p_H} = \left.\frac{\partial U/\partial x_L}{\partial U/\partial x_H}\right|_{x_H = Y_H, x_L = Y_L} = \frac{1-\lambda}{\lambda}\frac{Y_H}{Y_L}. \tag{2.15}$$

The expected per-worker output created by the high type firms is $q_{HH}\left(1 - (1 - \psi_H)^2\right)$, while the low type firms produce $(1 - q_{HH})\left(1 - (1 - \psi_L)^2\right)$ per worker. So, equation (2.15) is equivalent to

$$p_L = \frac{1-\lambda}{\lambda}\frac{q_{HH}}{1 - q_{HH}}\frac{1 - (1 - \psi_H)^2}{1 - (1 - \psi_L)^2}. \tag{2.16}$$

Summarizing we can define the equilibrium as follows:

[16] Due to the infinite size of the labor market, these events are independent.

Definition 2.1 *An equilibrium in the decentralized market is a tuple* $\{p_L,\ \theta_H,\ \theta_L, q_{HH}\}$ *such that the following four conditions hold:*

$$\psi_H^2 = \psi_L^2 p_L \tag{2.17}$$

$$p_L = \frac{1-\lambda}{\lambda}\frac{q_{HH}}{1-q_{HH}}\frac{1-(1-\psi_H)^2}{1-(1-\psi_L)^2} \tag{2.18}$$

$$\left(1-e^{-\phi_H}\right)(1-\psi_H) = c_H \tag{2.19}$$

$$\left(1-e^{-\phi_L}\right)(1-\psi_L) = \frac{c_L}{p_L} \tag{2.20}$$

Equation (2.17) represents the indifference condition for the workers, while equation (2.18) makes sure that the price of the low commodity equals the MRS. Equation (2.19) and (2.20) are the zero-profit conditions for the high and low type firms respectively. Next, we can show that there is a unique equilibrium.

Proposition 2.5 *In a decentralized market a unique equilibrium* $\{p_L^*,\ \theta_H^*,\ \theta_L^*, q_{HH}^*\}$ *exists* $\forall\ 0 < c_H, c_L < 1$.

Proof. See appendix. ■

2.4.3 Efficiency

Since we allow for free entry, we can now test whether the number and composition of vacancies is constrained efficient. Specifically, we assume that the social planner can again determine q_{HH}, q_{LL}, q_{HL}, and α, like in the basic model, but now he can also determine the number and composition of firms in the market, θ_H and θ_L. Using the same definitions for χ_{ij}^k as in section 2.2.4, we can write Y_i, i.e. total output created by type i firms, as follows:

$$Y_H = q_{HH}\chi_{HH}^H + q_{HL}\chi_{HL}^H \tag{2.21}$$

and

$$Y_L = q_{LL}\chi^L_{LL} + q_{HL}\chi^L_{HL}. \tag{2.22}$$

Next, denote the net value of the output per worker by V:

$$V = p_H Y_H + p_L Y_L - \theta_H c_H - \theta_L c_L.$$

The social planner is not concerned with redistribution issues. He just wants to maximize social welfare, i.e. the utility that can be obtained from V. This implies that he maximizes the indirect utility function associated to the Cobb-Douglas utility function specified in equation (2.12):

$$\max_{q_{HH},q_{LL},q_{HL},\alpha,\theta_H,\theta_L} \left(\frac{\lambda V}{p_H}\right)^{\lambda} \left(\frac{(1-\lambda)V}{p_L}\right)^{1-\lambda} \tag{2.23}$$

under the condition that $q_{HH} + q_{LL} + q_{HL} = 1$. Again, the price of the low commodity has to be equal to the marginal rate of substitution. Therefore, we can rewrite equation (2.23) as follows:

$$\max_{q_{HH},q_{LL},q_{HL},\alpha,\theta_H,\theta_L} (Y_H - \lambda\theta_H c_H - \lambda\theta_L c_L)\left(\frac{Y_L}{Y_H}\right)^{1-\lambda}. \tag{2.24}$$

The corresponding system of first order conditions cannot be solved analytically. Therefore, we use numerical optimization methods to derive the optimal values q^{**}_{HH}, q^{**}_{LL}, q^{**}_{HL}, α^{**}, θ^{**}_H, and θ^{**}_L. The results indicate that the optimal value for q^{**}_{LL} equals 0, i.e. the social planner does not let workers play LL.[17] The optimal values for $q^{**}_{HH} = 1 - q^{**}_{HL}$ are displayed in figure 2.5 for several values of c_L and $\lambda = 0.6$.[18] Each line shows two clear jumps. The first jump occurs where $c_H = c_L$, which can be explained by the behavior of α^{**}. This value is always equal to zero for $c_H < c_L$ and equal to one for $c_H > c_L$, because when a worker receives both a high and a low type offer, the planner wants the worker to fill the position that is more expensive to create. Ceteris paribus, this jump in α^{**} at $c_H = c_L$ increases the probability for a high firm to match and decreases the probability

[17] This conclusion even holds for c_L close to 1 and λ close to 0.5.

[18] Fixing λ at a different value, e.g. 0.9, changes the values of q^{**}_{HH}, q^{**}_{LL}, q^{**}_{HL}, α^{**}, θ^{**}_H, and θ^{**}_L, but none of the qualitative conclusions in this section.

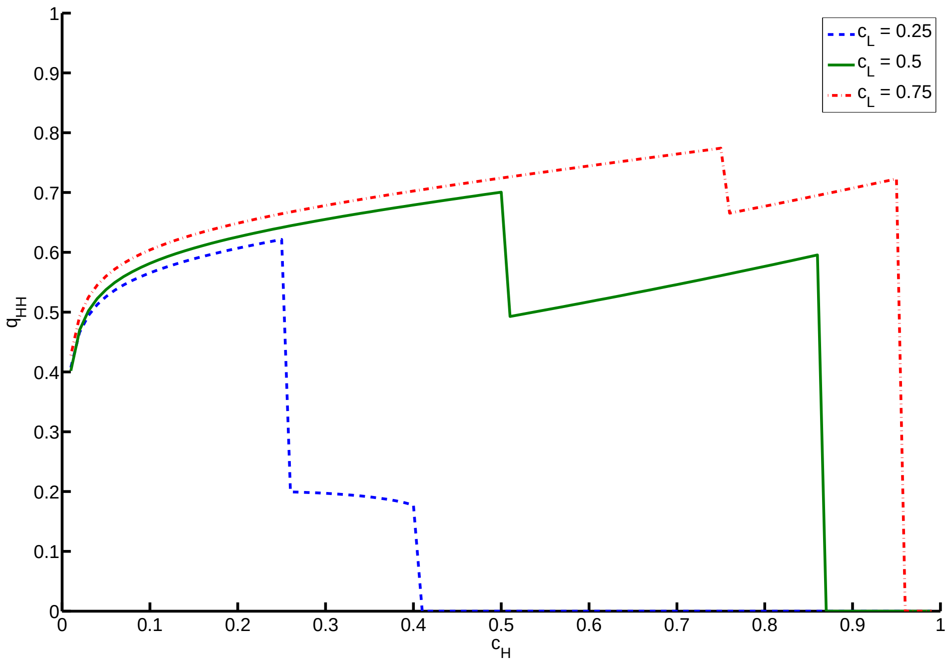


Figure 2.5: q^{**}_{HH} as a function of c_H for $\lambda = 0.6$ and several values of c_L.

for a low firm to match. Since the output the planner wants to create in the high and the low sector does however not change discontinuously, the positive jump in α^{**} must be neutralized by a negative jump in q^{**}_{HH}.

The second jump has no clear economic meaning. It is the result of the fact that the social welfare function is non-monotonic in its parameters. The value of c_H for which this second jump occurs is negatively related to λ. For large values of λ and c_L it can happen that this jump occurs before the point where $c_H = c_L$. In that case, there is only one jump.

Next, we turn to the important question whether there are too many or too few vacancies created in the decentralized market equilibrium. Albrecht et al. (2006) prove that in their model the market always opens more vacancies than the social planner if the number of applications is fixed, but that there can be either too many or too few vacancies if the number of applications is endogenous. In our model we focus on $a = 2$, but the composition of these applications over the sectors is endogenous both for the market and the planner. Unlike in Albrecht et al. (2006), the expected number of applications that a workers sends to a specific sector can now be a non-integer value, because he can play mixed strategies with respect to the sectors he applies to. Hence, the heterogeneity amongst the firms gives both the market and the social planner more freedom in choosing the optimal number of applications, even if the total number of applications is fixed.

Figures 2.6 and 2.7 respectively display the number of H-vacancies and the number of L-vacancies created by the market and the planner as a function of the entry cost for type H firms. The entry cost for type L firms is fixed at 0.5, while λ is still assumed to be 0.6.[19] The figures show that either too many or too few vacancies (both high and low) are opened in the decentralized market, depending on the values of the exogenous parameters. For low values of c_H, the market opens too many vacancies (both high and low) compared to the social optimum. The intuition is that because the posted wages are driven to zero, firms basically have monopsony power. The existence of ex post competition only partly offsets this. Albrecht et al. (2006) show for the identical-workers-and-jobs case that efficiency requires full ex ante and ex post competition.[20] When c_H approaches 1, the Albrecht et al. (2006) case is reversed, the social planner creates more vacancies than the market.

The intuition for the latter result is the following. If c_H approaches 1 in the decentralized market, no high firm is willing to enter, because its expected payoff is negative in that case. However, without supply of the high type commodity, workers can never obtain a positive utility and therefore the entire market collapses: there are no firms active in equilibrium. This result depends on our assumption that the cross partials of the utility function are positive. The $H-$firms do not internalize that increasing their output increases the value of $L-$output in particular when Y_H is low.

In order to check whether the market is constrained efficient, we compare the ratio between the utility obtained in the decentralized equilibrium (i.e. the indirect utility function evaluated at the equilibrium values) and the utility associated with the social planner's solution. This ratio is displayed by the dashed line in figure 2.8 for $\lambda = 0.6$. It shows that for small c_H, market utility is about 80% of what could be achieved. As c_H increases, the inefficiency goes up and when c_H approaches 1, the ratio of the utility obtained in the

[19] Different values of c_L and λ do not affect the main conclusions.

[20] In their directed search model, the possibility of ex post competition eliminates the ex ante competition for workers.

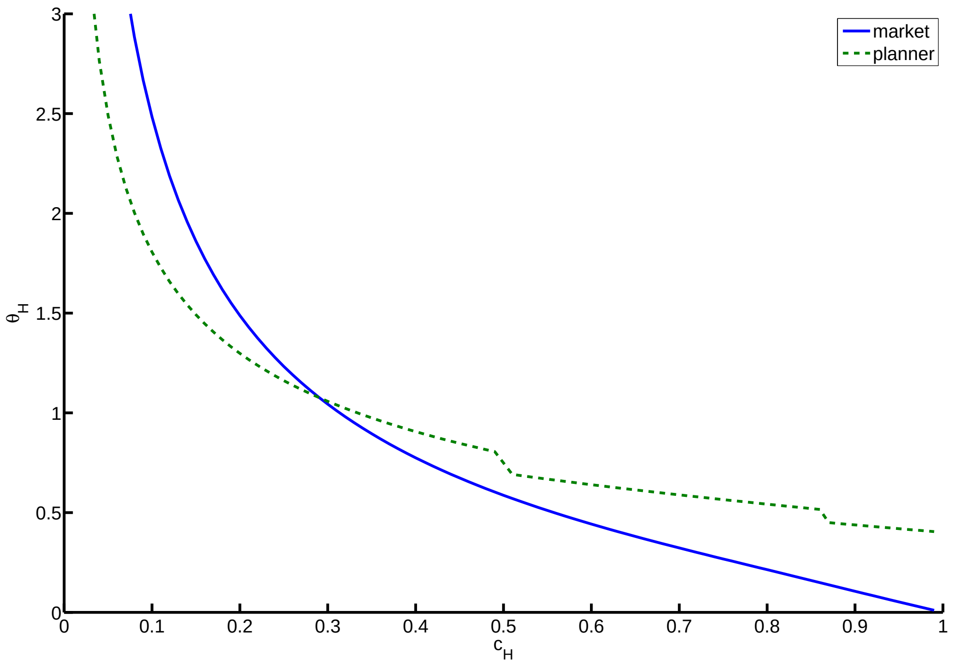


Figure 2.6: The number of high firms in the market as a function of c_H for $\lambda = 0.6$ and $c_L = 0.5$.

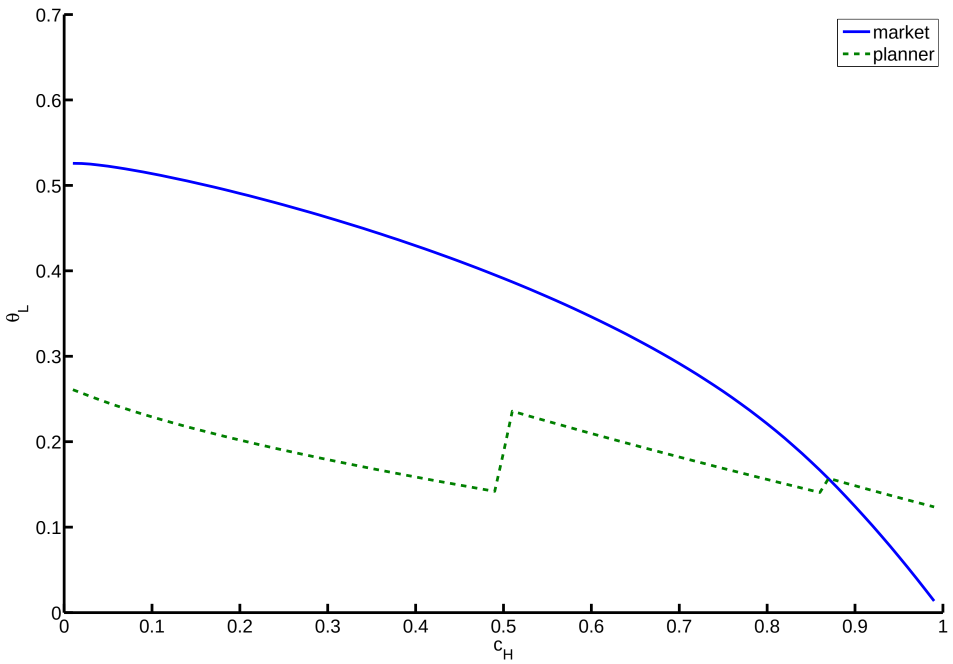


Figure 2.7: The number of low firms in the market as a function of c_H for $\lambda = 0.6$ and $c_L = 0.5$.

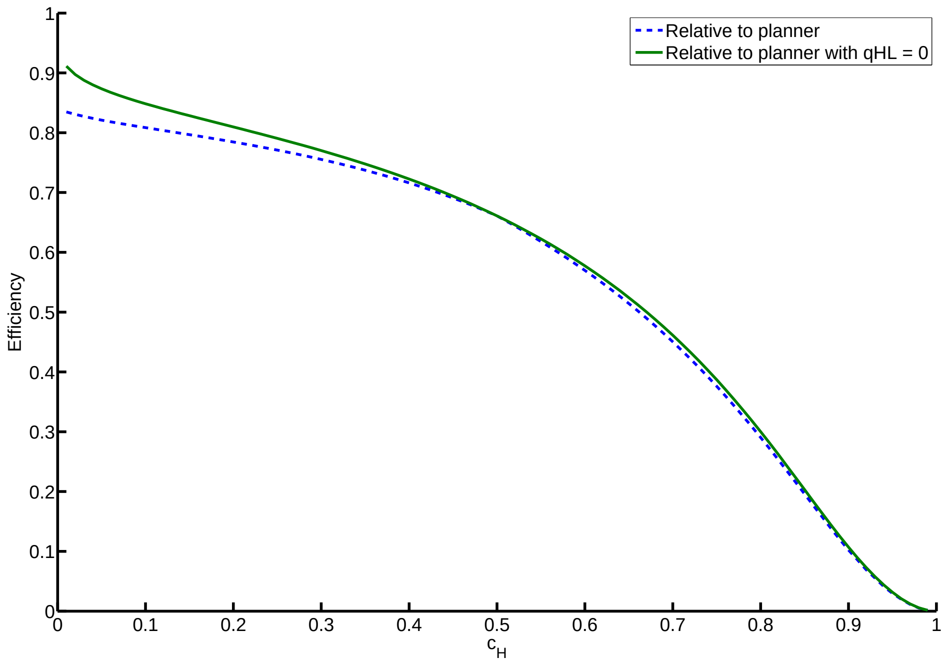


Figure 2.8: Efficiency of the decentralized equilibrium as a function of c_H for $\lambda = 0.6$ and $c_L = 0.5$.

market and under the planner goes to zero.

The intuition for this result is the same as above: in the decentralized market no vacancies are created if c_H approaches 1. The social planner however does create vacancies in that case. So, for c_H close enough to 1, the created output is virtually zero in the decentralized market but strictly positive under the social planner. This implies that the relative efficiency of the decentralized market equilibrium goes to 0.

To see to what extent this inefficiency is caused by the fact that the planner plays HL, we also consider a constrained planner who can only play (a mixture of) HH and LL. The efficiency of the decentralized equilibrium relative to this constrained planner's optimum is displayed in figure 2.8 by the solid line. The line shows that in this case the inefficiency is almost as large as in the case with the unconstrained planner. This is dramatically different from the model in section 2.2 where most of the market inefficiency was due to the fact that workers do not play HL.

To sum up, the market creates too many vacancies if the high-type-vacancy creation costs are low, while it creates too few vacancies if the high type vacancy creation costs are high. As a result of this, the expected number of applications that a high type vacancy receives in a decentralized market is larger than socially optimal for high values of c_H and smaller than optimal for low values of c_H. A similar pattern is found for the expected number of applications received by low type firms. If we restrict the planner to only play HH and LL, this conclusion still holds. Allowing for free entry almost completely eliminates the inefficiencies caused by the fact that workers do not play HL and $\alpha = 1$, but introduces new distortions: (i) since posted wages are 0, firms have monopsony power, (ii) H firms do not internalize that more output in the H-sector increases the value of output in the L sector, see (2.15). (i) dominates for low vacancy cost and (ii) dominates for high vacancy cost.

2.5 Conclusions

We presented a simple model where workers could apply to multiple, heterogeneous jobs to study whether workers hold socially efficient portfolios of applications. Workers do not apply to firms with the highest expected payoffs for an individual application but rather maximize the value of their portfolio. The resulting equilibrium is not efficient for two reasons. Workers want to maximize the productivity-weighted probability to get two job offers, while the planner aims to maximize the productivity-weighted number of matches. This conflict of interest results in too little matches and excessive unemployment. We showed that this result is not driven by the fact that search is random in our model. For a large share of parameter values the posted wages are also zero in the directed search version of our model as in Albrecht et al. (2006).

If we allow for free entry there is a second source of inefficiency. For high creation cost in the high productivity sector, the market creates too little vacancies. If entry cost are high, the risk of Bertrand competition makes firms stop entering the market at a point where the marginal social benefits are still positive. On the other hand if entry cost are low, vacancy creation is excessive because the absence of ex ante competition gives firms too much rents. The vacancy creation distortions can in principle be neutralized by an appropriately chosen firm tax or subsidy scheme. The workers' portfolio distortions are more severe. Governments may have instruments to make one of the sectors more attractive but this will only increase the fraction of workers who send both applications to this sector without increasing the fraction of workers that mixes between sectors.

2.A Proofs

2.A.1 Proof of Proposition 2.4

Proof. Suppose that all firms posting a wage equal to zero is not a directed search equilibrium. Then a profitable deviation must exist for either the high type firms or the low types firms. Consider a deviation by a high type firm first. Instead of 0 it posts a strictly positive wage: $w'_H > 0$. Workers now have two additional application strategies: they can send (i) one application to the deviant and the other one to a high firm or (ii) one application to the deviant and the other one to a low firm. Denote the former strategy by $H'H$ and the latter by $H'L$. The payoff of playing $H'H$ equals

$$\psi'_H \psi_H + \psi'_H (1 - \psi_H) w'_H \tag{2.25}$$

and the payoff of $H'L$ equals

$$\psi'_H \psi_L y_L + \psi'_H (1 - \psi_L) w'_H, \tag{2.26}$$

where ψ'_H is defined in the usual way and denotes the probability that an application to the deviant results in a job offer.

Since we consider a large labor market, a specific worker applies with probability zero to the deviant. So, the presence of a deviant does not affect the average number of applications received by the other non-deviant high or low firms. Therefore, the indifference condition $\psi_H^2 = \psi_L^2 y_L$ must still hold. By substituting $\psi_H = \psi_L \sqrt{y_L}$ in equation (2.25) and using the fact that $1 > \sqrt{y_L} > y_L$, one can easily see that $H'L$ is dominated by $H'H$.

In response to the deviation by one of the high firms, workers will adjust their application strategies such that they are indifferent between HH, LL and $H'H$. The new equilibrium is therefore defined by the following two equations:

$$\begin{aligned}\psi_H^2 &= \psi_L^2 y_L \\ \psi_H^2 &= \psi_H' \psi_H + \psi_H' (1-\psi_H) w_H'\end{aligned}$$

Let ϕ_H' denote the expected number of applications that the deviant receives. Then, by substituting $\psi_H' = \frac{1}{\phi_H'}\left(1-e^{-\phi_H'}\right)$ in the second condition and rearranging the result, we can derive the following relation between the posted wage w_H' and ϕ_H':

$$w_H' = \frac{1}{1-\psi_H}\left(\frac{\phi_H' \psi_H^2}{1-e^{-\phi_H'}} - \psi_H\right). \tag{2.27}$$

The first derivative of this function with respect to ϕ_H' equals

$$\frac{\partial w_H'}{\partial \phi_H'} = \frac{\psi_H^2}{\psi_H - 1}\frac{e^{-\phi_H'} + \phi_H' e^{-\phi_H'} - 1}{e^{-2\phi_H'} - 2e^{-\phi_H'} + 1} > 0 \; \forall \phi_H' > 0.$$

Hence, w_H' is a monotonic function of ϕ_H': the higher the wage set by the deviant, the higher the expected number of applications it receives. The fact that w_H' is monotonically increasing in ϕ_H' also implies that rather than deriving the optimal wage for a deviant, we can derive the optimal queue length. The one implies the other.

After substituting equation (2.27), the profit function for a high type deviant equals

$$\begin{aligned}\pi_H' &= \left(1-e^{-\phi_H'}\right)(1-\psi_H)\left(1-w_H'\right) \\ &= \left(1-e^{-\phi_H'}\right)(1-\psi_H)\left(1-\frac{1}{1-\psi_H}\left(\frac{\phi_H' \psi_H^2}{1-e^{-\phi_H'}} - \psi_H\right)\right).\end{aligned}$$

Differentiating this profit function with respect to ϕ_H' yields the following expression:

$$\frac{\partial \pi_H'}{\partial \phi_H'} = e^{-\phi_H'} - \psi_H^2,$$

which is a strictly decreasing function of ϕ_H' that equals zero for $\phi_H' = -2\log(\psi_H)$. Therefore, the profit function has a global maximum in this point. The corresponding value of w_H' follows from evaluating equation (2.27) in this maximum:

$$w_H' = \frac{\psi_H\left(\psi_H^2 - 2\psi_H \log(\psi_H) - 1\right)}{(1-\psi_H)^2(1+\psi_H)}. \tag{2.28}$$

This expression has the same sign as $\psi_H^2 - 2\psi_H \log(\psi_H) - 1$. The first derivative of this equation is equal to $2(\psi_H - \log\psi_H - 1)$, which easily can be shown to be positive for all ψ_H in the interval $(0,1)$. Together with the fact that $\lim_{\psi_H \to 1} \psi_H^2 - 2\psi_H \log(\psi_H) - 1 = 0$, this implies that the right hand side of equation (2.28) is negative $\forall \psi_H \in (0,1)$. Since we do not allow for negative wages, this optimal value of w_H' is not feasible. Given that the profit is strictly decreasing in $\phi_H' > -2\log(\psi_H)$ and that w_H' is strictly increasing in ϕ_H', the profit function maximization problem therefore has a boundary solution: the deviant maximizes its profit by posting $w_H' = 0$. This implies that the best response for a potential deviant is to also post w_H.

Now we perform the same analysis for a low type deviant. Suppose that it posts a wage $w_L' > 0$. In that case the payoff of playing LL' equals

$$\psi_L \psi_L' y_L + \psi_L'(1-\psi_L) w_L' = \psi_L' w_L' + \psi_L' \psi_L \left(y_L - w_L'\right)$$

and the payoff of HL' equals

$$\psi_H \psi_L' y_L + \psi_L'(1-\psi_H) w_L' = \psi_L' w_L' + \psi_L' \psi_H \left(y_L - w_L'\right),$$

where ψ_L' denotes the probability that an application to the deviant results in a job offer.

In a similar way as we described above, one can show that the strategy HL' is dominated by LL'. The new equilibrium is therefore defined by the following two indifference conditions:

$$\begin{aligned} \psi_H^2 &= \psi_L^2 y_L \\ \psi_L^2 y_L &= \psi_L \psi_L' y_L + \psi_L' (1-\psi_L) w_L' \end{aligned}$$

Let ϕ_L' denote the expected number of applications that the deviant receives. Then, by substituting $\psi_L' = \frac{1}{\phi_L'}\left(1-e^{-\phi_L'}\right)$ in the second condition and rearranging the result, we can derive the following relation between the posted wage w_L' and ϕ_L':

$$w_L' = \frac{1}{1-\psi_L}\left(\frac{\phi_L' \psi_L^2 y_L}{1-e^{-\phi_L'}} - \psi_L y_L\right). \tag{2.29}$$

The first derivative of this function with respect to ϕ_L' equals

$$\frac{\partial w_L'}{\partial \phi_L'} = \frac{\psi_L^2 y_L}{\psi_L - 1} \frac{e^{-\phi_L'} + \phi_L' e^{-\phi_L'} - 1}{e^{-2\phi_L'} - 2e^{-\phi_L'} + 1} > 0 \; \forall \phi_L' > 0.$$

Hence w_L' is a monotonic function of ϕ_L': the higher the wage set by the deviant, the higher the expected number of applications it receives.

The profit function for the deviant equals

$$\begin{aligned} \pi_L' &= \left(1-e^{-\phi_L'}\right)(1-\psi_L)\left(1-w_L'\right) \\ &= \left(1-e^{-\phi_L'}\right)(1-\psi_L)\left(1 - \frac{1}{1-\psi_L}\left(\frac{\phi_L' \psi_L^2 y_L}{1-e^{-\phi_L'}} - \psi_L y_L\right)\right). \end{aligned}$$

Differentiating this this profit function with respect to ϕ_L' yields the following expression:

$$\frac{\partial \pi_L'}{\partial \phi_L'} = e^{-\phi_L'}\left(1-(1-y_L)\psi_L\right) - \psi_L^2 y_L,$$

which is a strictly decreasing function of ϕ_L' that equals zero for $\phi_L' = -\log \kappa$, where $\kappa \equiv \frac{\psi_L^2 y_L}{1-(1-y_L)\psi_L}$. Therefore the profit function has a global maximum in this point. The

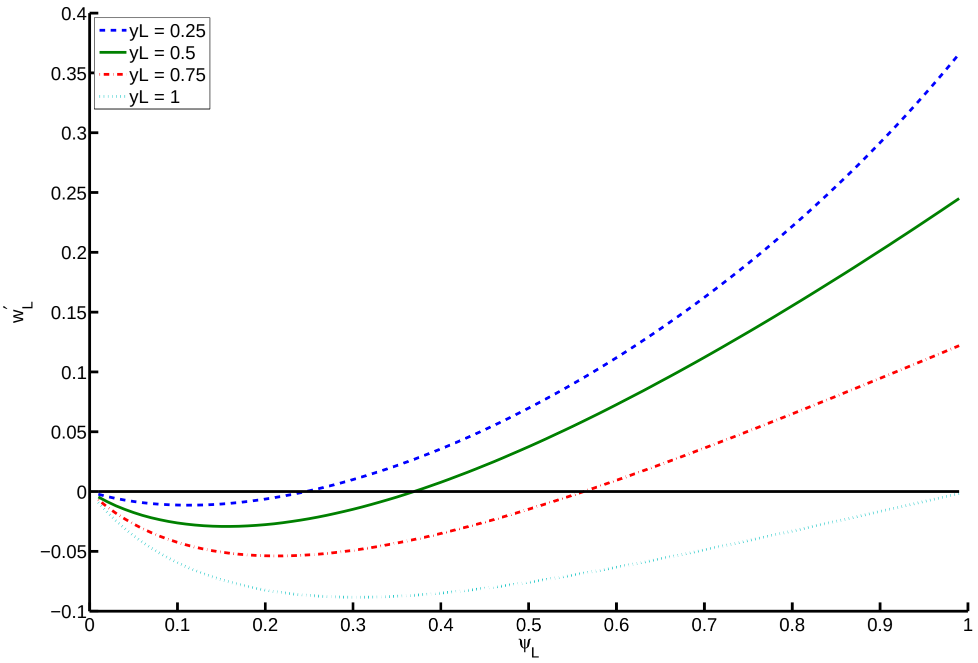


Figure 2.9: w_L' as a function of ψ_L for several values of y_L. Positive values of w_L' imply that a profitable deviation exists for a low type firm.

corresponding value of w_L' follows from evaluating equation (2.29) in this maximum:

$$w_L' = \frac{-\psi_L y_L}{1-\psi_L}\left(\frac{\psi_L \log\kappa}{1-\kappa}+1\right).$$

One can check that $\lim_{\psi_L\to 0} w_L' = 0$, $\lim_{\psi_L\to 0}\frac{\partial w_L'}{\partial \psi_L} = -y_L < 0$ and, by applying l'Hospital's Rule twice, $\lim_{\psi_L\to 1} w_L' = \frac{1-y_L}{2} > 0$ (see figure 2.9). Therefore, it depends on the equilibrium value ψ_L^* whether a profitable deviation exists. For ψ_L^* close to 0 the optimal value for w_L' is negative. Given the fact that $\frac{\partial \pi_L'}{\partial \phi_L'} < 0$ for $\phi_L' > -\log\kappa$ and that $\frac{\partial w_L'}{\partial \phi_L'} > 0 \; \forall \phi_L' > 0$, this implies that low type firms have no incentive to post a wage that is different from 0. On the other hand, for ψ_L^* close to 1, it is profitable for a low firm to deviate by posting a wage that is strictly positive. It is straightforward to show that both cases can occur. For example, $\psi_L^* \to 0$ if $\theta_H \to 0$, $\theta_L \to 0$ and $y_L \to 1$, while $\psi_L^* \to 1$ if $\theta_H \to \hat{\theta}_H$ where $\hat{\theta}_H$ is such that $\frac{\hat{\theta}_H^2}{4}\left(1-\exp\left(-\frac{2}{\hat{\theta}_H}\right)\right)^2 = y_L$.

■

2.A.2 Proof of Proposition 2.5

Proof. The function $1-e^{-\phi_H}$ is strictly positive and strictly increasing $\forall \phi_H > 0$. The same is true for the function $1-\psi_H = 1-\frac{1}{\phi_H}\left(1-e^{-\phi_H}\right)$. Therefore, the revenue for the high firm $\left(1-e^{-\phi_H}\right)(1-\psi_H)$ is a strictly increasing function of ϕ_H with

$$\lim_{\phi_H \to 0}\left(1-e^{-\phi_H}\right)(1-\psi_H) = 0$$

and

$$\lim_{\phi_H \to \infty}\left(1-e^{-\phi_H}\right)(1-\psi_H) = 1.$$

This implies that the condition (2.19) uniquely identifies a value $\phi_H^* > 0$ for any $0 < c_H < 1$.

Since $\left(1-e^{-\phi_L}\right)(1-\psi_L) < 1$, a necessary condition for condition (2.20) to hold is that $p_L > c_L$. Assume for the moment that p_L is exogenously given such that this condition is satisfied. In that case any value $0 < c_L < 1$ uniquely identifies a value ϕ_L^* as a function of p_L, i.e. $\phi_L^*(p_L)$. Since $\left(1-e^{-\phi_L}\right)(1-\psi_L)$ is strictly increasing in ϕ_L, $\phi_L^*(p_L)$ is strictly decreasing in p_L with $\lim_{p_L \to c_L}\phi_L^*(p_L) = \infty$ and $\lim_{p_L \to \infty}\phi_L^*(p_L) = 0$. Using this, it follows directly that $\psi_L(\phi_L^*(p_L))$ and $\psi_L(\phi_L^*(p_L))^2 p_L$ are both strictly increasing in p_L and that

$$\lim_{p_L \to c_L}\psi_L(\phi_L^*(p_L))^2 p_L = 0$$

and

$$\lim_{p_L \to \infty}\psi_L(\phi_L^*(p_L))^2 p_L = \infty.$$

This implies that given ϕ_H^* and $\phi_L^*(p_L)$ there exists a unique value $p_L^* > c_L$ such that the indifference condition is satisfied.

Let $\phi_L^* = \phi_L^*(p_L^*)$, $\psi_H^* = \psi_H(\phi_H^*)$ and $\psi_L^* = \psi_L(\phi_L^*(p_L^*))$. Then

$$\lim_{q_{HH} \to 0}\frac{1-\lambda}{\lambda}\frac{q_{HH}}{1-q_{HH}}\frac{1-(1-\psi_H^*)^2}{1-(1-\psi_L^*)^2} = 0,$$

while

$$\lim_{q_{HH}\to 1} \frac{1-\lambda}{\lambda}\frac{q_{HH}}{1-q_{HH}}\frac{1-(1-\psi_H^*)^2}{1-(1-\psi_L^*)^2} = \infty,$$

and

$$\frac{d}{dq_{HH}}\frac{q_{HH}}{1-q_{HH}} = \frac{1}{(1-q_{HH})^2} > 0.$$

The Intermediate Value Theorem implies that there exists a unique value $0 < q_{HH}^* < 1$ such that p_L^* equals the MRS $\frac{1-\lambda}{\lambda}\frac{q_{HH}^*}{1-q_{HH}^*}\frac{1-(1-\psi_H^*)^2}{1-\left(1-\psi_L^*\right)^2}$. Using q_{HH}^*, ϕ_H^* and ϕ_L^*, it is straightforward to determine θ_H^* and θ_L^*. Now, the equilibrium is defined by p_L^*, θ_H^*, θ_L^*, and q_{HH}^*.

■

3 Simultaneous Search and Search Intensity

3.1 Introduction

Many active labor market policies aim at increasing the search intensity of non-employed workers. Examples include (i) unemployment sanctions, like cuts in the benefits paid to the unemployed who do not engage in active job search (see Abbring et al., 2005), (ii) counseling and monitoring, like advising long term unemployed workers on how to draft application letters (see van den Berg and van der Klaauw, 2006), (iii) financial aids, like subsidizing child care in order to increase the number of actively searching workers (see Heckman, 1974; Graham and Beller, 1989), or (iv) re-employment bonus schemes (see Meyer, 1996). The evaluation of policy programs of this kind is not easy because, on the one hand, it is difficult to measure job search intensity directly and, on the other hand, a change in the search effort of the treatment group affects the wage distribution and matching rates for the non-treated workers as well so the general equilibrium effects can be substantial. In this essay we present a structural framework for the evaluation of public policies of this kind. We estimate the primitive parameters of an equilibrium search model with endogenous search intensity and free entry of vacancies. Those primitives are the search cost distribution, the value of home production and the capital cost of vacancy creation. Our estimates can then be used to calculate the socially optimal search intensities and the level of labor market tightness.

Specifically, we consider a discrete-time dynamic labor market with a continuum of

This chapter is based on Gautier, Moraga-González and Wolthoff (2007).

identical, infinitely-lived workers and free entry of vacancies. Firms enter the market and post wages to maximize profits. At each point in time, workers are either employed at one of the firms or non-employed. Employed workers stay in their job until their match with the firm is destroyed by some exogenous shock and they become non-employed again. Non-employed workers apply for jobs. Since search intensity is the policy parameter of interest, we explicitly model it as the number of job applications workers send out per period. For each application submitted, a worker incurs a search cost. This search cost differs amongst workers and is drawn from a common non-degenerate cumulative distribution function (cdf). As in Gautier and Moraga-González (2005) (who consider a one period version of this model with identical workers), wages, the number of applications, and firm entry are jointly determined in a simultaneous-moves game. For the usual reasons, as explained in Burdett and Judd (1983) and Burdett and Mortensen (1998), firms play mixed strategies and offer wages from a continuous wage offer distribution.

Rather than assuming an exogenous specification for a matching function (see the summary of empirical studies in Petrongolo and Pissarides, 2001), this is to our knowledge the first estimation of a labor search model where the matching process is not only endogenously determined by the firms and workers participation decisions, but also by the search efforts of heterogeneous workers. Therefore, in our model, the primitive parameters are not the elasticities of an exogenously specified matching function but the quantiles of the search cost distribution. As in Albrecht et al. (2006), our aggregate matching function is based on micro foundations and determined by the interplay between two coordination frictions: (i) workers do not know where other workers send their job applications and (ii) firms do not know which workers other firms make employment offers to. These two frictions operate in different ways for different distributions of worker search intensities and have implications on wage determination and firm entry. Working backwards from observed wages, we estimate the quantiles of the search cost distribution and the implied matching rates by maximum likelihood. To do this, we first derive an equilibrium relation between the accepted wage distribution and the wage offer distribution (which we do not directly observe). Then, we use the wage offer distribution together with data on labor market tightness and non-participation to estimate the distribution of search intensities and the matching probabilities. Since a worker continues to send applications till the marginal benefits of search equal the marginal cost, we can use this optimality condition to retrieve the magnitude of search costs for a given search intensity. This procedure works well and gives a good fit to the observed wage data suggesting that search cost heterogeneity alone can explain wage dispersion well. The model also performs well in predicting the matching rates for firms and workers. This is encouraging because unlike the wage distribution, those matching rates do not enter the likelihood function.[1]

To illustrate the difference between our model and models where either the wage distribution or search intensity is exogenous, consider the effects of a policy intervention such as an increase in the minimum wage. A priori, this policy makes search more attractive so one would expect all workers to search harder after the shock. In our model, however, very intensive search will be discouraged because the wage distribution becomes more compressed. Consequently, the matching rate, the job offer arrival rate and the wage distribution are not policy invariant. Moreover, the way these endogenous variables respond to policy changes depends on the primitive search cost distribution.

We also derive the worker's reservation wage in each labor market segment, which depends on the flow value of non-labor market time (i.e. home production and unemployment (UI) benefits), search costs and the wage distribution. Recently, Gautier and Teulings (2006) and Hornstein et al. (2007) argued that many search models cannot explain why reservation wages are substantially lower than the average or maximum wage, while at the same time unemployment or unemployment duration is low. In our model, unemployed workers who have a low search cost today realize that they can have a high search cost

[1] Hong and Shum (2006) were the first to present structural methods to retrieve information on search costs in consumer markets for homogeneous goods. Hortaçsu and Syverson (2004) extend their approach to a richer setting where price variation is not only caused by search frictions but also by quality differences across products. Moraga-González and Wildenbeest (2008) extend the approach of Hong and Shum (2006) to the case of oligopoly and present a maximum likelihood estimation method. Although we estimate search costs in a similar way, all these models are not directly applicable to the labor market since they deal with static models, do not capture the standard labor market frictions associated to rationing and assume firm entry is exogenous. For example, while all wage variation is attributed in those models to search cost variability, our dynamic approach relates wages to other factors like labor market tightness and makes reservation wages endogenous.

tomorrow. Therefore, they are willing to accept a low starting wage even though they have a large probability to receive one or more offers today.

The various policies mentioned above can be interpreted in this framework as aiming at either changing the shape of the search cost distribution or changing the marginal benefits of search. For example, one goal of subsidizing child care is to reduce the fraction of the labor force that does not search at all, while counsel ling unemployed workers is likely to lower the cost of writing effective application letters and increase the mean number of job applications. Besides policies that aim to directly affect search intensity, redistribution policies like UI insurance and minimum wages also affect search intensity indirectly. Without a suitable framework there is no way we can tell whether we should stimulate search intensity for all workers, only for particular groups or not at all.

We apply our model to the Dutch labor market. We find that in the decentralized market equilibrium, too few workers participate, while unemployed workers on average send too many job applications. The first result can be explained by a standard hold-up problem. Workers typically receive only part of the social benefits of their investments in search and therefore workers with high search cost invest too little in search. The second result on excessive search of the low-search cost workers is due to congestion externalities and rent seeking behavior. First, workers do not internalize the fact that sending more applications increases the probability that multiple firms consider the same candidate. Second, submitting more applications increases the expected maximum wage offer. A final source of inefficiency is excessive entry of vacancies. Given the search and participation strategies of the workers, the absence of ex ante competition (directed search) gives firms too much market power and under free entry this translates into excessive vacancy creation. Our estimates indicate that the three sources of inefficiency together lead to a market equilibrium output that is 10% to 15% lower than the socially optimal level of production. In the social optimum, more workers participate, the average search intensity is lower and more firms enter the market[2].

[2]The fact that the planner opens more vacancies than the market is due to the fact that the planner sets the participation rate higher. Given the planner's level of search intensity, the market would open too many vacancies.

Interestingly, the introduction of a moderate binding minimum wage can be desirable for three reasons: (i) it increases participation because the expected wage increases, (ii) it weakens the rent-seeking motive to send multiple applications because it compresses the wage distribution, (iii) it reduces excessive vacancy supply. We model UI benefits to be conditional on searching at least once (as is the case for many OECD countries).[3] The advantage of this UI scheme is that, while it increases the marginal benefits of sending one job application rather than zero, it does not give additional incentives to search too intensively, which keeps the congestion effects low. UI benefits can therefore also increase participation and reduce rent-seeking behavior. A final important advantage of the approach presented in this essay is that it allows us to study the interaction of policies that increase participation and policies that stimulate job creation, rather than study them in isolation.

The essay is organized as follows. Section 3.2 describes the theoretical model and section 3.3 shows how it can be estimated by maximum likelihood. Section 3.4 discusses our data and in section 3.5 we present our estimation results and discuss efficiency. Section 3.6 checks how robust the optimal policy is to relaxation of our simplifying assumptions. In this section we also allow the search cost functions of workers to be very general. Section 3.7 discusses related literature and section 3.8 concludes.

3.2 Model

3.2.1 Setting

Consider a discrete-time labor market with a continuum of identical firms and identical, infinitely-lived workers. Both are risk neutral. We normalize the measure of workers to 1 and we allow for free entry of firms, so the number of firms is endogenous. At each point in time, each worker is either employed at one of the firms or non-employed. The fractions of employed and non-employed workers at time t are denoted by e_t and $\overline{e}_t$ respectively,

[3] Formally, in order to be eligible for UI benefits in the Netherlands, workers must apply four times per month where an application is defined broadly, i.e. making a phone call also often qualifies. In this essay we consider the much smaller set of serious applications that could potentially lead to a job offer.

where $e_t + \bar{e}_t = 1$. Likewise, each job is either matched with a worker or vacant. The measure of firms with vacancies is denoted by v_t, while the measure of matched firms equals the measure of employed workers e_t. Employed workers stay in their job until their match with the firm gets destroyed by some exogenous shock; after this, the workers in question flow into non-employment and their jobs become vacant. We assume that a fraction δ of the matches is destroyed every period.

In our model, non-employed workers can decide whether they want to search for a job or not. This provides us with a meaningful distinction between unemployment and non-participation. The non-participants are the non-employed workers who decide not to search at all, while the unemployed are workers who happen to search at least once but fail to get a job. We discuss this in more detail below. In each period a fraction m_W of the non-employed workers flows to employment and a fraction m_F of the vacancies gets filled. As usual, the employment and vacancy dynamics are given by the following equations:

$$e_{t+1} = (1-\delta)e_t + m_W(1-e_t) \tag{3.1}$$

$$v_{t+1} = \delta e_t + v_t(1-m_F). \tag{3.2}$$

The fractions m_W and m_F are endogenous in our model and we will derive an expression for them in the next subsections. We make the usual assumption that the labor market is in steady state, meaning that the number of workers and firms in each state is constant over time, i.e. $e_t = e$ and $v_t = v\ \forall t$, where e and v are given by

$$e = \frac{m_W}{m_W + \delta} \tag{3.3}$$

and

$$v = \frac{\delta}{m_F} e. \tag{3.4}$$

A worker who is employed in a given period receives a wage w. The payoff of the firm that employs the worker equals $y - k - w$, i.e. the difference between the value of the output produced, y, a capital cost k and the wage paid to the worker. Non-participants have a payoff that is determined by two components: the value of their home production and the economic value of their leisure. We assume that, together, these amount to a quantity denoted by h. An unemployed worker additionally receives UI benefits denoted by b. These benefits along with the option value of search determine the worker's reservation wage w_R. The distinction between b and h will be important in section 5.3 where we discuss efficiency. Firms with an unfilled vacancy do not produce, but still have to pay the capital cost. Their payoff therefore equals $-k$. All agents discount future payoffs at a rate $1/(1+r)$.

We assume that a worker applies for jobs at the beginning of a period, but only learns whether she is accepted or not at the end of the period. In this setting, searching non-sequentially is optimal (Morgan and Manning, 1985). Sending several applications at a time reduces the risk of remaining unmatched and increases the chance of getting a juicy offer. We denote by $a(c)$ the number of applications a worker with search cost c sends out. Because of computational considerations, we impose a maximum S on the number of jobs to which a worker can apply for in a given period. Since S can be any finite number, this assumption is hardly restrictive.

Next, we turn to the specification of search costs. We assume that for each application submitted to a firm, a worker incurs a search cost $c > 0$. The total cost of sending a applications therefore equals $C(a) = ca$.[4] The search cost per application, c, differs amongst workers, but is drawn from a common, non-degenerate distribution $F_c(c)$, defined over the set $[0, \infty)$. One very useful simplification we make is that workers draw a new search cost every period. This captures the idea that the opportunity cost of job search is a random variable that is affected by things like having kids, health status, etc. The benefit of this assumption is that the reservation wage is the same for all workers. If we alternatively assumed search costs to be worker-specific, we would have to calculate search-cost-dependent reservation wages and this would make the model a lot more complicated. Since our aim is to estimate the cross-sectional search cost distribution we

[4] In section 3.6.3 we consider the general class of search cost functions $C(a)$ that are weakly increasing in a and we show that the main conclusions in terms of the difference between the desired and the actual distribution of applications per worker do not change much from the linear benchmark case that we consider here.

choose the simplest option.[5]

In related models of consumer search, e.g. Burdett and Judd (1983) and Moraga-González and Wildenbeest (2008), there usually is no rationing and each buyer is served. In a labor market model, the assumption of no rationing is unrealistic: firms typically hire only one or a few of the applicants for a certain job. To allow for rationing we consider an urn-ball matching process with multiple applications as in Albrecht et al. (2004). We extend their endogenous matching function to the case of heterogeneity in the number of applications that workers send out. The wage determination process is as in Gautier and Moraga-González (2005). The timing of events within a time period t is as follows:

1. Each firm posts a wage w. At the same time, each worker draws a search cost c, decides to how many jobs she wishes to apply for and sends her application letters to random vacancies.

2. Once job applications are received by the vacancies, each vacancy receiving at least one application randomly picks a candidate and offers her the job. Applications that are not selected are returned as rejections.

3. Workers that receive one or more wage offers accept the highest one as long as it is higher than the reservation wage. Other wage offers are rejected.

The number of job applications workers send out and the wages firms set are determined in a simultaneous-moves game. In the next subsections we discuss the workers' and firms' optimal strategies. We focus on symmetric equilibria, i.e. where identical firms have similar strategies. In the estimation procedure we use a sample of the flow from non-employment to employment.[6] This allows us to focus on the wage distribution for newly hired workers and to ignore the job-to-job transitions, which are an additional source for wage dispersion (see Burdett and Mortensen, 1998). This way we can isolate the search intensity contribution to wage dispersion and keep the model tractable.

[5] Our assumption may be restrictive in situations where some workers are permanently in a position to contact many employers, just because they have a good network of contacts, because they live in a location where there are many job opportunities, or because they possess the desirable social skills and working abilities.

[6] We assume that firms can post separate wages for employed and unemployed workers.

3.2.2 Workers' Problem

The strategy of an unemployed worker consists of a reservation wage and a number of job applications to send out to the vacancies. Since workers are ex ante identical, the reservation wage, denoted w_R, is the same for all workers. However, workers learn their search cost c before they start applying to the vacancies, so different workers may send out different numbers of applications. Let $a(c)$ be the applications that an unemployed worker with search cost c submits. Denote the fraction of non-employed workers sending a applications by p_a, $a = \{0, 1, 2, ..., S\}$. For a fraction p_0 of the workers, the search cost is so high that it is not profitable for them to search even once in this period. These workers become non-participants. The other workers (fraction $1 - p_0$) send at least one application and are therefore considered to be the unemployed who actually search for a job. Let u and n be the steady state fractions of unemployed and non-participating workers in the population, then we have:

$$n = p_0(1-e) \tag{3.5}$$

$$u = (1-p_0)(1-e). \tag{3.6}$$

Since search is random, all firms are equally likely to receive applications from the unemployed workers. This implies that the expected number of applications per vacancy is equal to the total number of applications divided by the number of vacancies:

$$\phi = \frac{(1-e)\sum_{a=1}^{S} a p_a}{v} = \frac{\sum_{a=1}^{S} a p_a}{\theta}, \tag{3.7}$$

where $\theta = v/(1-e)$ denotes labor market tightness. Due to the infinite size of the labor market, the actual number of applicants at a specific vacancy follows a Poisson distribution with mean ϕ.[7] Likewise, the number of competitors that a worker faces at a given

[7] This is not completely obvious because in a finite labor market more matches are realized for a given mean search intensity when the variance is zero. The key intuition why the number of applicants follows a Poisson distribution in the limit and why all that matters is the average search intensity is that the probability that any two workers compete for the same job more than once is zero when workers apply to a finite number of jobs. Consequently, the event that application i results in a job offer only depends on labor

firm also follows a Poisson distribution with mean ϕ. In case an individual worker competes with i other applicants for a job, the probability that the individual in question will get the job equals $1/(1+i)$. Therefore, the probability ψ that an application results in a job offer equals

$$\psi = \sum_{i=0}^{\infty} \frac{1}{i+1} \frac{\exp(-\phi)\,\phi^i}{i!} = \frac{1}{\phi}\left(1-\exp(-\phi)\right). \tag{3.8}$$

We assume that if two or more firms compete for the same worker, the worker picks the highest wage, as in Albrecht et al. (2004). The firms not chosen open the vacancy again in the next period.

Given the assumptions above, the number of wage offers that a worker receives follows a binomial distribution.[8] More precisely, for a worker who sends a applications the probability $\chi(j|a)$ to get $j \in \{0,1,2,...,a\}$ job offers equals

$$\chi(j|a) = \binom{a}{j} \psi^j (1-\psi)^{a-j} \tag{3.9}$$

We denote the fraction of non-employed workers that receive j job offers by q_j. This fraction is equal to the product of p_a (i.e. the fraction of non-employed workers sending a applications) and the probability that these a applications result in exactly j job offers, summed over all possible a:

$$q_j = \sum_{a=j}^{S} \chi(j|a)\, p_a. \tag{3.10}$$

This notation allows us to give a simple expression for the matching probability m_W that a non-employed worker flows into employment in the next period:

$$m_W = 1 - q_0 = 1 - \sum_{a=0}^{S} \chi(0|a)\, p_a = 1 - \sum_{a=0}^{S} (1-\psi)^a p_a. \tag{3.11}$$

In order to derive an expression for the reservation wage we specify two discrete time Bellman equations. The first defines the expected discounted lifetime income of a worker

market tightness and the total number of applications, and is independent of the event that application j results in a job offer.

[8] See Albrecht et al. (2006).

who is currently employed at a wage w, which we denote $V_E(w)$:

$$V_E(w) = w + \frac{1}{1+r}\left((1-\delta)V_E(w) + \delta V_{NE}\right), \tag{3.12}$$

where V_{NE} denotes the expected value of being non-employed. Hence, the value of employment equals the sum of the wage w and the discounted value of employment if the worker stays in the job next period (probability $1-\delta$) or the discounted value of non-employment if the match with the firm gets destroyed (probability δ).

Non-employed workers face a trade-off when deciding how many applications to send out. Applying to one more job is costly but it brings two sorts of benefits: one, it reduces the probability of remaining unmatched and two, it increases the likelihood to get a better paid job. Therefore, a non-employed worker with search cost c chooses the number of applications a in such a way that she maximizes her expected discounted lifetime payoff $V_{NE}(a|c)$:

$$\begin{aligned} V_{NE}(a|c) &= h + I_{a>0}b + \frac{1}{1+r}\sum_{j=1}^{a}\chi(j|a)\int_0^\infty \max\{V_{NE}, V_E(w)\}\,dF_w^j(w) \\ &\quad + \frac{1}{1+r}\chi(0|a)V_{NE} - ca. \end{aligned} \tag{3.13}$$

This expression describes the value of non-employment for a worker with search cost c who applies for a jobs, which equals the sum of home production h and the expected discounted payoff of her search strategy. If the worker sends a applications, then she receives j wage offers with probability $\chi(j|a)$. Each wage offer w is a random draw from a wage offer distribution F_w with corresponding density f_w.[9] In case the worker receives multiple job offers, she accepts the best one as long as that offer gives her a higher payoff than remaining non-employed, denoted V_{NE}. If the worker fails to find a job, she remains non-employed again in the next period. A necessary condition to receive UI benefits b is to actively search for a job (represented by the indicator function $I_{a>0}$), as is the case in most OECD countries. The total cost of sending a applications equals ca.

[9] We derive this wage offer distribution in the next subsection.

Let $a(c) = \arg\max V_{NE}(a|c)$ be the optimal search strategy of a worker with search cost c. Define $V_{NE}(c) = \max_a V_{NE}(a|c)$. Since, ex ante, the non-employed workers do not know the value of the search cost that they will draw in future periods, their expected value of non-employment is therefore equal to

$$V_{NE} = \int_0^\infty V_{NE}(c) dF_c(c). \tag{3.14}$$

By evaluating equation (3.12) at the reservation wage w_R and using the reservation wage property $V_E(w_R) = V_{NE}$, it follows that

$$V_{NE} = \frac{1+r}{r} w_R. \tag{3.15}$$

Substituting this expression back in (3.12) and rewriting gives

$$V_E(w) = \frac{1+r}{r+\delta}\left(w + \frac{\delta w_R}{r}\right). \tag{3.16}$$

Using (3.15) and (3.16), we can rewrite (3.13) as:

$$V_{NE}(a|c) = h + I_{a>0} b + \frac{w_R}{r} + \frac{1}{r+\delta}\zeta_a - ac, \tag{3.17}$$

where

$$\zeta_a = \sum_{j=1}^{a} \chi(j|a) \int_{w_R}^\infty (w - w_R) dF_w^j(w) = \int_{w_R}^\infty (w - w_R) d(1 - \psi + \psi F(w))^a \tag{3.18}$$

(the last equality follows from the binomial theorem). Next, we combine equations (3.14) and (3.17) to obtain an implicit expression for the worker's reservation wage:

$$w_R = h + \int_0^\infty \max_a \left(I_{a>0} b + \frac{1}{r+\delta}\zeta_a - ca\right) dF_c(c). \tag{3.19}$$

The reservation wage depends on the value of home production and the option value of search. One can easily show that this expression for the reservation wage satisfies

Blackwell's (1965) sufficient conditions for a contraction mapping. Therefore, a unique value for the reservation wage w_R exists.

Using integration by parts in (3.18), it is easy to see that the function ζ_a is monotonically increasing in a. Therefore, the workers' maximization problems induce a partition in the support of the search cost distribution as follows. There exists a worker with critical search cost denoted Γ_a such that she is indifferent between sending out a and $a-1$ job applications, i.e., $V_{NE}(\Gamma_a, a) = V_{NE}(\Gamma_a, a-1)$, $a = 1, 2, ..., S$. From the expressions above, it follows that Γ_a is equal to

$$\Gamma_1 = \frac{1}{r+\delta}\zeta_1 + b \tag{3.20a}$$

$$\Gamma_a = \frac{1}{r+\delta}\left(\zeta_a - \zeta_{a-1}\right),\ a = 2, ..., S, \tag{3.20b}$$

where the second term of equation (3.20a) reflects the fact that the worker becomes eligible to unemployment benefits b only when she searches at least once. It is straightforward to show that Γ_a is a decreasing function of a. This implies that workers continue searching as long as Γ_a is larger than their search cost c. Hence, the fractions p_a of workers sending a job applications satisfy the following conditions:

$$p_0 = 1 - F_c(\Gamma_1) \tag{3.21a}$$

$$p_a = F_c(\Gamma_a) - F_c(\Gamma_{a+1}),\ a = 1, 2, ..., S-1 \tag{3.21b}$$

$$p_S = F_c(\Gamma_S) \tag{3.21c}$$

3.2.3 Firms' Problem

In this subsection we derive the wage offer distribution for newly hired workers. A firm with a vacancy offers one randomly picked applicant (if present) a wage w. In order to be attractive to both the firm and the applicant, this wage should be higher than the worker's reservation wage w_R, but lower than the value of the output that will be produced in case of a match net of capital cost, $\widehat{y} = y - k$. Moreover, the wage has to be higher than the legal minimum wage $w_{\min}$. Define $\underline{w} = \max\{w_R, w_{\min}\}$. The firm faces the following trade-off within the interval $[\underline{w}, y-k]$: posting a lower wage increases its payoff $y-k-w$ conditional on the worker accepting the offer, but it also increases the probability that the worker rejects the offer and chooses to work for another firm.

For reasons similar to those in Burdett and Judd (1983) and Burdett and Mortensen (1998), there exists no symmetric pure strategy wage equilibrium.[10] Next, we characterize a mixed strategy equilibrium in wage offers to newly hired workers. Let $F_w(w)$ denote the equilibrium wage offer distribution. Firms that offer wages below $\underline{w}$ will never hire any worker. A firm that offers the lower bound of this distribution only hires a worker if she has no other offers. Since this occurs with a strictly positive probability, the lower bound of the wage distribution must be equal to $\underline{w}$. The exact shape of the equilibrium wage offer distribution follows from an equal payoff condition.

First, we derive the firms' Bellman equations. A firm that is matched to a worker produces output y and has to pay a capital cost k and a wage w. In the next period, the firm is still active with probability $1-\delta$; otherwise it has a vacancy again. Hence, the firms' value $V_F(w)$ of being matched with a worker earning a wage w is given by

$$V_F(w) = y - k - w + \frac{1}{1+r}\left((1-\delta)V_F(w) + \delta V_V\right). \tag{3.22}$$

where V_V denotes the lifetime expected payoff of a vacancy.

Next, consider a firm with a vacancy offering a wage w. The probability $m_F(w)$ to hire a worker at wage w equals the probability of offering the job to a worker who happens to get no other job offers, or only offers paying less than w:[11]

$$m_F(w) = \frac{1}{\theta}\sum_{j=1}^{S} jq_j F_w^{j-1}(w). \tag{3.23}$$

If the firm happens to match, it obtains a value $V_F(w)$ in the next period. On the other hand, if the firm fails to match (with probability $1-m_F(w)$), it gets V_V again. Hence, the

[10]To be precise, in the non-sequential search model of Burdett and Judd (1983)there is a pure-strategy equilibrium where workers are offered the minimum wage. This type of equilibrium is non-generic in the sense that it can only exist for particular search cost distributions in our model.

[11]See appendix 3.A.1 for the derivation.

value function of this firm, conditional on offering a wage w equals

$$V_V(w) = -k + \frac{1}{1+r}\left(m_F(w) V_F(w) + (1 - m_F(w)) V_V\right). \tag{3.24}$$

In equilibrium, each wage in the support of F_w must yield the same level of expected profits to a firm. Therefore, the following equal profit condition implicitly defines the equilibrium wage distribution F_w:

$$V_V(w) = V_V(\underline{w}).$$

Substituting equation (3.22) and (3.23), and simplifying the result, gives

$$\sum_{j=1}^{S} j q_j F_w^{j-1}(w) = q_1 \frac{y - k - \underline{w} - rV_V}{y - k - w - rV_V}. \tag{3.25}$$

We assume free entry of vacancies, i.e. unmatched firms enter the market as long as the expected payoff is positive. Hence, in equilibrium it must be the case that $V_V = 0$ and then equation (3.25) reduces to

$$\sum_{j=1}^{S} j q_j F_w^{j-1}(w) = q_1 \frac{y - k - \underline{w}}{y - k - w}. \tag{3.26}$$

This equation defines implicitly the equilibrium wage offer distribution. In appendix 3.A.2 we derive an expression for the density function of posted wages f_w. Evaluating (3.26) at the upper bound $\overline{w}$, where $F_w(\overline{w}) = 1$, gives :

$$\overline{w} = y - k - \frac{(y - k - \underline{w}) q_1}{\sum_{j=1}^{S} j q_j}, \tag{3.27}$$

which is strictly smaller than y, since $q_1 > 0$. Hence, firms always post wages below the productivity level. This is because a wage equal to y would give the firm a payoff of zero with probability one, while posting a lower wage gives a strictly positive expected payoff, since some applicants do not compare wages.

Using the free entry condition $V_V(w) = V_V(\underline{w}) = 0$ and (3.22), equation (3.24) reduces to

$$0 = -k + \frac{1}{r+\delta}\frac{1}{\theta}q_1\left(y - k - \underline{w}\right). \tag{3.28}$$

This expression implicitly determines the free-entry equilibrium number of vacancies in the market.

3.2.4 Efficiency

An interesting policy issue is whether the decentralized market equilibrium is efficient. To answer this question, we consider a social planner who decides in each period how many vacancies $v_t \geq 0$ are opened and how many applications workers send (or equivalently the values of $\{p_{0t}, p_{1t}, ..., p_{St}\}$) to maximize the present discounted value of future output, net of application and entry costs. Since search is costly, once we obtain the optimal partition $\{p_{0t}, p_{1t}, ..., p_{St}\}$ we can use equation (3.21) to assign the optimal number of applications to an individual with search cost c. In each period the employed workers produce y, while the non-employed produce h and incur a search cost c for each application they send. Each of the $v_t + e_t$ firms present in the market at time t has to pay the capital cost k. Let $\widetilde{h}$ denote

$$\widetilde{h} = h - \int_0^\infty a\left(c; p_{0t}, ..., p_{St}\right) c dF_c(c).$$

Then, the planner's maximization problem is given by

$$\max_{\{p_{0t},...,p_{St},v_t,e_{t+1}\}_{t=0}^\infty} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left(ye_t + \widetilde{h}(1-e_t) - k\left(v_t + e_t\right)\right),$$

where e_0 is exogenously fixed and e_{t+1} satisfies (3.1).

With respect to the matching function $m_W(.)$, we distinguish two different cases. First, we consider a social planner who is constrained in the sense that he cannot solve the coordination frictions in the market. This type of planner faces the same matching function as the market, which was given in equation (3.11). Second, we consider an unconstrained planner who can match workers and firms as desired. This type of planner generates a number of matches that equals the minimum of the number of unemployed and the number of vacancies:

$$m_W^U\left(p_{0t},\dots,p_{St},e_t,v_t\right)=\frac{\min\{u_t,v_t\}}{1-e_t}.$$

Comparing the market allocations under these two types of planner allows us to decompose the efficiency loss in the economy into a part that is directly due to frictions and a part that is due to distorted incentives.

Following Shimer (2004a) and Rogerson et al. (2005), we express the problem in a recursive way. Let $V(e)$ be the expected present value of future output when the current employment rate is e. Then, the following Bellman equation holds:

$$V(e)=ye+\max_{p_0,\dots,p_S,v}\left(\widetilde{h}(1-e)-k(v+e)+\frac{1}{1+r}V\left(e'\right)\right), \tag{3.29}$$

where e' denotes employment in the next period. Since our estimation method provides us with estimates of y, h, and $F_c(c)$, we can numerically solve this maximization problem for the two different matching functions and confront the market outcome with the social optima. We do this in section 3.5.3.

3.3 Maximum Likelihood Estimation

3.3.1 Likelihood

The model described in the previous section can be estimated by maximum likelihood. In this section we provide a short sketch of the non-parametric estimation procedure which is similar in nature to Moraga-González and Wildenbeest (2008). In particular, we describe how we can obtain estimates for $\{p_0,\dots,p_S\}$, which in turn provide us with the cutoff points $\{\Gamma_1,\dots,\Gamma_S\}$ of the search cost distribution. We refer to appendix 3.A.2 for details.[12]

We start by discussing the data that are required to estimate the model. First of all, we need cross-sectional wage data for newly hired workers who flow from unemployment

[12] Identification of search costs is discussed in Hong and Shum (2006) and in Moraga-González and Wildenbeest (2008). The latter paper argues that to identify the search cost distribution at points other than the cutoffs $\{\Gamma_1,\dots,\Gamma_S\}$ one would need to pool data from many markets each of them with the same underlying search cost distribution. Since we are estimating the model for the entire Dutch economy, we cannot follow that procedure here.

into employment. Our source for this sort of information is the Dutch AVO data set, which contains information on the Dutch Labor market. We discuss this data set in more detail in section 3.4. Secondly, we need some aggregate statistics on the labor market. In particular, we need information about the number of vacancies v in the market and about the fractions of employed (e), unemployed (u) and non-participating ($n = 1 - e - u$) individuals. Accurate data to estimate these variables are available from most statistical agencies and they are usually not only available for the labor market as a whole, but also for submarkets. Third, to calculate household production h, we need information about the level of the unemployment benefits b. The unemployment benefits can, without loss of generality, be defined as the product of a replacement rate ρ and the average wage. An estimate for ρ can easily be obtained from macro-data. Note that the replacement rate ρ only changes the decomposition of the reservation wage into b and h. Therefore, the estimates for $\{p_0, ..., p_S\}$ do not depend on the value of ρ.

Two other parameters have to be fixed exogenously: the maximum number of applications per period S and the discount factor $1/(1+r)$. One can easily test whether the estimation results are sensitive to the values chosen for these parameters, but in general this does not seem to be the case. For example, $S = 30$ and $S = 40$ give very similar results, because the difference in expected payoff between searching 30 or 40 times is negligible. Likewise, note that choosing a different value for the interest rate does not change the estimates of the search fractions p_a, the job offer probability ψ, the job offer fractions q_j, or the net productivity $\widehat{y} = y - k$. It only affects the scale of the search cost distribution $F_c(c)$.[13]

These parameters, the data, the structure of the model and the steady state assumption provide us with all the information we need to estimate the search cost distribution. The first step is to use (3.5) to identify p_0 as the ratio of the fraction of non-participants in the population to the fraction of non-employed. The other fractions p_a are estimated by maximizing the likelihood of the observed wages. The equilibrium value for the separation

[13]Equation (3.20) shows that $\Gamma_{a>1} = \dfrac{1}{r+\delta}(\zeta_a - \zeta_{a-1})$, where ζ_i does not depend on r or δ. Doubling $r+\delta$ therefore reduces these cutoff points by a factor 2. The effect on Γ_1 is slightly smaller, since Γ_1 includes the constant b.

rate δ follows from the steady state condition given in equation (3.3).

Note that, as in many models with on-the-job search, cross-sectional wages are not representative for the wages that are offered by the firms, but only for the wages that are accepted by the workers. High wage offers are more likely to be accepted than low wage offers, so the distributions of the offered wages and the accepted wages differ from each other. We denote the distribution of the accepted wages by $G_w(w)$ with associated density $g_w(w)$. Conditional on receiving at least one job offer, a worker will only accept a wage that is lower than some value w if all the j offers that she receives after sending a applications are lower than w. This means that $G_w(w)$ simply follows from $F_w(w)$ (see appendix 3.A.2 for a derivation).

Flinn and Heckman (1982) and Kiefer and Neumann (1993) suggest to use the lowest wage and the highest wage in the sample to estimate the lower end and the upper end of the support of the wage offer distribution.[14] Although this approach gives superconsistent estimates, we do not follow this suggestion, since these order statistics are quite sensitive to outliers. Instead, we estimate the net productivity $\widehat{y}$ and the lower bound $\underline{w}$ as parameters in our maximum likelihood problem. Together they imply a value for the upper bound $\overline{w}$ as was shown in equation (3.27).

A small fraction of the observations in our data set lies outside the interval $[\underline{w}, \overline{w}]$. We consider these observations to be the result of measurement error. We incorporate this measurement error in our model in the standard way (see e.g. Wolpin, 1987): the observed wage $\widetilde{w}$ depends on the true wage w and a multiplicative random error term ε with a log-normal distribution with parameters $\mu = 0$ and $\sigma^2 = \text{var}(\log(\varepsilon))$. We will estimate the value of σ. The density of the observed wages can now be obtained by integrating over all possible values of the error term. Let $g_{\widetilde{w}}(\widetilde{w})$ denote this density, then the likelihood of the sample is equal to the product of $g_{\widetilde{w}}(\widetilde{w}_i)$ for each individual i.[15] So, the maximum

[14] See also Donald and Paarsch (1993) for a discussion of the use of order statistics to estimate the bounds of distributions.

[15] See the appendix for an expression for $g_{\widetilde{w}}(\widetilde{w})$.

likelihood problem is given by

$$\max_{p_1,\dots,p_S,\sigma,\underline{w},\widehat{y}} \frac{1}{N}\sum_{i=1}^{N} \log g_{\widetilde{w}}\left(\widetilde{w}_i\right), \tag{3.30}$$

subject to the conditions $\sum_{a=0}^{S} p_a = 1$, $p_a \in [0,1]$ $\forall a$ and $w_{\min} \leq \underline{w} \leq \widehat{y}$. The derivations in the appendix show that the productivity and the capital cost only enter the expression for $g_{\widetilde{w}}\left(\widetilde{w}_i\right)$ as the difference $y-k$. Hence, the productivity and the capital cost are not separately identified: we can only obtain an estimate for the net productivity $\widehat{y}$. Ex post however, we can retrieve the value for k from equation (3.28). Subsequently, the productivity y simply equals the sum of $\widehat{y}$ and k.

As is common in these kinds of models, the reservation wage is only identified if it exceeds the minimum wage.[16] In that case $w_R = \underline{w}$. Otherwise, we can only derive some bounds on w_R (or alternatively one has to make parametric assumptions). The upper bound in that situation is given by $\underline{w}$, while the lower bound is defined by equation (3.19) and the restriction $h = 0$. We discuss these bounds in section 3.6.1.

The covariance matrix of the estimates is calculated by taking the inverse of the negative Hessian matrix evaluated at the optimum. The standard errors of the other variables, e.g. q_j, can be calculated by using the delta method.

3.3.2 Goodness of Fit

In our model, the density of accepted wages $g_w(w)$ has a flexible form: it can be strictly upward sloping, but also hump-shaped. This is shown in figure 3.1, which displays the wage density for two different search profiles $\{p_0, p_1, \dots, p_S\}$ while keeping the other parameters fixed to some arbitrarily chosen values (in particular, $\underline{w} = 0$, $y-k = 20$, $\theta = 1$). If many workers search very little, then a given firm with an applicant does not face much competition from other firms. As a result, a large part of the probability mass is at low wages. Conversely, if enough workers send many applications, then firms have an incentive to post relatively high wages as well. Hence, by choosing the right values

[16] Flinn and Heckman (1982) refer to this as the recoverability problem.

$\{p_0, p_1, ..., p_S\}$ we get a hump-shaped wage distribution. This flexibility is an important advantage compared to existing search models with identical workers and jobs, like for example Burdett and Mortensen (1998), because these models are unable to generate wage distributions similar to the hump-shaped ones observed in real-world markets.

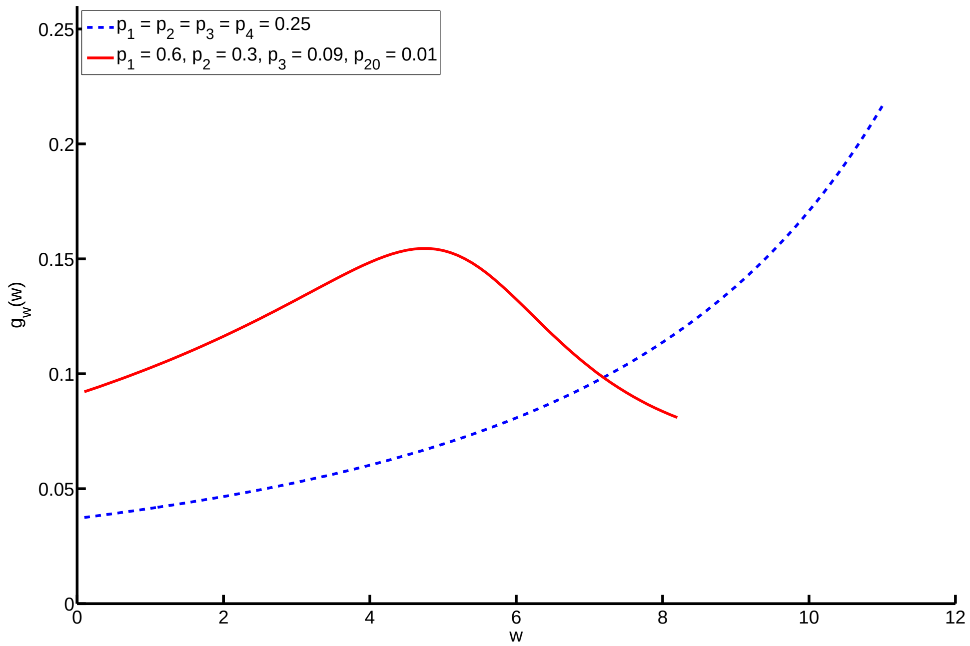


Figure 3.1: $g_w(w)$ for different values of the fractions $\{p_0, p_1, ..., p_S\}$

At first sight, since our wage distribution has S degrees of freedom, our model may seem so flexible that it can fit any observed wage distribution. Our model imposes however a lot of structure on the data. First, the fractions p_a are probabilities and must therefore be non-negative and sum to 1. Secondly, our explicit modeling of the contact process imposes restrictions on the relation between $\{p_0, p_1, ..., p_S\}$ and $\{q_0, q_1, ..., q_S\}$, see equation (3.10). For example, $q_j > 0$ implies that $q_i > 0$ for all $i < j$. This imposes structure on the expected payoff from searching a times, see (3.18), which in turn affects the shape of $F_w(w)$. Thirdly, the shape of $F_w(w)$ is further restricted by the equal profit condition of firms, given by equation (3.26). Fourth, workers with multiple offers choose the highest wage, which imposes conditions on the relation between $F_w(w)$ and $G_w(w)$, see (3.32) in appendix 3.A.2.

Because of these restrictions, it is not obvious that our model can generate a good fit. In the empirical analysis, we assess the fit of the model in three different ways. First, we compare the wage distribution implied by the model to a kernel estimate and check whether they are close to each other. This test alone is however not sufficient, since the maximum likelihood estimation is designed to match the wage distribution. Second, we evaluate the fit of the model by comparing the predicted firm-worker matching and separation probabilities to the actual ones. This comparison gives an indication whether the model can fit the average durations of unemployment and employment spells. Finally, we examine the magnitude of the estimate of σ. The value of σ provides a natural test on the fit of the model, because it indicates how much measurement error is required to get a good fit of the wage distribution. If we find a very large value for σ, then a large part of the variation in the data cannot be explained by the model, implying that the model performs relatively poor. If we however find a small value for the standard deviation, this can be seen as supporting evidence for the model.

3.3.3 Search Cost Distribution

Using the maximum likelihood estimates, we can derive cutoff points of the search cost distribution. In appendix 3.A.3 we show how we apply a change of variables to get an expression of ζ_a as a function of the estimates. The marginal gains from an additional application Γ_a can then be easily calculated from (3.20). These values serve as cutoff points of the search cost distribution $F_c(c)$, as is shown in equation (3.21).

Figure 3.2 illustrates how the search cost distribution can be estimated from the observed wage distribution. In this example we set $w_R = 0$. Note first that the expected maximum wage offer a worker may receive when applying for a jobs, ζ_a, corresponds to a point on $G_w(w)$ (panel 1). The shape of $G_w(w)$ determines the marginal benefits of search, Γ_a. For example, in a close-to-competitive economy where workers are the scarce factor, most job applications result in an offer so wages are close to net productivity. As a result, one should expect Γ_1 to be very large and $\Gamma_{a>1}$ to be close to zero. Figure 3.2 shows that the marginal benefits of applying to more than 1 job are positive but decreas-

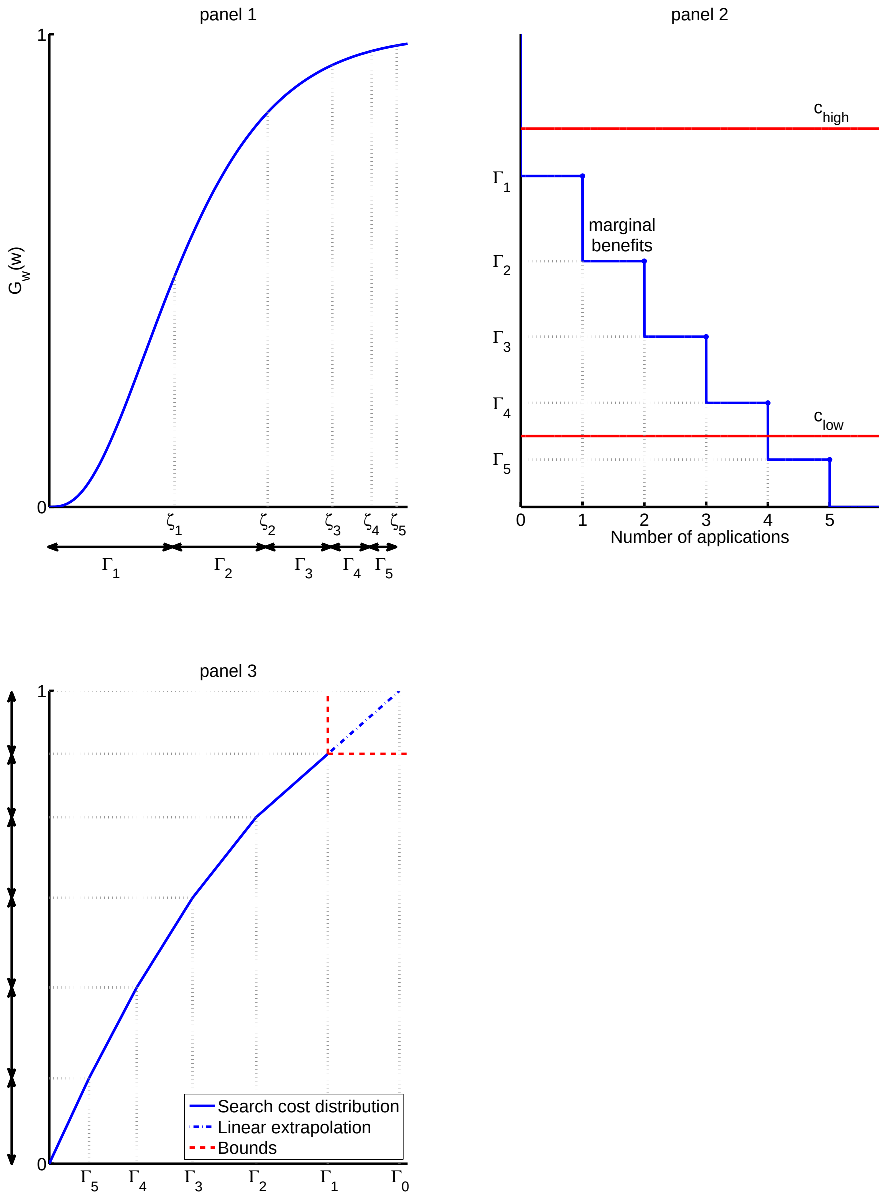


Figure 3.2: Relation between the wage distribution and the search cost distribution
For the ease of graphical exposition, the figure shows a special case in which $b = 0$, $w = 0$, $r + \delta = 1$, and $S = 5$. The figure is purely illustrative and no inferences about the actual distributions of wages and search costs should be made from it.

ing. A worker, realizing that her applications do not affect the wage distribution, takes $\Gamma_1, \Gamma_2, ..., \Gamma_S$ as given and chooses her search intensity such that the marginal gains of an additional application equal the marginal cost (panel 2).

An econometrician proceeds in exactly the opposite way. When he observes (or estimates) that a fraction p_0 of the workers does not search at all, he concludes that the search cost c of each of these individuals must have exceeded Γ_1. This provides him with one point of the search cost distribution $F_c(c)$, i.e. $p_0 = 1 - F_c(\Gamma_1)$ (panel 3). Similarly, by taking $F_c(\Gamma_a) - F_c(\Gamma_{a+1})$ one obtains the fraction of workers with search costs such that if they search a times or less, the marginal benefits exceed the marginal cost but if they search $a+1$ times, the marginal cost of search exceeds the marginal benefits. So, this determines p_a. In the estimation procedure, we start with the wage distribution which gives information on the fraction of workers who received j offers, (q_j). The structure of the model relates $\{q_0, ..., q_S\}$ to $\{p_0, ..., p_S\}$ and implies values for the marginal benefits of searching a times. The set $\{q_0, ..., q_S\}$ is chosen such that the wage distribution implied by the model is as similar as possible to the observed one. This procedure determines S points of the search cost distribution.

In section 3.5.3, where we solve the social planner's problem, we need the full distribution $F_c(c)$. A natural approximation of this distribution can be obtained by interpolating the S cutoff points. In this essay, we use linear interpolation. Note that we also have to extrapolate the distribution, because we do not know the distribution of the search costs among the non-participants. We only know that for this group the search cost c is larger than Γ_1, because otherwise they would have searched at least once. However, for the social planner it makes a difference whether the search cost of a specific non-participant is only slightly higher than Γ_1 or much higher. We start by using linear extrapolation: we assume that the search cost distribution keeps increasing linearly for $c > \Gamma_1$, with the same slope as just before Γ_1, until it reaches 1. In section 3.6.2 we relax this assumption by considering bounds on the search cost of non-participants (also shown in panel 3).

Finally, solving the social planner's problem requires values for unemployment benefits b and household production h. The value for b can be calculated from the replacement rate ρ and the average wage. To be precise,

$$b = \rho \int_{\underline{w}}^{\overline{w}} w dG_w(w).$$

This result together with (3.19) can be used to derive an estimate for h, which simply follows from the difference between the reservation wage and the option value of search.

3.4 Data and Empirical Issues

3.4.1 Parameters

We apply the model developed in the previous section to the Dutch labor market. The wage data that we use for estimation are described in detail in the next subsection. Here, we first explain how we obtain estimates for the exogenous parameters. We start by setting the maximum number of applications S equal to 30. As mentioned before, the estimation results are not sensitive to this specific value.

We use data from Statistics Netherlands to get a value for the replacement rate ρ. The Dutch government spent 4075.5 million euros on UI benefits in 2005. The stock of unemployed contained on average 305140 individuals in that year. Hence, on average 13350 euros were paid per individual. Since the average income amounted to 33000 euros, we set the average replacement rate ρ equal to 0.40. This is exactly the same value that Hornstein et al. (2007) use for the US.

In order to determine a reasonable value for the discount factor we first have to fix the length of a period in our model. For this, we rely on van Ours and Ridder (1993), who study vacancy durations in the Netherlands. They find that the time that elapses between posting and filling a vacancy conditional on having candidates is about four months. Given an annual interest rate of 5%, this implies that $1/(1+r) = 0.984$. It is worth stressing that the length of a period does not affect our estimates of the search intensities p_a, the probability of getting a job offer ψ, the fractions q_j of workers getting j offers, nor the net productivity $\widehat{y}$. It only affects the discount rate, which in turn only rescales the search cost distribution.

Values for the labor market statistics e, u, and n are also obtained from Statistics Netherlands. Data for these parameters are available for each combination of calendar year, gender, education, and age cohort. We use that information to calculate the values of p_0 for our sample, taking into account the composition of the sample. The number of vacancies v is calculated indirectly: it equals the product of the average labor market tightness $\theta/(1-p_0)$ (0.70 in our sample) and the unemployment rate u. In section 3.4.4 we present the numerical values for these parameters.

3.4.2 AVO Data Set

The source for the wage data that we use in the empirical application is the AVO data set[17] of the Dutch Labor Inspectorate, which is part of the Ministry of Social Affairs and Employment. The data are collected annually from the administrative wage records of a sample of firms. The sample period spans from 1992 to 2002. The sampling procedure consists of two stages. In October of each year, first a stratified sample of firms in the private sector is drawn from the Ministry's firm register. The strata are based on industrial sector and firm size (measured by the number of employees). In the second stage, workers are sampled from the administrative records of the firms. Information is collected at two points in time: one year before the sampling date and the sampling date itself. The number of workers sampled depends on the firm size, the number of workers who are newly hired, who have stayed in their job or who have quit the firm, and the number of workers covered by collective labor agreements. The data set contains sampling weights for both the firm strata and the employees. For the firm the weight is equal to the inverse of the probability that the firm is sampled, while for the workers it corresponds to the inverse of the probability that the worker was selected from all employees at the firm. Multiplying these values gives the weight that can be used to calculate sample statistics for the workers.

A consequence of the sampling design is that we do not observe flows that occur between the two sampling dates. Our assumption that the length of one period in the

[17] AVO is the Dutch acronym for *Terms of Employment Study*.

model equals four months implies that we only observe the time points $t = 0, 3, 6...$ in the AVO data set. Workers may have experienced other employment and unemployment spells between these moments of observation. Note that this is not a major problem for our analysis. The only assumption we must make is that the exact moment at which a newly hired worker entered his job does not affect his wage at the sampling date.

A big advantage of this administrative data set compared to survey data is its precision. Missing values are rare and some variables are observed in great detail.[18] For example, the data set distinguishes seven different wage components, which together add up to the total compensation for the worker. These components include personal bonuses etc.

Besides the wage information the data set also contains background characteristics of both workers and jobs. For workers, we have gender, age and educational level. For jobs we have information on industry, firm size and occupation. Furthermore, we observe what type of contract a worker has. Most workers are covered by a collective employment agreement (CAO), which is bargained over at the sector level, or by some leading firms within the sector. The Minister of Social Affairs and Employment can declare this agreement legally binding for all other firms in the same sector, implying that these firms must offer the same terms of employment to its employees. This is labeled AVV.[19] It is important to note that the existence of collective labor agreements does not rule out wage dispersion. A typical collective labor agreement provides many different salary scales and to a large extent firms can determine themselves according to which salary scale they will pay the newly hired worker. Furthermore, firms can also use bonuses and allowances to pay a worker a salary that exceeds the CAO wages. Finally, we have information on the job level at an eight-point scale.[20] The number of observations in job level 7 and 8 is relatively small. Therefore, we combine these workers with the ones in job level 6.

[18]Nevertheless, some measurement error seems present in the data. We discuss this topic in more detail in subsection 3.4.4.

[19]Some large companies have their own collective employment agreement. Besides that there are also workers who have a bilateral bargained wage contract. These workers are typically employed at higher positions in the firm.

[20]The lowest value (1) corresponds to jobs that consist of "very simple, continuously repeating activities, for which no education and only a little experience is required and which are performed under direct supervision". At the other end of the spectrum, the highest job level (8) implies "managing large companies or comparable departments or organizations " (Venema et al., 2003).

For our analysis, we select the workers who flow from unemployment to employment. As argued before, we can isolate the contribution of search frictions to wage dispersion in this way. We further restrict the sample by focusing on workers who work for at least 32 hours per week, which corresponds to 80% of a typical working week of 40 hours. We also exclude individuals below 23 years of age and above 65 years. Individuals above 65 face mandatory retirement and a lower minimum wage applies to workers below 23 years of age. Hence, both groups cannot be considered to be identical to the rest of the workers.

Because of missing variables, we cannot use the samples of 1992 to 1995 and 1999. Hence, we use data from six waves (1996 to 1998 and 2000 to 2002). We correct the wage data for inflation by using a wage index and calculate the hourly wage for each worker by dividing her monthly wage by the number of hours worked. In section 3.4.4 we give some descriptive statistics of the sample, but first we describe in the next subsection how we partition the labor market into five segments.

3.4.3 Segments

In the theoretical model we make two important assumptions about the labor market. First, we assume that, apart from measurement error, differences in search cost are the only source of wage dispersion amongst individuals. Secondly, we consider a labor market in which no new workers can enter and in which the matching probability only depends on the strategy of the agents that are present in the market. In reality, workers obviously earn different wages for many reasons. Therefore, we first create approximately homogeneous segments correcting for observed heterogeneity and then we assume that our model suits each of those segments. The more segments one creates, the more homogeneous the workers in a given segment will be; however, at the same time, the assumption that we do not allow the best worker in segment i to compete with the worst worker in segment $i+1$ becomes more restrictive. As a compromise, we construct five segments. We assume that these segments constitute separate labor markets within the economy and that each worker and each firm is active in exactly one of the five submarkets. Further, we assume that within a segment all workers and all firms are homogeneous.

In order to create the segments, we construct a worker skill index L_s and a job-complexity index L_c, as in Gautier and Teulings (2006). We create the skill index for the workers by regressing the logarithm of an individual's wage w_i, denoted by ω_i, on all his/her observable characteristics: gender, years of education, years of working experience[21] (also squared and cubed), interaction terms, and year dummies. Similarly, the job-complexity index is created by regressing ω_i on dummy variables for the sector, the type of contract for the job, the job level, the occupation type, and year dummies. Appendix 3.B provides details.

The estimation results of these regressions are displayed in table 3.1 and 3.2. The fit is good and most coefficients are in line with what is usually found in Mincerian type wage regressions. For example, an extra year of education increases log(wage) by 0.075 for school-leavers, but this effect is smaller for more experienced workers. In the job complexity regression, log(wage) is increasing in the job level. The correlation between the skill level and the complexity level is 0.58. Hence, there is positive assortative matching in the labor market: better skilled workers have more complex jobs. We create the segments accordingly. A straightforward way of achieving this, is by defining:

$$\Upsilon(L_s, L_c) = L_s L_c.$$

Next, we define the five segments as the quantiles of $\Upsilon(L_s, L_c)$.[22]

If we repeat the skill and the complexity regression for each of the segments separately, we observe indeed that the segments are much more homogeneous than the labor market as a whole. For example, performing the skill regression on the first segment gives an R^2 of only 0.048 while for the whole sample it is 0.358. This means that only a negligible fraction of the wage dispersion in this segment can be attributed to differences in

[21] As common in literature, we define work experience as a function of age and the years of schooling. To be precise, we assume the following relation: experience = (age - years of education - 6) / 50, where rescaling is applied for reasons of computational convenience.

[22] We have experimented with several other definitions of the segments as well. This did not change any of the main conclusions. The advantage of this one above, for example, defining Υ as $E[w|s,c]$ is that our measure is more conservative in the sense that less wage variation within segments can be explained by observable characteristics.

Variable	Estim.	Std.err.		Variable	Estim.	Std.err.	
Constant	1.192	0.116	*	**Year**			
				1997	-0.012	0.015	
Education				1998	0.010	0.014	
Years of education	0.075	0.008	*	2000	0.054	0.014	*
				2001	0.036	0.015	*
Gender				2002	0.041	0.016	*
Male	-0.047	0.032					
				Interaction effects			
Experience				Educ. × Experience	-0.110	0.066	
Experience	3.792	0.930	*	Educ. × Experience^2	0.404	0.183	*
Experience^2	-9.955	2.283	*	Educ. × Experience^3	-0.378	0.156	*
Experience^3	7.805	1.733	*	Male × Experience	0.349	0.355	
				Male × Experience^2	0.790	1.016	
Statistics				Male × Experience^3	-1.343	0.817	
Observations	5801						
R^2	0.358						

* = significant at 5% level.
Reference groups: female, 1996.

Table 3.1: Estimation results of the skill regression

human capital factors like education and experience. The complexity regression can explain a slightly larger part of the wage variation ($R^2 = 0.188$), but again considerably less than for the entire labor market ($R^2 = 0.475$). The same conclusion holds for the other segments. The only segment that calls for some circumspection in the interpretation of the results is the fifth. There the R^2 values of the skill and the complexity regression are 0.222 and 0.256 respectively, implying that a larger part of the heterogeneity is not filtered out.

3.4.4 Descriptive Statistics

In this subsection we present the labor market statistics that we use in the estimation of the model as well as some descriptive statistics of the AVO data set. A first issue is that we discard some observations in order to prevent that our estimate of σ is determined by outliers.[23]

[23] We calculate the 10th percentile $w_{0.1}$, the median $w_{0.5}$ and the 90th percentile $w_{0.9}$ of the wage distribution in each segment and we delete observations that are smaller than $w_{0.5} - \frac{3}{2}(w_{0.5} - w_{0.1})$ or larger than $w_{0.5} + \frac{3}{2}(w_{0.9} - w_{0.5})$. If observed wages were normally distributed, this procedure would lead to deleting 2.7% of the observations at both the top and the bottom of the distribution. However, the wage distributions

Variable	Estim.	Std.err.		Variable	Estim.	Std.err.	
Constant	2.251	0.023	*	**Job level**			
				Level 2	0.027	0.014	*
Sector				Level 3	0.166	0.014	*
Industry	-0.026	0.019		Level 4	0.327	0.016	*
Education	-0.013	0.028		Level 5	0.570	0.023	*
Construction	0.133	0.022	*	Level 6-8	0.743	0.046	*
Trade, reparation	-0.052	0.021	*				
Hotel, catering	0.055	0.031		**Occupation type**			
Transport, communic.	-0.049	0.023	*	Administrative	-0.031	0.012	*
Financial services	-0.001	0.031		Automation	-0.022	0.021	
Other services	-0.064	0.022	*	Commercial	-0.029	0.015	*
Health care	0.007	0.026		Service providing	-0.062	0.011	*
Culture, recreation	-0.052	0.036	*	Creative	-0.021	0.031	
				Management	0.193	0.039	*
Year							
1997	-0.025	0.013		**Coll. empl. agreement (CAO)**			
1998	0.001	0.013		AVV	0.081	0.015	*
2000	0.023	0.013		Company CAO	-0.047	0.019	*
2001	0.014	0.013		No CAO	-0.052	0.010	*
2002	0.051	0.015	*				
Statistics							
Observations	5801						
R^2	0.475						

* = significant at 5% level.
Reference groups: agriculture, 1996, industry CAO, 1-4 employees, level 1, simple technical activities.

Table 3.2: Estimation results of the complexity regression

After discarding the outliers we still observe that a small but strictly positive fraction (between 0.7% and 1.9%) of the workers in the lowest three segments earns a wage that is lower than the legal minimum wage, which equals 7.51 euros per hour. Given the strict enforcement of labor laws in the Netherlands, it seems highly unlikely that these workers actually earn such a low wage. Therefore, we interpret this phenomenon as evidence of reporting mistakes in either the monthly wage or in the number of worked hours. Our model can easily deal with this, since we explicitly allow for measurement error in the data.

The descriptive statistics of the Dutch labor market are displayed in table 3.3. As the

are skewed to the right and this results in a removal of slightly more observations in the right tail (on average 3.8%) than in the left tail (1.4%).

	Segment				
	1	**2**	**3**	**4**	**5**
Labor market states					
Employment (e)	0.479	0.615	0.736	0.790	0.875
Unemployment (u)	0.043	0.033	0.030	0.027	0.022
Non-participation (n)	0.477	0.351	0.234	0.183	0.103
Vacancies					
Number of vacancies (v)	0.030	0.023	0.021	0.019	0.015
Non-participation					
Number of non-participants (p_0)	0.917	0.914	0.885	0.870	0.826
Wage distribution					
Lowest observed wage	7.40	7.42	7.36	8.08	9.46
Highest observed wage	12.07	14.08	15.17	16.81	27.25
Mean observed wage	9.00	10.07	10.73	12.17	15.75

Table 3.3: Values of the exogenous parameters per segment

table shows, the labor market conditions are clearly increasing in the segment number. Compared to workers in a lower segment, workers in a given segment are (i) more likely to be employed and (ii) more likely to search for a job when non-employed. The table also presents the number of vacancies and some characteristics of the wage distribution for each of the segments. As one would expect, the average wage is strictly increasing in the segment index. In the next subsection we explore how these stylized facts affect the search strategy of the various types of workers and we estimate their search cost distributions.

Table 3.4 presents some descriptive statistics of the AVO data set. The table lists the mean and the standard deviation for several worker and job characteristics. A first observation is that workers in higher segments are better educated. The workers in segment 5 have on average completed almost seven years of education more than the workers in segment 1. This difference corresponds to more than 80% and is strongly significant. Higher segments contain relatively more men than women. There are no large differences in the average age across segments. Another observation is that workers in the higher segments work more often in the service sector and less often in trade or industry.

	Segment 1		Segment 2		Segment 3		Segment 4		Segment 5	
	mean	**s.d.**	**mean**	**s.d.**	**mean**	**s.d.**	**mean**	**s.d.**	**mean**	**s.d.**
Observations	1043		1179		1153		1070		1022	
Male	0.52	0.50	0.63	0.48	0.71	0.45	0.72	0.45	0.67	0.47
Education	8.05	1.88	10.42	1.88	11.72	2.16	12.93	2.23	14.75	1.38
Age	32.55	9.45	31.83	9.70	31.49	8.78	31.93	8.73	33.50	8.65
Sector										
Agriculture	0.02	0.12	0.03	0.18	0.03	0.18	0.02	0.13	0.02	0.14
Industry	0.30	0.46	0.24	0.43	0.25	0.44	0.18	0.39	0.15	0.36
Education	0.00	0.01	0.01	0.11	0.01	0.10	0.01	0.09	0.01	0.11
Construction	0.01	0.12	0.03	0.16	0.07	0.26	0.21	0.40	0.09	0.29
Trade, reparation	0.17	0.38	0.19	0.39	0.13	0.34	0.12	0.33	0.08	0.27
Hotel, catering	0.04	0.20	0.04	0.19	0.03	0.18	0.02	0.15	0.01	0.10
Transp., comm.	0.04	0.20	0.10	0.30	0.11	0.31	0.04	0.20	0.03	0.17
Financial services	0.01	0.08	0.02	0.14	0.03	0.17	0.06	0.23	0.06	0.24
Other services	0.27	0.45	0.19	0.39	0.23	0.42	0.22	0.42	0.37	0.48
Health care	0.10	0.30	0.13	0.34	0.09	0.28	0.09	0.29	0.13	0.34
Culture, recr.	0.03	0.18	0.02	0.13	0.01	0.11	0.02	0.14	0.04	0.19
Coll. empl. agreement										
Industry CAO	0.55	0.50	0.63	0.48	0.62	0.49	0.62	0.49	0.53	0.50
AVV	0.03	0.16	0.05	0.22	0.06	0.25	0.09	0.29	0.09	0.29
Company CAO	0.10	0.30	0.06	0.23	0.06	0.24	0.05	0.21	0.02	0.14
No CAO	0.32	0.47	0.27	0.44	0.25	0.44	0.25	0.43	0.36	0.48
Firm size										
1-4 empl.	0.02	0.16	0.03	0.16	0.05	0.21	0.04	0.20	0.02	0.15
5-9 empl.	0.08	0.27	0.06	0.24	0.11	0.31	0.09	0.29	0.09	0.29
10-19 empl.	0.12	0.33	0.13	0.33	0.13	0.33	0.13	0.34	0.09	0.28
20-49 empl.	0.22	0.42	0.18	0.38	0.17	0.37	0.21	0.41	0.12	0.33
50-99 empl.	0.09	0.29	0.11	0.32	0.11	0.31	0.10	0.29	0.12	0.33
100-199 empl.	0.10	0.31	0.13	0.34	0.12	0.33	0.09	0.28	0.09	0.28
200-499 empl.	0.14	0.35	0.16	0.36	0.11	0.31	0.14	0.35	0.14	0.34
$\geq$ 500 empl.	0.22	0.41	0.21	0.41	0.22	0.41	0.20	0.40	0.33	0.47
Job level										
Level 1	0.21	0.41	0.02	0.14	0.01	0.11	0.00	0.02	0.00	0.00
Level 2	0.57	0.49	0.20	0.40	0.07	0.25	0.01	0.12	0.00	0.02
Level 3	0.21	0.41	0.76	0.43	0.75	0.43	0.45	0.50	0.09	0.29
Level 4	0.00	0.00	0.01	0.11	0.17	0.38	0.51	0.50	0.33	0.47
Level 5	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.15	0.44	0.50
Level 6-8	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	0.35
Nature of occupation										
Simple technical	0.39	0.49	0.33	0.47	0.42	0.49	0.39	0.49	0.23	0.42
Administrative	0.08	0.28	0.20	0.40	0.15	0.36	0.19	0.40	0.13	0.34
Automation	0.01	0.08	0.01	0.10	0.03	0.16	0.05	0.22	0.08	0.27
Commercial	0.06	0.24	0.08	0.27	0.11	0.31	0.12	0.33	0.10	0.31
Service providing	0.46	0.50	0.38	0.48	0.29	0.45	0.19	0.40	0.25	0.43
Creative	0.00	0.05	0.01	0.08	0.01	0.11	0.03	0.18	0.13	0.33
Management	0.00	0.00	0.00	0.00	0.00	0.03	0.01	0.11	0.09	0.29

Table 3.4: Descriptive statistics per segment

3.5 Results

3.5.1 Market Equilibrium

We estimate the model for each of the five segments separately. The estimation results are shown in table 3.5. To ease the reading, the fractions p_a and q_j that appear in the table are reported conditional on searching at least once. The search intensity estimates for segments 1 and 2 are different than those for segments 3,4, and 5. In the first and second segments, the majority of the searchers sends out one job application per period. The remaining workers search almost always twice. In the three highest segments, this pattern is reversed. Most individuals searching for a job send out two applications, while a smaller group only searches once. In all five segments, a small fraction of the workers applies to many (i.e. 30) vacancies. The average number of applicants per vacancy varies between 2.3 and 3.3. This results in a job offer probability between 29% and 39%, implying that most workers get either zero or one job offer. Between 4% and 8% of the unemployed receive 2 offers.

We find that both the productivity of a match y and the capital cost k are monotonically increasing across segments. The net productivity $y-k$ is also increasing, except between segments 2 and 3, but the difference is only 0.8 and not statistically significant. The net output produced by a filled vacancy is 17.68 euros per hour in segment 1 and increases to 39.54 euros per hour in segment 5. This is approximately 2 to 2.5 times the average wage in each segment, implying that firms capture a considerable part of the total output. The estimate for the unemployment benefits b ranges from 3.60 euros per hour in the lowest segment to 6.26 euros per hour in the highest segment. We find that the legal minimum wage is binding in the two lowest segments, but not in the other three. Hence, in these latter three segments we can identify the reservation wage and obtain an estimate for h, the combined value of home production and utility derived from leisure. It turns out that h is an important component of the reservation wage. The estimates are between 6.11 and 6.52 euros per hour, which corresponds to 60%-80% of the reservation wage. For

	Segment 1		Segment 2		Segment 3		Segment 4		Segment 5	
	est.	**s.e.**	**est.**	**s.e.**	**est.**	**s.e.**	**est.**	**s.e.**	**est.**	**s.e.**
Applications (in %)										
p_1	57.9	8.4	63.7	5.4	39.3	18.2	37.9	1.18	39.1	10.9
p_2	40.1	8.1	35.4	5.3	59.7	17.9	61.4	11.7	58.4	10.5
...	...	...	...	...	...	...	...	...	...	...
p_{30}	2.0	0.4	0.9	0.2	1.0	0.3	0.7	0.2	2.5	0.5
Other ML parameters										
$\hat{y}$	17.68	0.89	26.96	1.5	26.15	2.04	30.70	1.72	39.54	2.72
$\underline{w}$	7.51	0.00	7.51	0.00	7.56	0.12	8.29	0.13	10.61	0.29
σ	0.012	0.005	0.007	0.003	0.031	0.012	0.026	0.013	0.074	0.016
Job offers (in %)										
q_0	56.5	1.2	52.0	0.6	51.2	0.6	49.9	0.4	57.1	1.2
q_1	37.1	1.7	41.7	1.0	40.6	1.2	41.6	0.7	35.5	1.4
q_2	4.4	0.3	5.4	0.4	7.2	0.6	7.7	0.5	4.9	0.4
...	...	...	...	...	...	...	...	...	...	...
q_4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
q_5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1
q_6	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.1
q_7	0.2	0.1	0.0	0.0	0.1	0.1	0.0	0.0	0.3	0.1
q_8	0.2	0.1	0.1	0.0	0.1	0.1	0.1	0.0	0.4	0.1
q_9	0.3	0.1	0.1	0.0	0.1	0.1	0.1	0.0	0.4	0.1
q_{10}	0.3	0.1	0.1	0.0	0.2	0.1	0.1	0.0	0.3	0.0
q_{11}	0.3	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.3	0.0
q_{12}	0.2	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.2	0.0
q_{13}	0.2	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.1	0.0
q_{14}	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.0	0.0
q_{15}	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Transition probabilities (in %)										
δ	3.9		2.6		2.0		1.7		1.1	
m_W	3.6		4.1		5.6		6.5		7.5	
m_F	62.2		68.6		69.7		71.5		61.3	
Other variables										
ϕ	2.824		2.307		2.692		2.607		3.314	
ψ (in %)	33.3		39.0		34.6		35.5		29.1	
b	3.60		4.02		4.27		4.83		6.26	
h	[0,7.08]		[0,6.74]		6.11		6.15		6.52	
k	96.7		274.1		295.2		395.2		541.3	
$E_{F_w}[w]$	8.87		9.91		10.46		11.86		15.11	
$E_{G_w}[w]$	9.01		10.06		10.67		12.09		15.65	
Statistics										
Obs.	1043		1179		1153		1070		1022	
LogL.	-1.352		-1.776		-1.940		-2.077		-2.641	
KS	3.25		1.26		1.59		1.53		1.29	

The presented fractions are conditional on searching at least once. The fraction of non-searchers (p_0) is displayed in table 3.3. Not reported fractions are equal to (or rounded down to) zero for all segments.

Table 3.5: Estimation results

segments 1 and 2, the minimum wage is binding and we can only identify an upper bound on the value of home production.

Maximization of the likelihood also provides us with an estimate for the density of accepted wages $g_w(w)$. This estimate can be used to calculate the expected wage $E_{G_w(w)}[w]$. The values obtained in this way are also displayed in table 3.5. They are very close to the values found in the data, which were presented in table 3.3. The expected wage offer $E_{F_w(w)}[w]$ is always slightly lower, reflecting the fact that lower wage offers are less likely to be accepted than higher ones.

Figure 3.3 provides a closer look at the fit of the model. There we compare the estimate for $g_{\widetilde{w}}(\widetilde{w})$ to a kernel estimate of the wage density.[24] The figure shows that our model indeed matches the wage distribution very well. We also formally test the fit of the model by performing a Kolmogorov-Smirnov (KS) test.[25] The values of the test statistic are shown in the last row of table 3.5. It turns out that for segment 2 and 5 the test statistic is below the critical value 1.36. Hence, the test does not reject the null hypothesis that the empirical cdf and the estimated cdf have the same distribution in these two segments. In the other segments, the test statistic is significant. This is however a common finding in the estimation of search models with many observations (see e.g. Postel-Vinay and Robin, 2002b).

The estimated match probability m_W for a non employed worker is lowest in segment 1 (3.6%) and highest in segment 5 (7.5%). For the job destruction rate δ, we find an opposite pattern. It is highest in segment 1, where a fraction 3.9% of the matches is destroyed in each period, and monotonically decreases to 1.1% in segment 5. Note that these are probabilities per period. In order to check whether they match the actual probabilities, we convert them to annual values and we average over the segments. Appendix 3.C gives the details. We find an annual aggregate matching probability for the workers that equals 14.1%. The annual matching probability conditional on search in the current period, implied by the model, is 50.1%. The annual aggregate firing probability implied

[24] We use a standard normal kernel with bandwidth $1.06 s_w n^{-1/5}$, where s_w denotes the standard deviation of w.

[25] We calculate the KS statistic as $\Delta\sqrt{N}$, where N is the number of observations and Δ is the maximum absolute difference between the estimated and the empirical distribution of the observed wages.

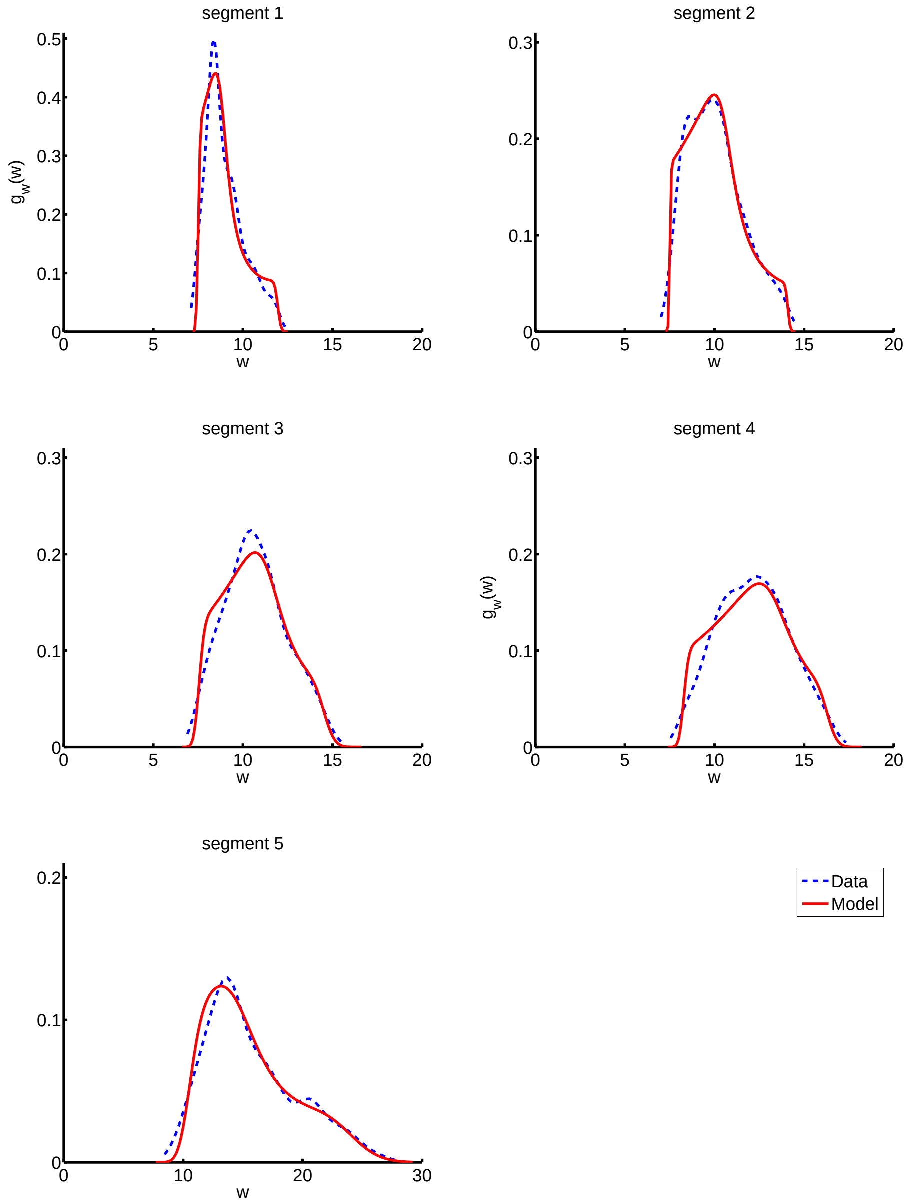


Figure 3.3: Estimated wage densities

by the model is equal to 5.2%.

These values are close to values given by other sources. Data of Statistics Netherlands[26] indicates that of the non-employed workers at the end of 1999, 11.0% was employed one year later. Broersma et al. (1998) report that over the period 1970-1995, the flow from unemployment to employment divided by the stock of unemployment in the previous period is 55%.[27] Van den Berg and van der Klaauw (2001) report three-month-unemployment-exit probabilities of 37%-45% for 1982-1994, which is roughly in line with our aggregate four-month-unemployment-exit probability $m_{W|U}$ of 46.5%. The model also performs well for the job loss rate δ. Using again data of Statistics Netherlands, we find that of the employed workers at the end of 1999, 5.2% was non-employed after a year. Our estimate matches this figure perfectly.

For firms we estimate matching probabilities, m_F, to be between 61.3% and 71.5%. The weighted average over the segments equals 66.9%. This value is in line with the matching probability given by van Ours and Ridder (1992). Using Dutch survey data, they find that 71% of the reported vacancies had been filled four months later. Den Haan et al. (2000) find exactly the same value (but on a quarterly basis) for the US labor market. The matching rates for firms and workers are variables that do not enter the likelihood function and consequently our estimation procedure is not designed to match them. So it is encouraging to see that the predicted values are close to the actual ones.

In order to determine to what extent the good fit of the wage distribution depends on the presence of measurement error, we judge the estimates for the standard deviation σ. We find values of σ between 0.007 (segment 2) and 0.074 (segment 5). The higher estimate of σ in segment 5 is in line with the fact that we still found considerable heterogeneity there. However, in general we can conclude that the degree of measurement error is small. The estimates for σ are of the same order of magnitude as the values found by van den Berg and Ridder (1998), who find standard deviations of 0.022 and 0.045. Dey and Flinn (2005) argue that the degree of measurement error that is required to provide a good fit of the model to the data can be considered to be an index of the degree of model misspecification. Such being the case, we conclude that our model gives an adequate de-

[26]http://statline.cbs.nl.

[27]This number is calculated from different tables in their paper as: $(UO/U) * (F_{ue}/U0) = 0.79 * 290/418 = 0.55$, where UO is total unemployment outflow and F_{ue} is the flow from unemployment to employment.

scription of the labor market.

The good fit is partly caused by the fact that our model allows the densities of accepted wages to be hump-shaped (also if we estimate the model without measurement error). This feature distinguishes our model from those described by, for example, Burdett and Mortensen (1998) and Gautier and Moraga-González (2005) which imply increasing densities. Another interesting result is that the wage distribution in each segment first-order stochastically dominates the distributions in all lower segments. Note that this was not directly visible in the raw data, where the lowest wages in the third segment were lower than in the second segment. By retrieving the true wage distribution, our model reveals that this is only the result of measurement error. In figure 3.4 we present the estimates for the search cost distributions. We find that search costs (measured in the same unit as the wages and the productivity) are in general higher in higher segments.

It is difficult to obtain direct information on the number of applications that workers send out. Van der Klaauw et al. (2003) have information on this variable for university graduates. The median number of applications per 4 months (one period in our model) in the period before a job was found is between 4 and 5 while our model predicts that it is close to 2 in the highest segment (which is the relevant one for university graduates).[28] So for this particular group we either underestimate the number of applications or overestimate the length of a period. Remember however that we make the simplifying assumption in our model that each application has an equal probability to be accepted while in reality workers may send some applications to jobs that are far above or below their league. Those applications may have a much lower acceptance probability, implying that the “effective" number of applications is less than 4 per period.

3.5.2 Mean-Min Ratio

In a recent paper Hornstein et al. (2007, hereafter HKV) argue that a large class of search and matching models is not able to explain the degree of wage dispersion that is observed in reality. Since our model belongs to the same class, we check to what extent our findings

[28] We thank Aico van Vuuren for kindly giving us this information.

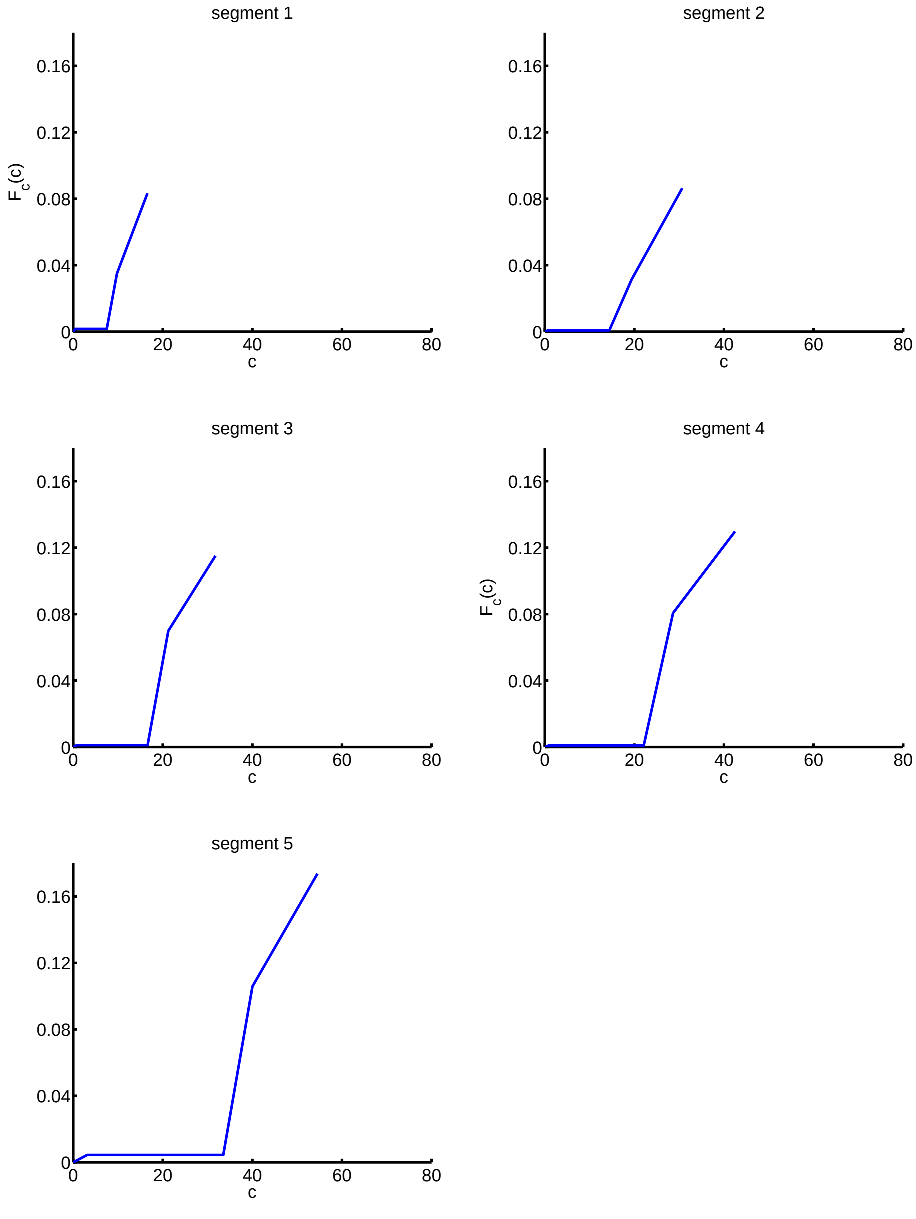


Figure 3.4: Estimated search cost distributions

are susceptible to this critique.

HKV discuss a specific measure of wage dispersion, which is defined as the ratio between the average wage and the lowest wage paid to employed workers. They show that a closed form expression for this mean-min ratio (Mm) can be obtained in a general class of search and matching models, without making any parametric assumption on the wage offer distribution. As defined before, ρ denotes the replacement rate, i.e. the ratio between b and the average wage. Then Mm is given by

$$Mm = \frac{1 + \frac{m_W}{r+\delta}}{\rho + \frac{m_W}{r+\delta}}.$$

They calibrate their model with US data on m_W and δ, which results in $Mm = 1.036$. However, at the same time they find that in US data sets with wage information the ratio between the average wage and the reservation wage is typically about 1.70 or larger. From this, they conclude that standard search models are not able to explain the observed combination of a low reservation wage and a high matching rate for unemployed workers. A similar point was made in Gautier and Teulings (2006) who focus on the ratio of the competitive and the reservation wage.[29] They argue that low unemployment rates imply small search frictions while substantial wage dispersion implies large search frictions.

In order to check the performance of our model in jointly explaining observed unemployment and wage dispersion, we set m_W and δ at the estimated values that we obtained in the previous subsection, while keeping ρ at 0.4 and r at 0.016. Then, we calculate the mean-min ratio as predicted by the market equilibrium and we compare this to what we observe in the data. The results of this are given in table 3.6. If we follow HKV by taking the fifth percentile of the wage distribution ($w_{5\%}$) in each segment as the reservation wage, then the mean-min ratio in the data varies between 1.215 (segment 1) and 1.598 (segment 5).[30] If we ignore the non-participants, we find that the matching probability $m_{W|U} = \frac{m_W}{1-p_0}$ is between 0.4 and 0.5 per period. This implies a mean-min ratio Mm_U that varies between 1.037 (segment 5) and 1.073 (segment 1). Those values are very close to the one found by HKV.

[29]They report substantially less wage dispersion than Hornstein et al. (2007). The difference is due to the fact that Gautier and Teulings (2006) correct for measurement error and unobserved heterogeneity.

[30]For optimal comparison with HKV, we do not throw away outliers here.

	Segment				
	1	**2**	**3**	**4**	**5**
Data					
w_{avg}	9.12	10.21	10.97	12.35	16.33
$w_{1\%}$	7.12	7.50	7.40	7.14	8.31
$w_{2\%}$	7.50	7.65	7.51	7.98	9.06
$w_{5\%}$	7.51	7.90	7.92	8.60	10.22
$Mp1$	1.281	1.361	1.481	1.729	1.965
$Mp2$	1.216	1.336	1.461	1.548	1.803
$Mp5$	1.215	1.293	1.385	1.435	1.598
Model					
m_W	0.036	0.041	0.056	0.065	0.075
$m_{W\|U}$	0.435	0.480	0.488	0.501	0.429
δ	0.039	0.026	0.020	0.017	0.011
r	0.016	0.016	0.016	0.016	0.016
ρ	0.400	0.400	0.400	0.400	0.400
Mm_{NE}	1.572	1.435	1.310	1.258	1.190
Mm_U	1.073	1.051	1.044	1.039	1.037

Table 3.6: Mean-min ratio in data and model

In our model however, the unemployed workers are a selective subsample of the total group of non-employed workers, namely the ones who happened to draw a low search cost in the current period and therefore have a large probability to receive a job offer in this period. They realize that in the next period they may draw a high search cost and they take this into account when they determine their reservation wage. We find that the mean-min ratio for the entire group of non-employed workers, Mm_{NE}, is between 1.190 (segment 5) and 1.572 (segment 1). Hence, in our model where workers have a positive probability to become a non-participant in the next period, a much larger part of wage dispersion can be explained by search frictions. The possibility of becoming non-participant and consequently obtaining a very low matching rate in the next period is consistent with a low reservation wage and a high transition rate from unemployment to employment.[31]

[31]In Albrecht and Vroman (2006), UI benefits fall over time. Their model is therefore also consistent with a low unemployment rate and substantial wage dispersion. In Albrecht and Axell (1984) workers are assumed to have different values of leisure. If there is enough heterogeneity in reservation values but at the same time, most workers accept most offers, there can be low unemployment together with substantial wage dispersion.

3.5.3 Efficiency

To check whether the Dutch labor market is constrained efficient, we solve the planner's problem for each of the five segments. We use the estimates of the search cost distribution $F_c(c)$, the productivity y, the capital cost k and the home productivity h that we obtained above and solve the Bellman equation (3.29) as in Shimer (2004a). Note that in the lowest two segments, we cannot identify the exact value of h. In those segments we set $h = 6.11$, the same value as in segment 3. The fact that h is almost the same in segments 3, 4 and 5 makes this assumption reasonable. Nevertheless, in section 3.6 we relax it and calculate the planner's solution using the estimated lower and upper bounds on h obtained for segments 1 and 2 (see table 5). There, we also check how sensitive our welfare analysis is to different search cost functions, and assumptions about the search cost of the non-employed.

A priori there is no simple answer to the question whether the number of applications sent by workers in the market equilibrium is too high or too low from a social planner's point of view. On the one hand, workers might underinvest in search since they face a standard hold-up problem: They only receive a part of the social benefits of their investments in search. On the other hand, workers might also send too many applications, since they only take into account their own expected payoff and ignore the congestion effects their applications cause in the market.

What about firm behavior? Albrecht et al. (2006) show that when all workers search two or more times, efficient entry requires full ex ante and full ex post (i.e. Bertrand) competition for workers. This is not the case in our model. There is no full ex ante competition, since the firm that offers the lowest wage in the market receives as many applications as the other firms, and there is no full ex post competition, because a firm that offers the job to a worker with (an) other offer(s) still has a positive expected payoff.

Table 3.7 presents the key parameters of both the constrained and the unconstrained planner's solution for each of the segments. We observe important differences between the market equilibrium and the constrained planner's strategy. First, the planner increases

	Market	Planner			Market	Planner	
		cstr.	uncstr.			cstr.	uncstr.
Segment 1				**Segment 2**			
p_0	91.7	83.9	76.0	p_0	91.4	83.3	74.8
p_1	4.8	16.1	24.0	p_1	5.5	16.7	25.2
p_2	3.3	0.0	0.0	p_2	3.1	0.0	0.0
...	...	...	...	...	...	...	...
p_{30}	0.2	0.0	0.0	p_{30}	0.1	0.0	0.0
θ	0.058	0.087	0.240	θ	0.060	0.076	0.252
$\frac{v}{u}$	0.700	0.541	1.000	$\frac{v}{u}$	0.700	0.455	1.000
$v+e$	0.510	0.681	0.843	$v+e$	0.639	0.744	0.895
Output	501.0	575.4	693.9	Output	721.6	822.4	995.0
Gain		15%	39%	Gain		14%	38%
Segment 3				**Segment 4**			
p_0	88.5	81.6	74.9	p_0	87.0	80.8	74.4
p_1	4.5	18.4	25.1	p_1	4.9	19.2	25.6
p_2	6.9	0.0	0.0	p_2	8.0	0.0	0.0
...	...	...	...	...	...	...	...
p_{30}	0.1	0.0	0.0	p_{30}	0.1	0.0	0.0
θ	0.081	0.082	0.251	θ	0.091	0.084	0.256
$\frac{v}{u}$	0.700	0.446	1.000	$\frac{v}{u}$	0.700	0.437	1.000
$v+e$	0.757	0.802	0.915	$v+e$	0.809	0.829	0.928
Output	840.7	936.9	1076.5	Output	1039.9	1141.2	1294.9
Gain		11%	28%	Gain		10%	25%
Segment 5							
p_0	82.6	67.3	60.9				
p_1	6.8	32.7	39.1				
p_2	10.1	0.0	0.0				
...	...	...	...				
p_{30}	0.4	0.0	0.0				
θ	0.122	0.145	0.391				
$\frac{v}{u}$	0.700	0.445	1.000				
$v+e$	0.890	0.935	0.975				
Output	1600.1	1801.3	1944.9				
Gain		13%	22%				

The fractions p_a are percentages. Omitted values are equal to (or rounded down to) zero.

Table 3.7: Comparison of the market equilibrium with the constrained (cstr) and unconstrained (uncstr) planner's solution

participation: a considerable group of non-employed workers (6%-15%) should send one rather than zero applications. Second, the planner decreases the number of workers sending two or more applications. These workers (3%-10% of the non-employed) have low search costs, which makes it profitable for them to send so many applications. As described above however, they do not to take into account that their large number of applications increases the probability that multiple firms consider the same candidate, which is socially wasteful. Given the estimated values of capital and application cost and our matching technology, it is socially not desirable that workers send many applications.[32] These two results have an important effect on the optimal stock of vacancies. Given the increase in participation and the lower congestion in the market, the planner finds it optimal to increase labor market tightness.[33] For the unconstrained planner we find optimal strategies that are quite similar to the constrained planner's solution, except that the unconstrained planner wants to increase the participation of workers even more since coordination frictions can be eliminated fully.

Finally, table 3.7 reports the present value of output for the market and the planner. For both the market and the planner we set the initial value of the state variable e at the market steady state level. We define the efficiency of the labor market as the ratio between the market and planner's present value. It turns out that the constrained planner generates a 10% to 15% higher output than the market, while the unconstrained planner does on average about 30% better. This result allows us to decompose the total efficiency loss into a part caused by wrong incentives and a part caused by coordination frictions. We find that in the lowest 4 segments wrong incentives account for about 35% to 40% of the total inefficiency while the coordination frictions contribute 60% to 65%. In the

[32] Although for some of the values of θ that we find, the social benefits of a second application are positive, they do not outweigh the marginal cost. The social benefits of sending more than two applications are usually negative (the positive effect of reducing the probability that a vacancy has no applicants is smaller than the negative effect of increasing the probability that multiple vacancies compete for the same candidate).

[33] An interesting question is whether, given market equilibrium search intensities, entry is insufficient or excessive. If we calculate the optimal number of firms for search intensities set equal to those in the market equilibrium, we find excessive entry. Correcting for firms' entry incentives only yield quite modest welfare gains. The bulk of welfare gains are thus derived from increasing participation, lowering congestion and increasing firm entry together.

highest segment the pattern is reversed. Wrong incentives explain 60% and coordination frictions account for the remaining 40% of the inefficiency. Figure 3.5 shows the planner's transition path for segment 3.[34]

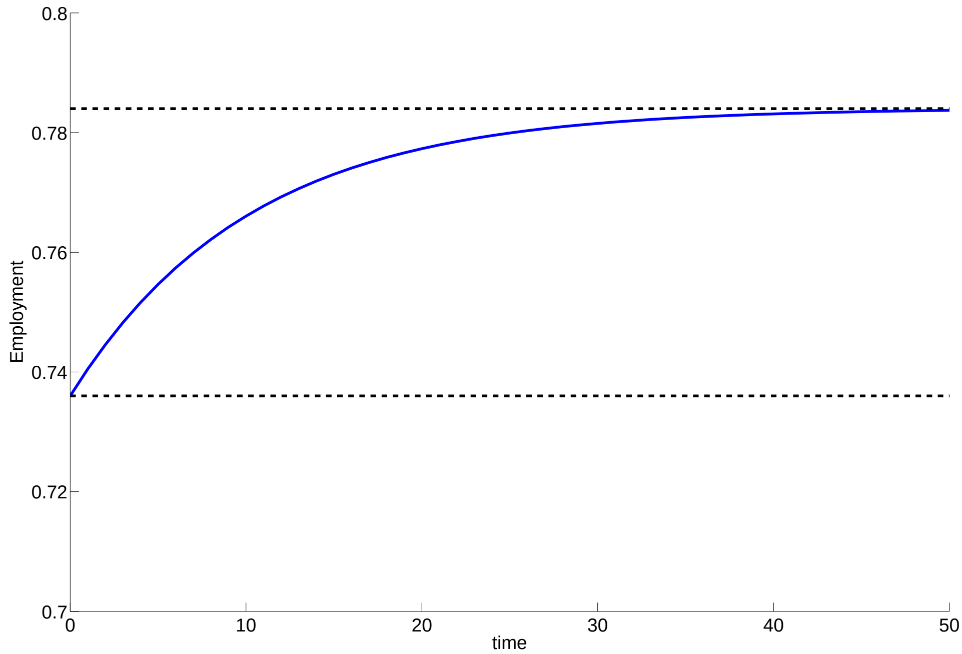


Figure 3.5: The planner's transition path

Our efficiency results also shed new light on the desirability of a binding but moderate minimum wage. It has the potential to help correct all three inefficiencies that are present in our model. First, by increasing the average wage it makes more workers search once rather than zero times, so it increases participation. Second, by compressing the wage distribution it reduces the incentives to search more than once, decreasing rent seeking behavior and coordination frictions. Third, it reduces excessive entry of vacancies. Unemployment benefits that are conditional on searching at least once have similar effects. Hence, a marginal increase in either the minimum wage of the UI benefits is welfare improving.[35]

[34]Substituting the optimal values for p_a and θ in the difference equation (3.1) gives an eigenvalue between 0 and 1, so convergence is monotone (see Shimer, 2004a).

[35]Due to numerical constraints we are not able to derive the socially optimal minimum wage or UI benefits. Another reason for only considering marginal changes in the minimum wage or UI benefits is that we cannot rule out multiple equilibria. If we considered large changes in those parameters, the model could

3.6 Robustness

In our main analysis we made some simplifying assumptions, namely: (i) the value of household production in segments 1 and 2 that could not be identified (because the minimum wage is binding there) is equal to the value of household production in segment 3, (ii) the irrecoverable part of the search cost distribution for the non-participants can be obtained by linear extrapolation, and (iii) the search cost functions $C(a)$ are linear. To what extent do these assumptions affect our main results? To answer this, we relax (i)-(iii) in the subsequent subsections. Finally, in section 3.6.4 we discuss how other potential sources of heterogeneity, which we ignored to keep the model tractable, would affect our results and how factors that we left out would affect the magnitude of search costs.

3.6.1 Value of Home Production

In table 3.7 we fixed home production h in the lowest two segments, where the minimum wage was binding, to 6.11, i.e. the same value as in the third segment. Instead, we could use bounds for h. The lower bound would be zero and the upper bound would be the value of h for which the reservation wage is equal to the minimum wage, i.e. $h = 7.08$ for segment 1 and $h = 6.74$ for segment 2. The different values for the home production influence the estimated search cost distributions, as is shown in figure 3.6. Not surprisingly, we find that the estimated search costs are higher for lower values of h. After all, lower values of h imply larger benefits of search. In order to have the same values $p_0, ..., p_S$ (that maximize the likelihood) in equilibrium, the costs of search must be higher as well in that case.

Hence, the value of h affects the planner's solution in two ways: directly by changing the contribution of a non-employed worker to total output, and indirectly via the estimated search cost distribution. Table 3.8 shows that the latter effect dominates. The constrained planner sets p_0 at a higher value for the lower bound of h than for the upper bound. The main conclusion that participation should be increased and that a small fraction of

jump to a different equilibrium.

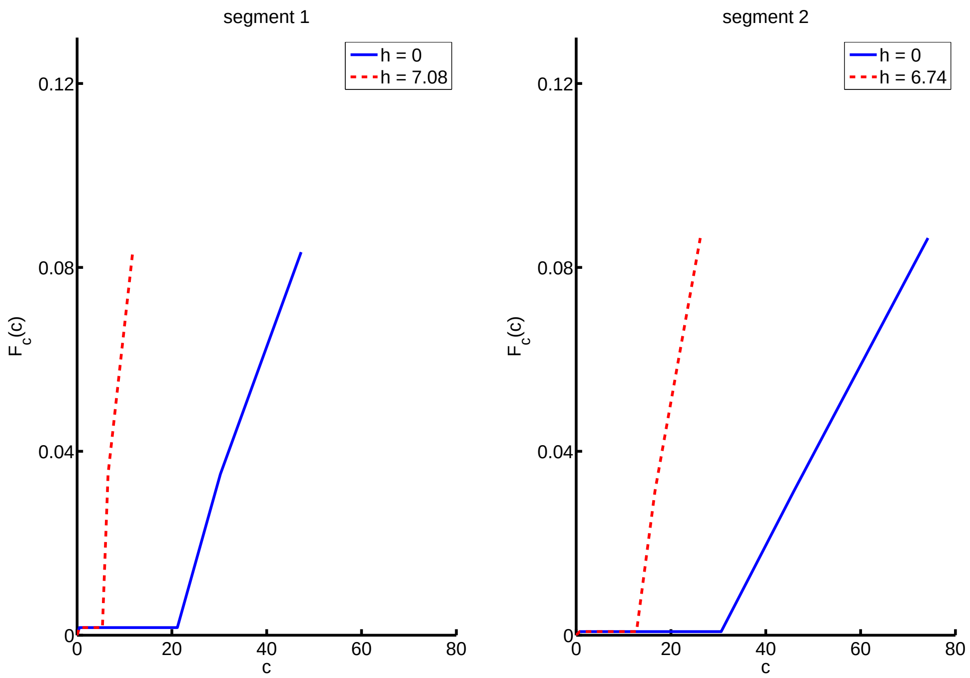


Figure 3.6: Estimated search cost distributions for segment 1 and 2 using bounds on the value of household production

workers sends too many applications however remains. Furthermore, the efficiency loss due to wrong incentives is similar to what was found in table 3.7.[36] We can conclude that the assumption about home production in the lowest two segments does not affect our main conclusions.

3.6.2 Search Cost for Non-Participants

Since the search costs of workers who decide not to search are in principle irrecoverable, we made a parametric assumption, namely that their search cost could be obtained by linear extrapolation. In this subsection we relax this assumption by considering bounds for the search cost of non-participants. The planner generates a higher output as the search cost of the non-participants is lower. The upper bound to the planner's solution is therefore obtained when all non-participants have a search cost that is equal to Γ_1, i.e. the marginal benefit of the first application. They cannot have a lower search cost, since otherwise

[36] Note that a change in h also affects the calculated value for the market output. The relative difference between market and planner's output is therefore the most informative measure.

	Market	Planner		Market	Planner
Segment 1 ($h = 0$)			**Segment 1** ($h = 7.08$)		
p_0	91.7	85.9	p_0	91.7	82.6
p_1	4.8	14.1	p_1	4.8	17.4
p_2	3.3	0.0	p_2	3.3	0.0
...	...	...	...	...	...
p_{30}	0.2	0.0	p_{30}	0.2	0.0
θ	0.058	0.107	θ	0.058	0.086
$\frac{v}{u}$	0.700	0.757	$\frac{v}{u}$	0.700	0.496
$v+e$	0.510	0.701	$v+e$	0.510	0.685
Output	227.1	341.3	Output	544.3	616.1
Gain		50%	Gain		13%
Segment 2 ($h = 0$)			**Segment 2** ($h = 6.74$)		
p_0	91.4	87.1	p_0	91.4	82.1
p_1	5.5	12.9	p_1	5.5	17.9
p_2	3.1	0.0	p_2	3.1	0.0
...	...	...	...	...	...
p_{30}	0.1	0.0	p_{30}	0.1	0.0
θ	0.060	0.075	θ	0.060	0.079
$\frac{v}{u}$	0.700	0.583	$\frac{v}{u}$	0.700	0.439
$v+e$	0.639	0.727	$v+e$	0.639	0.753
Output	489.9	602.2	Output	744.7	847.6
Gain		21%	Gain		14%

The fractions p_a are percentages. Omitted values are equal to (or rounded down to) zero.

Table 3.8: Effect of bounds for household production on the planner's solution

becoming a non-participant would not have been a utility maximizing choice. On the other hand, the lower bound to the planner's solution arises when all non-participants have an infinitely large search cost. Table 3.9 presents both bounds to the constrained planner's solution. In order to ease comparison, the table also displays again the market equilibrium and the planner's solution in case of linear extrapolation, as in table 3.7.

Not surprisingly, the planner keeps p_0 equal to the value in the market equilibrium, when all non-participants have infinitely large search costs. Moreover, he changes the number of vacancies only marginally. Like in the linear case however, the planner does not want workers to send more than one application. The average gain relative to the market equilibrium is about 10%, which is not very different from the value found with

	Market	Planner			Market	Planner	
		max. cost	min. cost			max. cost	min. cost
Segment 1				**Segment 2**			
p_0	91.7	91.7	0.0	p_0	91.4	91.4	0.0
p_1	4.8	8.3	100.0	p_1	5.5	8.6	100.0
p_2	3.3	0.0	0.0	p_2	3.1	0.0	0.0
...	...	...	...	...	...	...	...
p_{30}	0.2	0.0	0.0	p_{30}	0.1	0.0	0.0
θ	0.058	0.046	0.476	θ	0.060	0.040	0.410
$\frac{v}{u}$	0.700	0.546	0.476	$\frac{v}{u}$	0.700	0.466	0.410
$v+e$	0.510	0.515	0.955	$v+e$	0.639	0.596	0.962
Output	501.0	561.9	633.4	Output	721.6	805.0	889.4
Gain		12%	26%	Gain		12%	23%
Segment 3				**Segment 4**			
p_0	88.5	88.5	0.0	p_0	87.0	87.0	0.0
p_1	4.5	11.5	100.0	p_1	4.9	13.0	100.0
p_2	6.9	0.0	0.0	p_2	8.0	0.0	0.0
...	...	...	...	...	...	...	...
p_{30}	0.1	0.0	0.0	p_{30}	0.1	0.0	0.0
θ	0.081	0.052	0.405	θ	0.091	0.057	0.403
$\frac{v}{u}$	0.700	0.454	0.405	$\frac{v}{u}$	0.700	0.443	0.403
$v+e$	0.757	0.713	0.969	$v+e$	0.809	0.763	0.973
Output	840.7	926.7	986.2	Output	1039.9	1132.8	1187.0
Gain		10%	17%	Gain		9%	14%
Segment 5							
p_0	82.6	82.6	0.0				
p_1	6.8	17.4	100.0				
p_2	10.1	0.0	0.0				
...	...	...	...				
p_{30}	0.4	0.0	0.0				
θ	0.122	0.080	0.404				
$\frac{v}{u}$	0.700	0.462	0.404				
$v+e$	0.890	0.880	0.983				
Output	1600.1	1781.0	1848.2				
Gain		11%	16%				

The fractions p_a are percentages. Omitted values are equal to (or rounded down to) zero.

Table 3.9: Effect of bounds on non-participant's search cost on the planner's solution

linear extrapolation. If we assume that all non-participants have search cost Γ_1, then it is optimal for the planner to let everybody search once ($p_1 = 1$). He increases θ to about

0.4 in each of the segments. The gain compared to the market now varies between 14% (segment 4) and 26% (segment 1).

In all segments, the minimum and maximum search cost case that we consider provide reasonably narrow bounds on the planner's solution. Moreover, both the minimum and maximum case are actually unrealistically extreme. We know that in the labor market non-participants with low and high search costs coexist. A large fraction of the non-participants is unable to work for various reasons, like disability or because they follow an education. At the same time however, certain non-participants can be considered to be marginally attached (MA) to the labor force. Jones and Riddell (1999) show for example that a small fraction of the Canadian non-participants has a positive probability to flow into employment.[37] Hence, the linear extrapolation case, in which some non-participants have relatively low search cost, while others have high search costs, describes the labor market a lot better than the two bounds. Nevertheless, it is encouraging to see that the bounds, despite being unrealistically extreme, are reasonably close to this linear case.

3.6.3 Search Cost Function

In section 3.2 we assumed that different workers can have different search costs but that the search cost technology is linear in a for all workers. So, we considered functions $C(a) = ca$, with $c > 0$. In reality, this assumption might not hold. $C(a)$ will be concave if workers invest a lot of time in drafting the first application letter but spend less time on the subsequent ones. On the other hand, $C(a)$ will be convex if workers have easy access to a small number of vacancies (e.g. via their network of friends and colleagues), but have to search really hard to find other job openings. Because of this, we relax the linearity assumption in this subsection and consider a very general class of search cost functions.

We allow different workers to have different shapes of the search cost function: in every period each worker i draws a search cost function $C_i(a)$ from a given collection

[37] If those workers are included in unemployment they would consist of 25-30% of the unemployed. For the Netherlands, an upper bound estimate of the number of marginally attached workers is 7.5% of the labor force (calculated as all non-disabled workers in 1999 below age 54 who are available, but not necessarily immediately, for 12 hours or more, including school leavers, see Bijsterbosch and Nahuis, 2000).

of search cost functions. We only make two very weak assumptions about the shape of $C_i(a)$: $C_i(a)$ is (i) equal to zero for $a = 0$ and (ii) weakly increasing in a. Although the workers know the collection of search cost functions from which they draw, the econometrician does not observe this. Therefore, we have to make one more assumption. Note that the collection of search cost functions does not directly enter the maximum likelihood problem, but determines which sets of $\{p_0, ..., p_S\}$ are feasible. Suppose for example that each search cost function $C_i(a)$ consists of a (stochastic) fixed cost for the first application and that all other applications can be sent for free. This would imply that workers either send 0 or S applications and never $1, 2, ..., S-1$ applications. Hence, only solutions in which $p_0 + p_S = 1$ would be possible in that case. Since this solution generates a lower likelihood than the solution that we obtain, this collection of search cost functions is not feasible.

In fact, no collection of search cost functions could generate a higher likelihood value than we obtain, since our linear search cost functions did not restrict the set $\{p_0, ..., p_S\}$ in any way. Hence, we impose that the collection of search cost functions supports the ML estimates found in section 3.5. This implies that we allow some workers to have a search cost function that consists of a fixed cost only, but not too many. Some workers must have a different search cost function, otherwise, no worker would send one or two applications, while the ML estimates indicate that we need a substantial amount of such workers in order to fit the data well.

This condition on the collection of search cost functions implies that the solution that we find for the market equilibrium does not change, because it maximizes (3.30) and remains feasible. The planner's solution will change, but it can be bounded. First, note that we can identify for a worker who applied to $\widehat{a} > 0$ jobs that the total cost she makes is below a certain threshold, namely the sum of the marginal benefits of searching $\widehat{a}$ times: $C_i(\widehat{a}) < x_{\widehat{a}} = \sum_{l=1}^{\widehat{a}} \Gamma_l$. Since the total search costs are weakly increasing in a, the cost for this worker of searching $a \in \{1, ..., \widehat{a}-1\}$ times is also at least 0 and at most $x_{\widehat{a}}$. Hence, unlike in the linear case, we cannot rule out that a worker who sends 20 applications can pay more for sending 5 applications than a worker who actually sent 5 applications.

Similarly for $a > \widehat{a}$, the total search costs are at least $\sum_{l=\widehat{a}+1}^{a} \Gamma_l$. For example, consider a worker who applied once and assume that the marginal benefits of sending 2 applications equal 15. The total cost of sending 1 application must have been at least 0 and therefore the total cost of sending 2 applications for this worker must be at least 15 (otherwise the worker would have sent two applications).

For each segment we can now calculate a 31 by 31 matrix where cell ij contains the minimum amount of search cost of sending j applications for a worker who has actually sent i applications. We can do the same for the maximum search cost. Using those matrices, the planner can then determine his solutions. Note that all workers now have different search costs than in the baseline linear case. As a consequence, the option value of search changes, which results in a different estimate for the value of home production.[38] This does not affect the market output, since the change in h is exactly offset by the change in expected search costs in equilibrium. However, it does not affect the planner's solution. Hence, we let the planner take the new estimate for h into account when he determines his solution. Table 3.10 gives the results for our set estimates and what strikes is that the bounds are quite tight. In fact, they are very similar to the bounds found in table 3.9, where we only relaxed the search cost of the non-participants.[39]

Obviously, we again find that the lower the search costs are, the higher output is and the smaller the desired fraction of non-participants p_0 is. For the high search cost case, the planner's solution is close to the market equilibrium. The planner lets only a small fraction of the workers search and it is optimal that these workers send one application. Finally, the planner lets the workers who searched thirty times become non-participants. Letting these workers search once (as in the low search cost case) is not a good strategy, since the (maximum) cost for them of applying once is the same as the cost of applying thirty times.

[38]This was not the case in the previous subsection. There only the search costs of non-participants changed. Since they do not search, the option value remains unchanged.

[39]The bounds are not necessarily wider than in table 3.9, because the value of h has changed.

	Market	**Planner**	
		max. cost	**min. cost**
Segment 1			
h	6.11	6.36	4.89
p_0	91.7	91.8	0.0
p_1	4.8	8.2	100.0
p_2	3.3	0.0	0.0
...	...	...	...
p_{30}	0.2	0.0	0.0
θ	0.058	0.047	0.468
$\frac{v}{u}$	0.700	0.573	0.468
$v+e$	0.510	0.518	0.955
Output	501.0	544.5	631.6
Gain		9%	26%
Segment 2			
h	6.11	6.58	3.69
p_0	91.4	91.4	0.0
p_1	5.5	8.6	100.0
p_2	3.1	0.0	0.0
...	...	...	...
p_{30}	0.1	0.0	0.0
θ	0.060	0.041	0.412
$\frac{v}{u}$	0.700	0.481	0.412
$v+e$	0.639	0.599	0.962
Output	721.6	782.1	886.9
Gain		8%	23%
Segment 3			
h	6.11	6.68	2.30
p_0	88.5	88.6	0.0
p_1	4.5	11.4	100.0
p_2	6.9	0.0	0.0
...	...	...	...
p_{30}	0.1	0.0	0.0
θ	0.081	0.055	0.409
$\frac{v}{u}$	0.700	0.483	0.409
$v+e$	0.757	0.721	0.970
Output	840.7	889.1	981.4
Gain		6%	17%
Segment 4			
h	6.15	7.02	0.34
p_0	87.0	87.1	0.0
p_1	4.9	12.9	100.0
p_2	8.0	0.0	0.0
...	...	...	...
p_{30}	0.1	0.0	0.0
θ	0.091	0.055	0.408
$\frac{v}{u}$	0.700	0.424	0.408
$v+e$	0.809	0.755	0.974
Output	1039.9	1083.5	1180.5
Gain		4%	14%
Segment 5			
h	6.26	10.61	-4.35
p_0	82.6	83.1	0.0
p_1	6.8	16.9	100.0
p_2	10.1	0.0	0.0
...	...	...	...
p_{30}	0.4	0.0	0.0
θ	0.122	0.084	0.411
$\frac{v}{u}$	0.700	0.497	0.411
$v+e$	0.890	0.883	0.984
Output	1600.1	1737.5	1840.5
Gain		9%	15%

The fractions p_a are percentages. Omitted values are equal to (or rounded down to) zero.

Table 3.10: Planner's solution for a general class of search cost functions

We can conclude that if we consider a very general class of search frictions, the bounds on the planner's solution hardly change. Although the estimate of the value of home production changes, the main message remains that given our endogenous matching process, participation is generally too low and unemployed workers should not send too many applications.

3.6.4 Other Heterogeneity and the Search Costs' Magnitude

Our estimates of the search costs are quite large. The total incurred search cost per worker in segment 3 is estimated to be around 22,970 euros on average. The explanation for this is that the expected duration of jobs in combination with a low discount rate makes the expected benefits of job search very large. The only way to fit the estimated search strategies is to make job search expensive. We want to emphasize that in our model search costs should not be interpreted as the cost of a stamp, but include all the costs (opportunity costs, informational costs, etc.) that a worker must make to receive job offers. Below, we argue that this estimate of total search cost is sensitive to misspecification but that our main conclusion that participation is too low and that a small fraction of the workers sends out too many applications is a lot less sensitive to misspecification. We start with summarizing some potential sources of misspecification.

First of all, the estimated search costs would be substantially lower if the discount rate were higher because it reduces the benefits of search and accordingly the estimates of the search cost that are necessary to fit the wage distribution. Note however that the discount rate does not have an effect on the estimates for the search intensity fractions p_a, the job offer probability ψ, and the productivity y.

Second, allowing for on-the-job search would decrease expected job duration and increase our estimate of home production. The first effect is similar to increasing the job destruction rate δ. This again does not affect the estimates of the main parameters like the search fractions p_a, but it does change the scale of the search cost distribution downwardly. The second effect is caused by the fact that the option value of search during unemployment relative to search during employment goes down. In order for the reservation wage to be equal to the lowest wage in the segment, h would have to go up.

More serious concerns are related to unobserved heterogeneity. In our model, there are three reasons why identical workers earn different wages. First, workers differ in their search cost and accordingly in their search intensity. Second, some workers are more lucky than others in terms of the number of offers they receive. Third, some workers are more lucky than others with an equal number of offers in terms of the magnitude of their highest offer. However, the only source of worker heterogeneity before the realizations of offers is known is in terms of search cost. How do other potential sources of heterogeneity (e.g. in productivity, in home production, in search efficiency or in knowledge about the locations of vacancies), that we ignore to keep the model tractable, affect the search cost estimates?

First note that some sources of heterogeneity easily translate into search cost. For example, there is no fundamental difference between a worker who can easily obtain information about the location of jobs and a worker with low search cost. But, other sources of heterogeneity may affect our search cost estimates. We make the strong assumption that within a segment all workers have the same productivity, y. One can argue that this assumption does not hold. For example, some non-participants might have a very low productivity (the disabled and early retired workers). Such unobserved heterogeneity in productivity makes some of the wage dispersion that we find spurious, i.e. not caused by differences in search intensity. However, allowing for measurement error mitigates this effect (at the cost of a higher estimate for σ).

A similar story holds for home production or disutility of work. Some individuals have a very high value of h (e.g. individuals with young kids whose partner works). Heterogeneity in h translates into heterogeneity in reservation wages as in Albrecht and Axell (1984), but Eckstein and Wolpin (1990) show that only considering heterogeneity in h gives a poor fit of the wage data. In our model ignoring heterogeneity in h would not affect wage dispersion (as long as firms cannot condition their wage offer on the value of h), but it implies that our estimate of the search cost is too large for those workers who have large values of h.

Finally, it is important to emphasize that the effect of misspecification on our welfare analysis is smaller than on the magnitude of search cost because we use the same values of search cost for the market as for the social planner. In general, a planner would prefer to match a worker with a high expected wage in market equilibrium, rather than a worker with a low expected wage, no matter whether this high wage is (i) the result of low search cost or (ii) the result of high unobserved skills.

To illustrate the effect of a change in the level of search cost, we calculate what happens in segment 3 if we quadruple the discount rate r (i.e. from 5% to 20% per year). This approximately rescales the search cost distribution downwardly by a factor 2. It turns out that the constrained social planner then wants p_0 to be equal to 82.6%, compared to 81.6% with the actual search cost estimates. The optimal value of θ changes marginally from 0.082 to 0.083. The estimate of the market inefficiency (4%) is lower than before (11%), reflecting the fact that the planner now assigns less weight to future output. Overall, we conclude that the shape of the distribution of search costs is more important for our conclusions than the magnitude.

3.7 Related Literature

In this section we relate our essay to the existing literature. First, our model is very similar to the noisy search model of Burdett and Judd (1983) where (in labor market terminology) workers can receive multiple offers. As in Kandal and Simhon (2002) we allow for the possibility that applications are rejected. We extend their model by endogenizing search intensity and the distribution of job offers. Since we allow for coordination frictions in the matching process, increasing the average search intensity does not make the model converge to the Walrasian equilibrium like in their model. Stern (1989) also estimates a simultaneous job search model but he has an exogenous wage offer distribution.

Albrecht and Axell (1984) also get wage dispersion due to worker heterogeneity. Their heterogeneity is in terms of reservation wages while ours is in terms of search costs which gives us a continuous rather than a discrete wage distribution. Bontemps et al. (2000) and

Mortensen (2003) focus on heterogeneity on the firm side. Bontemps et al. (1999) have heterogeneity on both the worker and firm side. Introducing firm heterogeneity in the Burdett and Mortensen (1998) model of on-the-job search gives a good fit of the wage distribution. All the introduced heterogeneity in the above mentioned papers is motivated by the fact that wage data do not fit the mixed-strategy wage distributions implied by the models. Burdett and Mortensen (1998), Burdett and Judd (1983) and Gautier and Moraga-González (2005) all fail to produce hump-shaped distributions. We show that simply allowing for ex post heterogeneity in search cost gives a very good fit of the wage distribution. Basically, the fat right tail within a segment suggests that there is a small fraction of workers with low search cost, receiving many offers.

There are various models with endogenous search intensity. Benhabib and Bull (1983) consider the optimal number of applications in a partial search model with an exogenous wage distribution where, as in our model, workers take the highest offer. In Mortensen (1986), workers can increase the job offer arrival rate by spending more time on search. Bloemen (2005) estimates this model and van der Klaauw et al. (2003) estimate an extension of this model on a sample of university graduates where they allow search intensity before graduation to be time-varying. Christensen et al. (2005) estimate a wage posting model where workers can make investments to increase the job offer arrival rate. The congestion externalities of multiple applications that are present in our model are absent in their model. Albrecht et al. (2004) derive a matching function with multiple applications. More applications make it less likely that a vacancy has no applicants but more likely that multiple firms consider the same candidate.[40] The matching rate is determined by the interaction between those coordination frictions. The aggregate matching function is typically first increasing and then decreasing in average search intensity. This essay extends this matching framework by allowing for heterogeneity in search cost and is the first one which estimates it simultaneously with the wage distribution. This is important for policy analysis because wage policies affect search intensity and policies that affect search

[40]In current work, it has become standard to define search intensity by the number of simultaneous job applications workers send out, see Albrecht et al. (2003, 2006), Gautier and Moraga-González (2005), Gautier and Wolthoff (2006), Galenianos and Kircher (2008), Kircher (2007), Shimer (2004b), and Chade and Smith (2006).

intensity will also affect the wage distribution.

In principle, our model allows a non-employed worker to be in any of 30 different search states, each referring to the number of applications she sends. In the macro search literature, the focus has been more on the distinction between two states: participation and non-participation. We have defined non-participation as workers who do not apply to any job but one could alternatively make a distinction between workers sending many or few applications. For simplicity we only consider heterogeneity in search cost to drive participation and search intensity but in Pries and Rogerson (2004) variations in market productivity drive the participation decision while in Pissarides (2000) and Garibaldi and Wasmer (2005) variations in home productivity determine participation. In Frijters and van der Klaauw (2006), (true) duration dependence of unemployment can push the reservation wage below the value of home production.

There are many other structural estimates of search models, we mention just a few. Besides the ones mentioned above, Eckstein and Wolpin (1990) have estimated the Albrecht-Axell model, van den Berg and Ridder (1998) estimate the Burdett-Mortensen model and Postel-Vinay and Robin (2002b, 2004) estimate an on-the-job search model with Bertrand competition between the poaching and the incumbent firm. To our knowledge, there does not exist previous work estimating a labor market version of the Burdett and Judd (1983) model with rationing as in Gautier and Moraga-González (2005), which is what we do here.

Finally, there are a couple of other papers that study the general equilibrium effects of labor market policies that increase search intensity. Flinn (2006) estimates a matching model with Nash bargaining and finds potential positive welfare effects of a binding minimum wage. This is consistent with our findings. Davidson and Woodbury (1993) and Blundell et al. (2003) study the general equilibrium effects of giving a subset of workers a wage bonus or subsidy. Both find huge offsetting equilibrium effects and the latter even find a sign reversal since jobs taken by the treatment group would in the absence of the treatment be filled by non-treated workers. Lise et al. (2003) calibrate their equilibrium search model to data from the control group and then simulate a Canadian income assistance program within the model. They show that the model mimics the transition rate of the treatment group but that the total welfare effects are reversed when the general equilibrium effects are taken into account. A similar methodology is applied in Todd and Wolpin (2006).

We estimate the equilibrium model from the beginning and then compare the optimal search intensity distribution with the observed one and find that non-participation is too high and unemployed workers search to intensively. In our model, wage subsidy or counseling schemes for a subset of currently unemployed workers will increases their search intensity and individual employment probabilities but at the same time reduce the employment probabilities for other workers. Our results suggest that active labor market programs can best be targeted at the weakly attached workers, i.e. the ones who are non-participant but who are close to the margin of participating.

3.8 Conclusions

We have presented a discrete-time dynamic labor market model with a continuum of identical, infinitely-lived workers and free entry of vacancies. Unlike most of the literature, we have explicitly defined the search intensity as the number of applications that workers send out per period. As such, the model provides a framework for the evaluation of public policies intended to increase job search intensity. The model has been estimated by maximum likelihood using wages of newly hired workers and gives a good fit. We have found that in all segments most unemployed workers search once or twice in a four month period while a small fraction of the job seekers (between 0.7 and 2.5%) applies to thirty jobs. We have also shown that the decentralized market produces a welfare level that is about 10% to 15% below the constrained planner's outcome who takes the coordination frictions, value of home production, productivity and the search cost distribution as given. The planner would like some workers to search more, others to search less and increase firm entry. Especially, applying for two or thirty jobs is socially wasteful.

An important assumption in our model was that firms that fail to hire their candidate cannot offer the job to the next candidate. Albrecht et al. (2006) show that allowing for firms to make shortlists of workers is very tedious. It does reduce coordination frictions and consequently increases the matching rates but recall does not eliminate the coordination frictions because a firm with four candidates can still loose all of them to competing firms. Allowing for complete recall as in Kircher (2007) will increase the social benefits of sending multiple applications. We also discussed a number of other sources of misspecification but in general this does not change the qualitative conclusion that participation is too low and that a small fraction of unemployed workers searches too intensively.

Compared to other empirical equilibrium search models in the literature, we have modeled the matching process and search intensity with a lot more detail but in other respects our model is simpler. For example, since workers are ex ante identical, unemployment duration follows a geometrical distribution while in reality there typically is negative duration dependence. One way to get positive duration dependence in our framework is by assuming the heterogeneity in search cost to be worker specific such that high-search-cost workers receive fewer offers in each period in expectation. This is a research avenue we plan to pursue in future work.

3.A Derivations

3.A.1 Matching Probability

A firm matches if it offers its candidate a higher wage than all other firms competing for the same worker. The probability for a firm to have at least one applicant is equal to $1-e^{-\phi}$. The conditional probability that the candidate has sent a applications is given by $\frac{ap_a}{\sum_{i=1}^{S} ip_i}$. The $a-1$ other applications of the candidate result in $j \in \{0,1,...,a-1\}$ other job offers with probability $\chi(j|a-1)$, which are all lower with probability $F_w^j(w)$. Hence, the matching probability of a firm offering w is given by

$$m_F(w) = \left(1-e^{-\phi}\right) \sum_{a=1}^{S} \frac{ap_a}{\sum_{i=1}^{S} ip_i} \sum_{j=0}^{a-1} \chi(j|a-1) F_w^j(w).$$

By using $\left(1-e^{-\phi}\right)=\phi\psi=\frac{\sum_{i=1}^{S} ip_i}{\theta}\psi$ and the definition of $\chi\left(j|a-1\right)$, we can simplify this expression as follows

$$\begin{aligned}
m_F\left(w\right) &= \phi\psi\sum_{a=1}^{S}\frac{ap_a}{\sum_{i=1}^{S} ip_i}\sum_{j=0}^{a-1}\binom{a-1}{j}\psi^j\left(1-\psi\right)^{a-1-j}F_w^j\left(w\right)\\
&= \frac{1}{\theta}\sum_{a=1}^{S} ap_a\sum_{j=0}^{a-1}\binom{a-1}{j}\psi^{j+1}\left(1-\psi\right)^{a-1-j}F_w^j\left(w\right)\\
&= \frac{1}{\theta}\sum_{a=1}^{S}\sum_{j=1}^{a} ap_a\binom{a-1}{j-1}\psi^j\left(1-\psi\right)^{a-j}F_w^{j-1}\left(w\right)\\
&= \frac{1}{\theta}\sum_{j=1}^{S} jq_jF_w^{j-1}\left(w\right).
\end{aligned}$$

3.A.2 Likelihood

The first step in the estimation of the model is to calculate the fraction of non-searchers p_0. From equation (3.5), it follows that it equals the ratio of the fraction of non-participants in the population to the fraction of non-employed:

$$p_0=\frac{n}{1-e}. \tag{3.31}$$

The other fractions p_a are estimated by maximizing the likelihood that the observed wages are generated by our model. Note that a distribution for p_a, together with v, u and the urn-ball type of matching function that follows from (3.7) and (3.8), implies a job offer probability ψ. This job offer probability is the key parameter in the mapping from the number of applications p_a to the number of job offers q_j. Given estimates for the net productivity $\widehat{y}$ and the lower bound $\underline{w}$ of the support of the wage offer distribution $F_w\left(w\right)$, we can calculate the upper bound $\overline{w}$ of the support by using equation (3.27). Then, we can solve equation (3.26) to get the full wage offer distribution.

Note that in the data one typically does not observe all wage offers, but only the wage offers that have been accepted by the workers. Since workers can compare wage offers, high wage offers are more likely to be accepted than low wage offers. This implies that the distributions of the offered wages and of the accepted wages differ from each other. Let

$G_w(w)$ denote the distribution of the wages accepted by the non-employed workers. In order to derive an expression for $G_w(w)$, consider a worker who receives $j > 0$ job offers. She will only accept a wage that is lower than some value w if all her j job offers are lower than this value. As a result, the following relationship between $G_w(w)$ and $F_w(w)$ holds:

$$G_w(w) = \frac{\sum_{j=1}^{S} q_j F_w^j(w)}{1-q_0}. \tag{3.32}$$

It is straightforward to show that $G_w(w)$ first-order stochastically dominates $F_w(w)$. Taking the first derivative of this expression with respect to w gives the density of the accepted wages:

$$g_w(w) = \frac{\sum_{j=1}^{S} jq_j F_w^{j-1}(w) f_w(w)}{1-q_0}, \tag{3.33}$$

where $f_w(w)$ denotes the density function of the posted wages. An expression for this density can be derived by applying the implicit function theorem to equation (3.26). This yields

$$f_w(w) = \frac{\sum_{j=1}^{S} jq_j F_w^{j-1}(w)}{(y-k-w)\sum_{j=2}^{S} j(j-1) q_j F_w^{j-2}(w)}.$$

These equations show that $g_w(w)$ only depends on the productivity and the capital cost via the difference $y-k$. Hence, in the maximum likelihood estimation only the net productivity $\widehat{y}$ is identified. Ex post however, we can obtain estimates for y and k by using the equality $y = \widehat{y} + k$ and by rewriting the free entry condition (3.28) in the following way:

$$k = \frac{1}{r+\delta}\frac{1}{\theta} q_1 (\widehat{y} - \underline{w}).$$

As we explain in the main text, we estimate the lower bound $\underline{w}$ of the support of the wage distribution as a parameter in the maximum likelihood procedure. Together with the estimate for $\widehat{y}$, this implies a value for the upper bound $\overline{w}$. In order to explain observations outside the bounds of the support, we allow for measurement error. To be precise, we assume that the observed wage $\widetilde{w}$ depends on the true wage w and a random error term ε in a multiplicative way:

$$\widetilde{w} = w\varepsilon.$$

The error term ε has a log-normal distribution with parameters $\mu = 0$ and $\sigma^2 = \text{var}(\log(\varepsilon))$. We estimate σ as a parameter in the maximum likelihood procedure. The density of the observed wages can then be obtained by integrating over all possible values of the error term. If a wage $\widetilde{w}$ is observed, the error term must have been in the interval $[\widetilde{w}/\overline{w}; \widetilde{w}/\underline{w}]$. Hence, the density of the observed wages $g_{\widetilde{w}}(\widetilde{w})$ is equal to

$$g_{\widetilde{w}}(\widetilde{w}) = \int_{\widetilde{w}/\overline{w}}^{\widetilde{w}/\underline{w}} g_w(\widetilde{w}/\varepsilon) \frac{1}{\varepsilon} \eta(\varepsilon) \, d\varepsilon, \tag{3.34}$$

where $1/\varepsilon$ is the Jacobian of the transformation, $\eta(\varepsilon)$ denotes the log-normal density and $g_w(w)$ is given by (3.33). The integral in this equation must be calculated numerically, since it depends on $F_w(w)$, for which no explicit expression exists. Assuming independence of the N observations, the maximum likelihood problem is then given by

$$\max_{p_1,\ldots,p_S,\sigma,\underline{w},\widehat{y}} \frac{1}{N} \sum_{i=1}^{N} \log g_{\widetilde{w}}(\widetilde{w}_i),$$

subject to the conditions $\sum_{a=0}^{S} p_a = 1$, $p_a \in [0,1]$ $\forall a$ and $w_{\min} \leq \underline{w} \leq \widehat{y}$.

3.A.3 Search Cost Distribution

The maximum likelihood estimation provides us with estimates for $p_0, \ldots, p_S, \underline{w}$ and y. Using these estimates, we can derive cutoff points of the search cost distribution according to equation (3.21). This requires the calculation of the marginal gains from search Γ_a as given by equation (3.20). Note that this variable depends on the integral $\int_{w_R}^{\infty} w dF_w(w)^j$. To simplify the calculation of this integral, we apply a change of variables.

First, invert equation (3.26) and denote the inverse function of $F_w(w)$ by $w(z)$

$$w(z) = y - k - \frac{(y - k - \underline{w}) q_1}{\sum_{j=1}^{S} j q_j z^{j-1}}. \tag{3.35}$$

Then, we can write:

$$\int_{w_R}^{\infty} w dF_w^j(w) = \int_{\underline{w}}^{\overline{w}} w dF_w^j(w) = \int_0^1 j w(z) z^{j-1} dz,$$

Substituting this in equation (3.18) gives

$$\zeta_a = \sum_{j=1}^{a} \chi(j|a) \int_0^1 j \left(\widehat{y} - w_R - \frac{(\widehat{y} - \underline{w}) q_1}{\sum_{i=1}^{S} i q_i z^{i-1}} \right) z^{j-1} dz$$

The marginal gains from an additional application can then be calculated from (3.20). The equilibrium value for the separation rate δ that we need in this calculation follows from the steady state condition given in equation (3.3):

$$\delta = \frac{(1-q_0)(1-e)}{e}.$$

This procedure gives us S cutoff points $(\Gamma_1, ..., \Gamma_S)$ of the search cost distribution $F_c(c)$. For some purposes, e.g. for assessing the efficiency of the market equilibrium, we need an estimate of the full distribution (i.e. for every possible value of c). On the interval $[0, \Gamma_1]$ we obtain this estimate by using linear interpolation between the cutoff points:

$$F_c(c) = \sum_{j=i+1}^{S} p_j + \tfrac{p_i}{\Gamma_i - \Gamma_{i+1}} (c - \Gamma_{i+1}) \quad \forall c \in [\Gamma_{i+1}, \Gamma_i) \text{ and } i = \{1, ..., S\},$$

where we define $\Gamma_{S+1} = 0$.

For $c > \Gamma_1$ we assume that the search cost distribution keeps increasing linearly, with the same slope as just before Γ_1, until it reaches 1. Hence, on this interval $F_c(c)$ is given by

$$F_c(c) = \begin{cases} 1 - p_0 + \frac{p_1}{\Gamma_1 - \Gamma_2}(c - \Gamma_1) & \forall c \in [\Gamma_1, \Gamma_0) \\ 1 & \forall c \geq \Gamma_0 \end{cases}, \text{ where } \Gamma_0 = \Gamma_1 + \frac{p_0}{p_1}(\Gamma_1 - \Gamma_2).$$

In order to solve the planner's problem we also need estimates for the unemployment benefits b and the household production h. The value for b equals the product of the replacement rate ρ and the average wage:

$$b = \rho \int_{\underline{w}}^{\overline{w}} w dG_w(w) = \rho \left(w_R + \frac{1}{1-q_0} \sum_{a=1}^{S} p_a \zeta_a \right).$$

Use this equation and (3.19) and applying the same simplifications as above gives the following expression for home production.

$$h = w_R - \int_0^\infty \max_a \left(I_{a>0} b + \frac{1}{r+\delta} \zeta_a - ca \right) dF_c(c) \tag{3.36}$$

Next, partition the support of $F_c(c)$ into the intervals $[\Gamma_{S+1}, \Gamma_S)$, $[\Gamma_S, \Gamma_{S-1})$, ..., $[\Gamma_2, \Gamma_1)$, $[\Gamma_1, \Gamma_0]$, where Γ_{S+1} and Γ_0 are the lower bound and the upper bound of the support of $F_c(c)$. Due to the linear interpolation, $f(c)$ is constant on each of these intervals. Let f_a denote the value of $f(c)$ on the interval $[\Gamma_{a+1}, \Gamma_a)$. Then the following expression holds:

$$f_a = \frac{F(\Gamma_a) - F(\Gamma_{a+1})}{\Gamma_a - \Gamma_{a+1}} = \frac{p_a}{\Gamma_a - \Gamma_{a+1}}.$$

Substituting this in (3.36), we can write

$$\begin{aligned} h &= w_R - \sum_{a=1}^{S} \int_{\Gamma_{a+1}}^{\Gamma_a} \left(I_{a>0} b + \frac{1}{r+\delta} \zeta_a - ca \right) \frac{p_a}{\Gamma_a - \Gamma_{a+1}} dc \\ &= w_R - \frac{1}{r+\delta} \sum_{a=1}^{S} p_a \zeta_a + \frac{1}{2} \sum_{a=1}^{S} a p_a (\Gamma_a + \Gamma_{a+1}) - b(1-p_0) \\ &= w_R - \frac{1}{r+\delta} \sum_{a=1}^{S} p_a \left(\zeta_a - \frac{1}{2} a (\zeta_{a+1} - \zeta_{a-1}) \right) - b \left(1 - p_0 - \frac{1}{2} p_1 \right), \end{aligned}$$

where we define $\zeta_{S+1} = \zeta_S$ to simplify notation. This relates h to variables that we can estimate or directly observe.

3.B Labor Market Segments

In order to create the segments, we construct a worker skill index L_s and a job-complexity index L_c, as in Gautier and Teulings (2006). For the worker skills we assume the following linear relationship

$$\omega = X_s \beta_s + \varepsilon_s,$$

where β_s is a vector of coefficients and ε_s is an error term. The matrix X_s contains the explanatory variables: gender, years of education, years of working experience[41] (also squared and cubed), interaction terms, and year dummies. Next, we define the skill L_s of an individual as the predicted value following from this regression:

$$L_s = X_s \hat{\beta}_s,$$

where $\hat{\beta}_s = (X_s' X_s)^{-1} X_s' \omega$. Likewise, we construct a complexity measure for the jobs. We regress the logarithm of the wage paid by firm for this job on several job and firm characteristics:

$$\omega = X_c \beta_c + \varepsilon_c,$$

where X_c includes a constant, dummy variables for the sector, the type of contract for this job, the job level, occupation, and year dummies.[42] The complexity L_c of the job is defined as the predicted value of the regression:

$$\begin{aligned} L_c &= X_c \hat{\beta}_c \\ &= X_c \left(X_c' X_c\right)^{-1} X_c' \omega. \end{aligned}$$

3.C Flow Probabilities

As discussed in section 3.4, we assume that a year consists of three periods. This implies that a worker can flow from employment in year τ (time t) to non-employment in year $\tau+1$ (time $t+3$) in four different ways. She can loose her job at the beginning of either period $t+1$, $t+2$ or $t+3$, and remain non-employed after that. Alternatively, she can loose her job at $t+1$, get a new job at $t+2$ and loose it again at $t+3$. Hence, the yearly

[41] As common in literature, we define work experience as a function of age and the years of schooling. To be precise, we assume the following relation: experience = (age - years of education - 6) / 50, where rescaling is applied for reasons of computational convenience.

[42] Although we also observe the size of the firm, we do not include this variable in the job complexity regression to avoid endogeneity problems.

separation rate $\delta_{W,3}$ for the workers is given by

$$\begin{aligned}\delta_{W,3} &= \delta\,(1-m_W)^2 + (1-\delta)\,\delta\,(1-m_W) + (1-\delta)^2\,\delta + \delta m_W\,\delta \\ &= \delta\left(\delta^2 + m_W^2 + 2\delta m_W - 3\delta - 3m_W + 3\right).\end{aligned}$$

Expressions for the yearly separation rate of firms ($\delta_{F,3}$) and the annual matching probability for workers ($m_{W,3}$) and firms ($m_{F,3}$) can be derived in a similar way. The per-period matching probability conditional on search is given by

$$m_{W|U} = 1 - \frac{q_0 - p_0}{1-p_0} = \frac{1-q_0}{1-p_0}.$$

The annual matching probability conditional on search in the current period therefore equals

$$\begin{aligned}m_{W|U,3} \quad &= \quad m_{W|U}\,(1-\delta)^2 + \left(1-m_{W|U}\right) m_W\,(1-\delta) \\ &\quad + \left(1-m_{W|U}\right)(1-m_W)\,m_W + m_{W|U}\,\delta m_W.\end{aligned}$$

In order to be able to compare the estimated probabilities to the actual ones, we aggregate over the segments. For this we need to know the relative size s_i (i.e. the total mass of workers) for each of the segments. Normalize the size of the first segment to 1. Note that we defined the segments in such a way that the expected number of people flowing from non-employment to employment is the same in each one of them. Hence, the relative size s_i of segment $i > 1$ is defined by

$$m_{W,3,i}\,(1-e_i)\,s_i = m_{W,3,1}\,(1-e_1),$$

where $m_{W,3,i}$ denotes the annual matching probability and e_i the employment rate in segment i. Using this expression, the total number of matches formed in the market is then equal to $\sum_{i=1}^{5} m_{W,3,i}\,(1-e_i)\,s_i$ and the total number of non-employed workers equals $\sum_{i=1}^{5}(1-e_i)\,s_i$. So, the aggregate annual matching probability $\overline{m_{W,3}}$ can be calculated as follows:

$$\overline{m_{W,3}} = \frac{\sum_{i=1}^{5} m_{W,3,i} \left(1 - e_i\right) s_i}{\sum_{i=1}^{5} \left(1 - e_i\right) s_i}.$$

Aggregating $m_{W|U,3}$ can be done in a similar way. Likewise, the following expression holds for the aggregate annual separation probability $\overline{\delta_{W,3}}$:

$$\overline{\delta_{W,3}} = \frac{\sum_{i=1}^{5} \delta_{W,3,i} e_i s_i}{\sum_{i=1}^{5} e_i s_i},$$

where $\delta_{W,3,i}$ is the yearly separation rate in segment i.

The aggregate annual matching rate for the firms can be calculated according to

$$\overline{m_{F,3}} = \frac{\sum_{i=1}^{5} m_{F,3,i} v_i s_i}{\sum_{i=1}^{5} v_i s_i},$$

where v_i and $N_{F,i}$ respectively denote the fraction of vacancies and the measure of firms in segment i.

4 Simultaneous Search and On-the-Job Search

4.1 Introduction

Even within narrowly defined occupations in narrowly defined industries, there exists a lot of wage dispersion under workers with similar characteristics (see e.g. the references in chapter 1 of Mortensen, 2003, for empirical evidence). Two seminal contributions that explain this are Burdett and Judd (1983) and Burdett and Mortensen (1998).[1] In both papers, wage dispersion arises because firms realize that there is a chance that they compete with other firms for their candidate but they do not know exactly with how many firms and with which firms. In the Burdett-Judd model this is because the number of job offers that a worker receives is a random variable and in Burdett-Mortensen this uncertainty arises because some candidates are already employed and their current wage can be considered to be the offer that must be beaten. Both papers derive a continuous wage distribution but unfortunately, the density is increasing while in the data it is typically hump-shaped with a long right tail. In this essay, I show that by combining elements from both these models, a hump-shaped wage distribution can be derived.

Specifically, I construct a model of simultaneous search in which workers can search on-the-job. Like Pissarides (2000) and Mortensen (2000), I assume that each firm has

This chapter is based on Wolthoff (2008).

[1] Other examples include Albrecht and Axell (1984), Lang (1991), Acemoglu and Shimer (2000), and Postel-Vinay and Robin (2002b). In fact, Burdett and Judd (1983) develop a consumer search model to explain price dispersion. Their model can however be adapted to the labor market without changing its fundamental properties, see e.g. Gautier and Moraga-González (2005). I stick to labor market terminology to keep the discussion coherent.

exactly one position, that is either vacant or filled. An important new element in my model is that I increase the firms' strategy space in two ways: (i) I allow firms to choose a candidate from the pool of applicants based on their previous employment state and (ii) I allow firms to condition their wage offer on the previous wage of their candidate if she had one. The motivation for this setup is the following. In general it is very easy for firms to learn the current employment state of an applicant, for example from her curriculum vitae or from the references that she provided. It is in the firm's interest to use this information when making invitations for a job interview. *Ceteris paribus*, unemployed workers have worse outside options and are therefore cheaper than workers that have a job already. Ideally, firms also would like to know the wages of their employed applicants, but since workers and references do not report this it seems impossible to obtain this information at this stage of the recruitment process.[2]

However, as soon as a suitable candidate has been found, the first question in the final interview before making an offer usually is: "What is your current wage?". In fact, it is in the interest of firms to ask this question. In the Burdett-Mortensen model some contacts do not result in a match, because a firm offers too little to a worker who was earning a high wage already. But, since all wages in equilibrium are strictly lower than the productivity of the match, a positive surplus could have been shared, if only the firm had known how much the worker was earning in her current job. Clearly, workers earning high wages suffer the most from this lack of communication and are happy to announce their salary. I assume that all workers truthfully reveal how much they earn. After all, if the firm is not sure whether the worker tells the truth, it can ask for previous employment contracts.

In the Burdett-Mortensen model, where one worker and one firm meet at a time, revelation of the wages would destroy wage dispersion. Firms would offer a wage that is only marginally higher than the worker's current salary and the Diamond (1971) paradox with

[2]Analyzing a model in which firms can observe the wages of their applicants before they have to select one is very interesting from a theoretical point of view, but also technically difficult. In a model with continuous support of the wage distribution, this would create an infinite number of worker types, implying that firms need to develop a selection strategy for each of the infinitely many different pools of applicants that they can have. This could result in endogenous segmentation of the labor market, where firms are indifferent between all workers in a segment, but only offer the job to worker from an expensive segment if no cheaper applicants show up.

its degenerate wage distribution would prevail again. However, in a world of simultaneous search this is not the case. Firms take into account the potential competition of other firms for the same worker and continue to play a mixed strategy. I show that my model leads to a wage density that is similar to what typically is found in data. It has continuous support, a unique interior mode, and a long right tail, even if all agents are fully homogeneous. This is an important improvement compared to the Burdett-Mortensen model, which generates a strictly upward sloping earnings density. Most papers, like Bowlus et al. (1995), van den Berg and Ridder (1998), Bontemps et al. (1999, 2000), and Postel-Vinay and Robin (2002b), have used heterogeneity on the firm and/or worker side to obtain a better fit of the data. Although my model can easily be extended to include such heterogeneity, it does not require it to get the right shape of the wage distribution. The model is therefore suitable for a careful empirical assessment of the question which fraction of wage dispersion can be attributed to simultaneous search, on-the-job search, and productivity differences respectively.

Several other papers are related to what I do here. Carrillo-Tudela (2008) also describes a model with on-the-job search in which firms can observe the employment state of the workers. Firms can condition their wage offers on this state but not on the actual wage level. This results in a degenerate wage distribution for the unemployed and a upward sloping wage density, like in Burdett-Mortensen, for the employed. As argued above, allowing firms to condition on the current wage as well seems a realistic and important extension. A second difference concerns the matching technology. In Carrillo-Tudela (2008), workers get job offers from firms according to a Poisson process, i.e. one offer at a time. I use an urn-ball matching technology based on micro-foundations, in which workers can get multiple job offers.[3] The aggregate matching function is determined by the interplay between two coordination frictions: (i) workers do not know where other workers send their job applications and (ii) firms do not know which workers other firms make employment offers to.

[3] In current work, it has become standard to analyze a framework in which workers can send out simultaneous job applications, see Albrecht et al. (2003, 2004, 2006), Shimer (2004b), Gautier and Moraga-González (2005), Gautier and Wolthoff (2006), Chade and Smith (2006), Galenianos and Kircher (2008), Kircher (2007), and Gautier et al. (2007).

Delacroix and Shi (2006) study on-the-job search in a directed search setting. They find a wage distribution with a finite number of mass points as support. Together these mass points form a wage ladder and workers choose to only apply to firms that offer a wage that is one rung higher than their current wage level. My model generates a wage distribution that has a similar ladder structure. Not all wage levels can be reached directly from unemployment and it requires several job-to-job transitions to earn a really high wage. This is fundamental difference compared to models based on the Burdett-Mortensen framework, where even the highest wage in the economy can be obtained directly after an unemployment spell. Unlike the model by Delacroix and Shi (2006) however, my model allows for variation in the speed with which workers climb the ladder. Some workers experience larger wage increases between two jobs than others.

A few authors also obtain a hump-shaped wage density. For example, Mortensen (2000) creates endogenous productivity differences across jobs by introducing match specific investments by firms. Burdett et al. (2008) extend the Burdett-Mortensen framework by allowing workers to accumulate human capital while employed. They show that this results in a wage distribution that has a long Pareto tail. My model is simpler in this respect. The wage dispersion exclusively follows from a trade-off in the wage setting mechanism. Firms want to minimize wage payments, but realize that they can only hire the worker if their offer beats both the current employer of the worker and other recruiting firms.

This essay is organized as follows. Section 4.2 describes the setting of the model and section 4.3 solves for the equilibrium. In section 4.4 I characterize the earnings density and discuss its properties. Section 4.5 concludes.

4.2 Model

4.2.1 Setting

I consider a labor market with a continuum of identical firms and a continuum of identical workers. Time is discrete and I focus on the steady state. Workers can supply one indivisible unit of labor. Hence, each worker is either employed at one of the firms or unemployed. Likewise, each firm has one job, which can be in two states: filled by a worker and producing, or vacant. I normalize the measure of workers to 1 and denote the fractions of employed and unemployed workers by $1-u$ and u respectively. The measure of firms with vacancies is denoted by v and is endogenously determined by free entry.

A worker who is employed in a given period gets a payoff that is equal to her wage w. On the other hand, unemployed workers have a payoff equal to h, consisting of unemployment benefits, home production, and the value of leisure. A firm that gives employment to a worker produces an output y, but has to pay the worker's wage w. Hence, the firm's payoff equals $y-w$. Firms with a vacancy do not produce, but pay a vacancy cost $k>0$. Their payoff therefore equals $-k$.

Firms maximize the expected discounted future value of output minus capital and wage costs, while workers maximize the expected discounted value (hereafter, 'value') of future wage payments. All agents discount future payoffs at rate $1/(1+r)$ and are infinitely-lived and risk-neutral. As is common in the search literature, I introduce frictions in the market by considering equilibrium strategies that are symmetric and anonymous. This implies that all workers use the same application strategy, which they cannot condition on the firms' identities. Similarly, all firms use identical strategies, which cannot be based on the workers' identities.

Firms move first by deciding whether they want to enter the market. A firm that enters incurs the vacancy cost k as long as it is unmatched. Next, workers search for new jobs by applying to these vacancies. A worker applies to jobs at the beginning of a period, but only learns whether she is accepted or not at the end of the period. In such a setting, searching non-sequentially is optimal (see Morgan and Manning, 1985). Sending multiple applications can have two positive effects. First, it reduces the risk of not finding a new job. The second positive effect requires wage dispersion. If not all wage offers are the same, sending several applications increases the chance of getting a juicy offer. In case of wage dispersion, employed workers have an incentive to search as well, because they might be able to find a better paying job.

In each period, workers apply to all vacancies that they observe. The number of observed vacancies is however stochastic. It follows a Poisson distribution with mean $\alpha_U > 0$ for unemployed workers and $\alpha_E > 0$ for employed workers.[4] I allow for different 'vacancy observation rates' on and off the job to keep the model as general as possible. Analyzing the case with fully homogeneous workers is however straightforward by imposing $\alpha_U = \alpha_E$.

A firm with a vacancy observes the employment state of each applicant. Based on this, it selects one candidate. Hence, the firm can choose whether it wants to contact an unemployed or an employed worker. Let λ denote the probability with which the firm selects an unemployed worker. The choice is trivial in case the firm has only one type of applicants, but if both types show up, the firm may use a pure strategy, i.e. $\lambda \in \{0,1\}$, or mix between both types of workers, $\lambda \in (0,1)$. Note that the firm can observe the employment state, but cannot discriminate further, implying that within the group of workers with the same state, it selects a candidate randomly. Other applications are returned as rejections.

The firm contacts the candidate and during this contact it learns her current wage x. Conditional on this, it makes her a wage offer w, to which it commits. Let F_x be the distribution of wage offers to workers currently earning a wage x, with corresponding support $\mathscr{F}_x$. The distribution of wage offers to unemployed workers and its support are denoted by F_h and $\mathscr{F}_h$ respectively. The distributions F_x and F_h are equilibrium objects which are derived in section 4.3.1. They are common knowledge, but in case firms play a mixed strategy, workers do not know ex ante which wage a specific firm will offer. Firms commit to not counter outside offers received by the worker.

After learning the result of their applications, workers accept their best wage offer, as long as this gives a higher expected future payoff than remaining in the current job or state. All other wage offers are rejected. In line with literature, see e.g. Pissarides (2000), I assume that in each period a fraction $\delta \in (0,1)$ of the employed workers experiences an exogenous job destruction shock. The workers in question cannot search in that period and flow back into unemployment. Their former jobs become vacant.

[4] Endogenizing search intensity, like in Kaas (2007) or Gautier et al. (2007), would be a nice extension of the model, but complicates the analysis a lot without affecting the main conclusions of this essay. Therefore, I consider $\alpha_U > 0$ and $\alpha_E > 0$ to be exogenously given.

Although unemployed workers technically do not earn a salary, I will in the remainder of this chapter often refer to them as 'workers earning h' for reasons of convenience. I define a function $s(x) \in \{U, E\}$, which maps the 'wage' x into the employment state of the worker. Hence,

$$s(x) = \begin{cases} U & \text{if } x = h \\ E & \text{if } x > h. \end{cases}$$

I will often suppress the argument of s, to keep notation simple as simple as possible.

The strategies of the workers and the firms together imply a steady state earnings distribution $G(w)$, representing the fraction of employed workers earning a wage lower than w. I denote the corresponding density by $g(w)$ and its support by $\mathscr{G}$. In some cases I will refer to both employed and unemployed workers. For this purpose, define $\mathscr{G}' = \mathscr{G} \cup h$. I characterize the earnings distribution in section 4.4, after describing the matching technology, the workers' and firms' strategies, and the market equilibrium.

4.2.2 Matching Technology

The urn-ball matching framework that I use here, has been analyzed many times before in the literature. The first ones to use this micro-foundation of the matching process were Butters (1977) and Hall (1977). Albrecht et al. (2003) and Albrecht et al. (2004) extend the model by allowing for simultaneous search. In their specification the number of applications is a exogenous, discrete and finite number, which leads to relatively complicated binomial probabilities. Kaas (2007) circumvents this problem by introducing a continuous parameter, which is the mean of a Poisson process that determines the actual number of applications. I follow this approach here.

In every period, all unemployed and all employed not hit by the job destruction shock observe a fraction of the vacancies. Unemployed workers observe each of the $\widehat{v}$ vacancies with probability $\frac{\alpha_U}{\widehat{v}}$, while employed workers workers observe each of them with probability $\frac{\alpha_E}{\widehat{v}}$, where $\widehat{v} \to \infty$, keeping the ratio of $\widehat{v}$ over the number of workers fixed to v. In order to keep notation simple, I assume that $\alpha_U \geq \alpha_E$. The observations are independent across workers and vacancies, implying that the actual number of applications that a worker of type $s \in \{U, E\}$ observes, follows a Poisson process with mean α_s. Workers apply to all vacancies that they observe. All firms are equally likely to receive applications, which means that the expected queue length, i.e. the expected number of applications per vacancy, is equal to the total number of applications sent out divided by the number of vacancies. I distinguish between applications from unemployed and from employed workers. So, let ϕ_s denote the expected queue length formed by type $s \in \{U, E\}$ applicants, then the following expressions hold:

$$\phi_U = \frac{u\alpha_U}{v}$$

and

$$\phi_E = \frac{(1-\delta)(1-u)\alpha_E}{v}.$$

Due to the infinite size of the labor market, the actual number of applicants of type s at a specific vacancy follows a Poisson distribution with mean ϕ_s. Moreover, the number of competitors of type s that a worker faces at a given firm follows a Poisson distribution with mean ϕ_s as well.

The candidate selection proceeds in two steps. The firm first selects a type (U, E) of worker and after that it selects a candidate within that type. Consider an unemployed worker who has applied to a firm. In order to get a job offer, two things must happen. First, the firm must decide that it will offer the job to an unemployed applicant and second the worker has to be selected from the firm's pool of applicants without a job. The firm will select an unemployed worker with probability 1 if no employed workers show up (probability $e^{-\phi_E}$) and with probability λ otherwise. Conditional on this decision the worker has a probability $\frac{1}{n+1}$ to be selected, where n is her number of competitors, i.e. other applicants of the same type. Therefore, the probability to be selected equals

$$\sum_{n=0}^{\infty} \frac{1}{n+1} \frac{e^{-\phi_s}\phi_s^n}{n!} = \frac{1}{\phi_s}\left(1-e^{-\phi_s}\right).$$

Hence, the job offer probability for an application send by an unemployed worker equals

$$\psi_U = \left(e^{-\phi_E} + \lambda\left(1 - e^{-\phi_E}\right)\right)\frac{1}{\phi_U}\left(1 - e^{-\phi_U}\right).$$

In a similar way, I find that the probability that an application by an employed worker results in a match is given by

$$\psi_E = \left(e^{-\phi_U} + (1-\lambda)\left(1 - e^{-\phi_U}\right)\right)\frac{1}{\phi_E}\left(1 - e^{-\phi_E}\right).$$

Given the number of applications that a worker sends, the number of wage offers that she receives follows a binomial distribution with success probability ψ_s.[5] Initially, the worker does not know how many applications she will observe. The probability that she observes exactly n is given by $e^{-\alpha_s}\frac{\alpha_s^n}{n!}$. Hence, the ex ante probability to get j job offers is equal to

$$\begin{aligned}\chi_s(j) &= \sum_{n\geq j} e^{-\alpha_s}\frac{\alpha_s^n}{n!}\begin{pmatrix} n \\ j \end{pmatrix}\psi_s^j\left(1-\psi_s\right)^{n-j} \\ &= e^{-\alpha_s\psi_s}\frac{\left(\alpha_s\psi_s\right)^j}{j!}. \qquad (4.1)\end{aligned}$$

Hence, the number of job offers follows a Poisson distribution with mean $\alpha_s\psi_s$. One can interpret $\alpha_s\psi_s$ as the job offer arrival rate with which employed and/or unemployed workers get wage offers, like in Burdett and Mortensen (1998). However, workers might now get multiple job offers simultaneously, since time is discrete. The exact number of job offers is stochastic. Some workers are lucky and get multiple job offers, while others are unfortunate and remain unmatched. In this sense, the model is close to the noisy search model of Burdett and Judd (1983).

[5] See Albrecht et al. (2006).

4.2.3 Strategies and Payoffs

A worker takes the matching technology and the firms' strategies as given and decides whether she wants to accept or reject the (best) wage offer that she gets. She chooses the action that maximizes her expected discounted future payoff. This payoff depends on whether she is unemployed or employed. I construct a Bellman equation for each state. Let V_U denote the value of unemployment and $V_E(x)$ the value of being employed at wage $x \in \mathscr{G}$. The immediate payoff of an unemployed worker equals h. She observes vacancies at rate α_U, which can result in $j \in N$ job offers from the distribution F_h. She accepts the best wage offer, if she gets one, as long as the associated payoff $V_E(w)$ is higher than the payoff of remaining unemployed and rejects otherwise. Hence, the value of unemployment equals

$$V_U = h + \frac{1}{1+r}\left(\sum_{j=1}^{\infty} \chi_U(j) \int_{w\in\mathscr{F}_h} \max\{V_E(w), V_U\}\, dF_h^j(w) + \chi_U(0) V_U\right). \tag{4.2}$$

A similar expression holds for the value of employment. The immediate payoff is now equal to the worker's current wage x. If not hit by the job destruction shock, she sends $n \sim Poi(\alpha_E)$ applications which can result in $j \in \mathbb{N}$ job offers. Again, the worker accepts the (best) offer if it gives a higher payoff than rejecting it. Hence, $V_E(x)$ equals

$$V_E(x) = x + \frac{1}{1+r}\widetilde{V}_E(x), \tag{4.3}$$

where the continuation value $\widetilde{V}_E(x)$ equals

$$\begin{aligned}\widetilde{V}_E(x) &= (1-\delta)\sum_{j=1}^{\infty}\chi_E(j)\int_{w\in\mathscr{F}_x}\max\{V_E(w), V_E(x)\}\, dF_x^j(w) \\ &\quad + (1-\delta)\chi_E(0) V_E(x) + \delta V_U.\end{aligned} \tag{4.4}$$

Formally, the strategy of workers can be characterized by acceptance sets, consisting of the wage offers that they are willing to accept. A worker rejects all wage offers that are not part of her acceptance set. I simplify notation by conjecturing that workers follow a reservation wage strategy. A worker currently earning $x \in \mathscr{G}'$, accepts her best wage offer if it is higher than her reservation wage $w_R(x)$ and rejects it otherwise.[6] I show in section 4.3.2 that such a strategy is indeed optimal from the worker's point of view.[7]

Firms take the strategies of the workers and the other firms as given and choose (i) whether they want to enter the market, (ii) their selection probability λ and (iii) their wage offer distributions F_x, such that they maximize their expected discounted future payoff. Again, I construct two Bellman equations. Denote the firm's value of giving employment to a worker at wage w by $V_F(w)$ and the value of having a vacancy by V_V. A firm hiring a worker at wage w has an instant payoff of $\pi_w = y - w$. In the next period, the match terminates if a job destruction shock occurs (with probability δ) or if the worker moves to a better paying job (with endogenous probability $(1-\delta)\xi(w)$). The job opening at the firm becomes vacant again in that case. Otherwise, the firm continues to be matched. Hence, the value function $V_F(w)$ equals

$$V_F(w) = y - w + \frac{1}{1+r}\left((1-\delta)\xi(w)V_V + (1-\delta)(1-\xi(w))V_F(w) + \delta V_V\right). \quad (4.5)$$

A firm that has a vacancy does not produce, but has to pay the vacancy cost. So, its immediate payoff equals $-k$. Its continuation value depends on the candidate it selects and the wage offer it makes to this worker. Consider a firm with a vacancy that offers a wage w to an applicant of which it knows that she currently earns $x \in \mathscr{G}$'. Let $m_F(w|x)$ denote the firm's matching probability in this case. Then the firm's continuation value conditional on its wage offer equal

$$\widetilde{V}_V(w|x) = m_F(w|x)V_F(w) + (1 - m_F(w|x))V_V. \quad (4.6)$$

[6] Workers that get an offer equal to their reservation wage are indifferent between accepting and rejecting the offer. As a tie-breaking rule I assume that workers always reject in this event of measure zero.

[7] In writing down the Bellman equation for employed workers, I implicitly assume that their labor market status is observed by the firms to which they apply before they obtain the wage payment by their current employer. This guarantees that workers with a very low wage do not want to quit their current job in order to have better chances in their search for a new job.

The firm faces a trade-off. Offering a higher wage increases the matching probability $m_F(w|x)$, since workers can compare offers. However, simultaneously it lowers the value $V_F(w)$ of the future match. Firms will offer wages such that they maximize equation (4.6). This determines distribution the wage offer distribution F_x. I denote the maximum value that firm can obtain by $\widetilde{V}_V(x)$.

A firm with both employed and unemployed applicants can choose to which type it wants to offer the job. If the firm selects an unemployed worker, its continuation value equals $\widetilde{V}_V(U) = \widetilde{V}_V(h)$. In case the firm chooses an employed worker, it is unsure about the payoff it will get, since the current wage of the worker is still unknown at the moment of selection. The continuation value therefore equals the expected value of $\widetilde{V}_V(x)$.

$$\widetilde{V}_V(E) = \int \widetilde{V}_V(x)\, dG(x).$$

The firms with both types of applicants, selects the type that gives the highest profit. Hence, it has a continuation value $\widetilde{V}_V(U,E)$ which equals

$$\widetilde{V}_V(U,E) = \max\left\{\widetilde{V}_V(U), \widetilde{V}_V(E)\right\}.$$

Ex ante, a firm do not know whether it will get (i) no applications, (ii) only unemployed applicants, (iii) only employed applicants, or (iv) both unemployed and employed applicants. The continuation value before the applications are sent therefore equals

$$\begin{aligned}\widetilde{V}_V &= e^{-\phi_U} e^{-\phi_E} V_V + \left(1 - e^{-\phi_U}\right) e^{-\phi_E} \widetilde{V}_V(U) + e^{-\phi_U}\left(1 - e^{-\phi_E}\right)\widetilde{V}_V(E) \\ &\quad + \left(1 - e^{-\phi_U}\right)\left(1 - e^{-\phi_E}\right)\widetilde{V}_V(U,E).\end{aligned}$$

This expression allows me to write the value of a vacancy as follows.

$$V_V = -k + \frac{1}{1+r}\widetilde{V}_V.$$

I assume free entry of firms. Hence, firms will decide to enter the market as long as V_V is positive.

After describing the payoff and strategies of both workers and firms, I now define an equilibrium as follows.

Definition 4.1 *A market equilibrium is a tuple* $\left\{v, \{F_x\}_{x\in[h,y]}, \lambda, \{w_R(x)\}_{x\in[h,y]}\right\}$ *such that*

1. *Profit maximization:* $\widetilde{V}_V(w|x) = \widetilde{V}_V(x) \equiv \max_{w'} \widetilde{V}_V(w'|x)$ *for all* $w \in \mathscr{F}_x$*, and for all* $x \in \mathscr{G}'$.

2. *Optimal entry decision:* $\begin{cases} V_V = 0 & \text{if } v > 0 \\ V_V \leq 0 & \text{if } v = 0 \end{cases}$

3. *Optimal candidate selection:* $\begin{cases} \widetilde{V}_V(U) \leq \widetilde{V}_V(E) & \text{if } \lambda = 0 \\ \widetilde{V}_V(U) = \widetilde{V}_V(E) & \text{if } \lambda \in (0,1) \\ \widetilde{V}_V(U) \geq \widetilde{V}_V(E) & \text{if } \lambda = 1 \end{cases}$

4. *Optimal reservation wage:* $\begin{cases} \widetilde{V}_E(w) \geq \widetilde{V}_U & \text{for all } w \geq w_R(h) \\ \widetilde{V}_E(w) \leq \widetilde{V}_U & \text{for all } w < w_R(h) \\ \widetilde{V}_E(w) \geq \widetilde{V}_E(x) & \text{for all } w \geq w_R(x), \text{ and for all } x \in \mathscr{G} \\ \widetilde{V}_E(w) \leq \widetilde{V}_E(x) & \text{for all } w < w_R(x), \text{ and for all } x \in \mathscr{G} \end{cases}$

4.3 Market Equilibrium

4.3.1 Firms' Candidate Selection and Wage Setting

First, firms decide whether they will enter the market. The free entry condition implies that entry will take place as long as the value V_V of having a vacancy is positive. Hence, in any equilibrium with a positive measure of firms V_V equals 0. Consider a firm giving employment to a worker at wage w. The Bellman equation for its value function is given by (4.5). Solving this equation for $V_F(w)$ and substituting $V_V = 0$ yields

$$V_F(w) = \frac{(1+r)(y-w)}{r+\delta+(1-\delta)\xi(w)}. \tag{4.7}$$

On the other hand, a firm with a vacancy that offers a wage w to an applicant currently earning x, has a continuation value given that is given by equation (4.6). The firm's aim is to maximize this value with respect to w. In section 4.3.2 I show that all workers follow a reservation wage strategy. Workers never accept wage offers below their reservation wage $w_R(x)$, but are willing to accept higher offers. This implies that the matching probability $m_F(w|x)$ and consequently the firm's payoff equal zero for $w < w_R(x)$.

In the interval $(w_R(x), y]$, a firm faces a trade-off. A higher wage offer lowers the future per-period profit, but is more likely to be accepted by the candidate. To be precise, the worker will accept the wage offer if it is higher than all j other wage offers. If a firm offers the job to a worker, the conditional probability that she has sent n applications equals $\frac{np_s(n)}{\sum_{n=1}^{\infty} np_s(n)} = \frac{np_s(n)}{\alpha_s}$,where $p_s(n) = e^{-\alpha_s}\frac{\alpha_s^n}{n!}$. The probability that the $n-1$ other applications result in exactly j job offers is given by $\begin{pmatrix} n-1 \\ j \end{pmatrix} \psi_s^j (1-\psi_s)^{n-1-j}$. For $w > w_R(x)$, $m_F(w|x)$ therefore equals

$$\begin{aligned} m_F(w|x) &= \sum_{n=1}^{\infty} \frac{ne^{-\alpha_s}\frac{\alpha_s^n}{n!}}{\alpha_s} \sum_{j=0}^{n-1} \begin{pmatrix} n-1 \\ j \end{pmatrix} \psi_s^j (1-\psi_s)^{n-1-j} F_x^j(w) \\ &= \sum_{n=1}^{\infty} e^{-\alpha_s} \frac{\alpha_s^{n-1}}{(n-1)!} (1-\psi+\psi F_x(w))^{n-1} \\ &= e^{-\alpha_s \psi_s (1-F_x(w))}. \end{aligned} \tag{4.8}$$

Hence, the number of other, better offers follows a Poisson distribution with mean equal to $\alpha_s \psi_s (1-F_x(w))$. Next, I show that the number of $F_x(w)$ is continuous with connected support.

Lemma 4.1 *Given $v > 0$ and $w_R(h) < y$, in any market equilibrium, $F_x(w)$ is continuous with connected support.*

Proof. The proof is identical to the one given in lemma 1 of Gautier and Moraga-González (2005), which extends lemma 1 of Burdett and Judd (1983). The intuition is as follows. The matching technology implies that some workers compare wages because they get at least two job offers, while others do not have this possibility since they only receive one job offer. This feature implies that if all firms offer the same wage $w < y$, a deviant can do better by offering a marginally higher wage, which allows it to attract workers that compare multiple job offers. At the same time, posting $w = y$, giving a match payoff of zero, is dominated by posting $w_R(x)$ since there is a strictly positive probability that the candidate does not compare wages. Hence, firms post wages according to a mixed strategy. ■

This lemma allows me to derive the following result.

Proposition 4.1 *Given $v > 0$ and $w_R(h) < y$, in any market equilibrium, firms post prices according to*

$$F_x(w) = \begin{cases} 0 & \text{if } w \leq w_R(x) \\ \frac{1}{\alpha_s \psi_s} \log\left(\frac{y - w_R(x)}{y - w}\right) & \text{if } w \in \mathscr{F}_x = (w_R(x), \overline{w}(x)],\ \text{for all } x \in [h, y] \\ 1 & \text{if } w > \overline{w}(x), \end{cases} \tag{4.9}$$

where $\overline{w}(x)$ equals

$$\overline{w}(x) = y - \exp(-\alpha_s \psi_s)(y - w_R(x)). \tag{4.10}$$

Proof. First, note that the infimum of the support of $F_x(w)$ must equal $w_R(x)$. Offers below $w_R(x)$ are rejected and give a payoff of zero. On the other hand, if the infimum of the support would be strictly larger than $w_R(x)$, the firm posting the lowest wage in the market could decrease its offer and make a higher profit. Second, let $\overline{w}(x)$ denote the upper bound of the support of $F_x(w)$, hence $\mathscr{F}_x = (w_R(x), \overline{w}(x)]$.[8] The equilibrium definition now implies that the payoff for the firm must be the same for each $w \in \mathscr{F}_x$.

[8] Since we assume that workers reject wage offers that are equal to their reservation wage, $w_R(x)$ is not in the support. Changing this assumption does not affect any conclusions, but complicates notation since then both unemployed and employed workers could have a per-period income of h.

Hence, an expression for the wage offer distribution $F_x(w)$ for $w \in \mathscr{F}_x$ follows from the condition

$$\widetilde{V}_V(w_R(x)|x) = \widetilde{V}_V(w|x).$$

Substituting equations (4.6), (4.7), and (4.8), and simplifying the result gives

$$\frac{e^{-\alpha_s\psi_s}(y - w_R(x))}{r + \delta + (1-\delta)\xi(w_R(x))} = \frac{e^{-\alpha_s\psi_s(1-F_x(w))}(y-w)}{r+\delta+(1-\delta)\xi(w)}.$$

Solving for $F_x(w)$ yields

$$F_x(w) = \frac{1}{\alpha_s\psi_s}\log\left(\frac{r+\delta+(1-\delta)\xi(w)}{r+\delta+(1-\delta)\xi(w_R(x))}\cdot\frac{y-w_R(x)}{y-w}\right).$$

Firms always offer wages that are acceptable to the worker, i.e. above $w_R(x)$. This implies that $\xi(x) = 1 - e^{-\alpha_E\psi_E}$ and therefore the expression for the equilibrium wage offer distribution reduces to

$$F_x(w) = \frac{1}{\alpha_s\psi_s}\log\left(\frac{y-w_R(x)}{y-w}\right).$$

The upper bound $\overline{w}(x)$ of the support of $F_x(w)$ follows from solving $F_x(\overline{w}(x)) = 1$, which gives

$$\overline{w}(x) = y - e^{-\alpha_s\psi_s}(y - w_R(x)).$$

■

Next, I turn to the selection probability λ. A firm that receives applications from both unemployed and employed workers can choose which type it wants to select. This choice is not trivial. Unemployed workers are cheaper because they have worse outside options. But as a result, competition for these workers is higher, which reduces the probability that the job offer will result in a match. Let $\widetilde{V}_V(U)$ denote the firm's continuation value after offering the job to an unemployed worker. The equal profit condition implies

$$\widetilde{V}_V(U) = \frac{(1+r)e^{-\alpha_U\psi_U}(y - w_R(h))}{r+\delta+(1-\delta)(1-e^{-\alpha_E\psi_E})}.$$

Firms that select an employed worker do initially not know her current wage. Therefore, the continuation value $\widetilde{V}_V(E)$ follows from taking the expectation of $\widetilde{V}_V(x)$.

$$\begin{aligned}\widetilde{V}_V(E) &= \int \widetilde{V}_V(x)\, dG(x).\\ &= \frac{(1+r)\, e^{-\alpha_E \psi_E}\left(y - \int w_R(x)\, dG(x)\right)}{r+\delta+(1-\delta)\left(1-e^{-\alpha_E \psi_E}\right)}.\end{aligned}$$

Firms having both types of applicants compare $\widetilde{V}_V(U)$ and $\widetilde{V}_V(E)$ to decide which candidate to select. Both $\widetilde{V}_V(U)$ and $\widetilde{V}_V(E)$ depend on λ through the job offer probabilities ψ_U and ψ_E.

Proposition 4.2 *Given $v > 0$, an equilibrium value for λ exists.*

Proof. The existence of an equilibrium value for λ is guaranteed by Kakutani fixed point theorem. Consider a firm that has to decide on λ. If all other firms choose $\lambda' \in [0,1]$, the firm either (i) strictly prefers unemployed workers ($\widetilde{V}_V(U) > \widetilde{V}_V(E) \Rightarrow \lambda = 1$), (ii) strictly prefers employed workers ($\widetilde{V}_V(U) < \widetilde{V}_V(E) \Rightarrow \lambda = 0$), or (iii) is indifferent between both types ($\widetilde{V}_V(U) = \widetilde{V}_V(E) \Rightarrow \lambda \in [0,1]$). This best-response function has a fixed point, which determines the equilibrium value of λ. ■

4.3.2 Workers' Reservation Wage

The strategy of workers consists of the decision whether or not to accept the best wage offer w given their current wage or home production x and the firms' strategies. Conjecture that the workers' value function of employment $V_E(x)$ is strictly increasing in x. This implies that workers follow a reservation wage strategy. The reservation wage $w_R(x)$ is defined by the reservation wage property

$$\begin{cases} V_U = V_E(w_R(h)) \\ V_E(x) = V_E(w_R(x)) \quad \text{for all } x \in \mathcal{G}. \end{cases} \tag{4.11}$$

Hence, $w_R(x) = x$ for all $x \in \mathcal{G}$, meaning that employed workers accept all wage offers higher than their current wage. This is not necessarily the case for unemployed workers. They take into account that if they accept a job, their job offer arrival rate will decrease. In order to derive the reservation wage $w_R(h)$ of unemployed workers, I first derive explicit expressions for the workers' value functions

Lemma 4.2 *Given a reservation wage* $w_R(x)$ for all $x \in \mathcal{G}'$ and $v > 0$, *in any market equilibrium, the workers' value functions are given by*

$$V_U = h + \frac{\left(\frac{\delta}{r} + \Upsilon_U\right) w_R(h) + \left(\frac{1-\delta}{r}(1-\Upsilon_E) + 1 - \Upsilon_U\right) y}{1 + r - (1-\delta)\Upsilon_E} \tag{4.12}$$

$$V_E(x) = (1+r)\frac{x + \frac{\delta}{r} w_R(h) + \frac{1-\delta}{r}(1-\Upsilon_E) y}{1 + r - (1-\delta)\Upsilon_E} \tag{4.13}$$

where

$$\Upsilon_s = e^{-\alpha_s \psi_s}(1 + \alpha_s \psi_s).$$

Proof. See appendix 4.A.1. ■

Note that $V_E(x)$ is indeed strictly increasing in x, which confirms that a reservation wage strategy is optimal for the workers. Evaluating equation (4.13) in $w_R(h)$ and equating it to equation (4.12) gives the solution for the workers' reservation wage. The results on the worker's reservation wages are summarized in the following proposition.

Proposition 4.3 *Given* $v > 0$, *in any market equilibrium, the workers' reservation wage* $w_R(x)$ is given by

$$w_R(x) = \begin{cases} \frac{(1+r-(1-\delta)\Upsilon_E)h + (\delta(1-\Upsilon_E) + \Upsilon_E - \Upsilon_U)y}{1+r+\delta-\Upsilon_U} & \text{for } x = h \\ x & \text{for all } x \in \mathcal{G}. \end{cases} \tag{4.14}$$

Proof. See appendix 4.A.2. ■

It is straightforward to check that $w_R(h) > h$ since $\alpha_U \psi_U > \alpha_E \psi_E$. In other words, workers are choosy because they realize that their job offer arrival rate will fall after accepting a job. This result is in line with Burdett and Mortensen (1998).

4.3.3 Equilibrium Characterization

By combining the equilibrium elements derived above, we can now prove the following result.

Proposition 4.4 *A market equilibrium with a positive measure of firms, i.e.* $v > 0$*, exists for* $k < \frac{1}{r+\delta}(y-h)$. For $k \geq \frac{1}{r+\delta}(y-h)$, the market collapses.

Proof. Proposition 4.1, 4.2, and 4.3 describe the market equilibrium in case a positive measure of firms enters the market. Hence, in order to complete the derivation the entry decision of firms has to be considered. First, consider a situation in which the number of firms tends to zero, i.e. $v \to 0^+$. This implies that $\phi_s \to \infty$, $\psi_s \to 0$, and $\Upsilon_s \to 1$. As a result, the reservation wage of an unemployed worker goes to h, i.e. $w_R(h) \to h$. In fact, the wage offer distribution becomes degenerate at h, since $\overline{w}(h) \to h$. This implies that $\widetilde{V}_V \to \frac{1+r}{r+\delta}(y-h)$, and thus that $V_V \to -k + \frac{1}{r+\delta}(y-h) > 0$. Hence, as long as $k < \frac{1}{r+\delta}(y-h)$ some firms will enter the market. If $k \geq \frac{1}{r+\delta}(y-h)$ firms do not want to enter and the market collapses. Next, consider a situation in which the number of firms is much larger than the number of workers, i.e. $\phi_s \to 0$. Then $V_V \to -\frac{1+r}{r}k < 0$. Since V_V is continuous in v, there exists a value for v such that $V_V = 0$. Hence, a market equilibrium exists.[9] ■

In the market equilibrium, wages range from the reservation wage $w_R(h)$ up to the net productivity y. This contrasts the models by Burdett and Judd (1983) and Burdett and Mortensen (1998), where the upper bound of the wage density is strictly smaller than the productivity level. A second difference compared to both models, is that not all wage levels can be reached directly from unemployment. In the first job after unemployment,

[9]Note that although proving the existence of an equilibrium is straightforward, establishing whether it is unique or not is very complicated. The main reason for this is the fact that $\widetilde{V}_V(E)$ depends on the earnings distribution $G(w)$, for which no closed-form expression exists. See section 4.4 for more details.

a worker can never earn more than $\overline{w}(h)$, which is strictly lower than $y-k$. A worker who earns that upper bound and receives a job offer in the next period, can never earn more than $\overline{w}(\overline{w}(h))$ in that new job. By mathematical induction, one can show that the maximum wage $\hat{w}_{U,n}$ a worker can earn in her n^{th} job after unemployment is equal to

$$\hat{w}_{U,n} = \overline{w}^{n}(h) = y - e^{-\alpha_U \psi_U - \alpha_E \psi_E (n-1)} (y - w_R(h)) \text{ for } n \in \mathbb{N} \backslash \{0\}.$$

Hence, the wage distribution can be seen as a wage ladder. Workers have to climb one rung (i.e. find a job in $(\hat{w}_{U,n-1}, \hat{w}_{U,n}]$) before they can climb the next. In this respect, the model is similar to Delacroix and Shi (2006), who describe such a ladder in a directed search framework. However, an important difference exists between their model and my model. They find a wage distribution with a finite number of mass points as support. Workers choose to only apply to firms that offer a wage that is one rung higher than their current wage level. Hence, the speed with which workers climb the ladder is fixed. My model allows for variation in this speed. Some workers can experience larger wage increases between two jobs than others, which is in line with what one typically observes in reality.

The equilibrium unemployment level u can be calculated by equating inflow and outflow. In each period a fraction δ of the employed workers loses its job. Hence, inflow equals $\delta(1-u)$. Workers leave unemployment if they get at least one job offer, implying that outflow is equal to $u(1-e^{-\alpha_U \psi_U})$. Solving for u yields

$$u = \frac{\delta}{1 - e^{-\alpha_U \psi_U} + \delta}. \tag{4.15}$$

4.4 Earnings Distribution

4.4.1 Analytical Expression

In this section I describe the earnings density $g(w)$, i.e. the steady state cross-sectional wage density in the market equilibrium. In order to derive the earnings density, I consider the set $\mathscr{L}(w)$ of employed workers earning a wage lower than w. By definition the probability that a worker is included in this set is given by $G(w)$. I exploit the fact that in steady state inflow into and outflow from $\mathscr{X}(w)$ should be equal. Note that workers never move to jobs paying a lower wage, implying that inflow only occurs from unemployment. Further, note that unemployed workers flow into $\mathscr{X}(w)$ if the best wage offer they get is lower than w. Hence, inflow into the set of workers earning less than w, denoted by $I(w)$, equals

$$I(w) = u \sum_{j=1}^{\infty} \chi_U(j) F_h^j(w). \tag{4.16}$$

On the other hand, outflow from $\mathscr{X}(w)$ can occur for two reasons. All employed workers are subject to a job destruction shock at rate δ, in which case they flow back to unemployment. The ones that are not hit by the shock have the opportunity to leave the group by getting a job offer paying more than w. So, outflow $O(w)$ equals

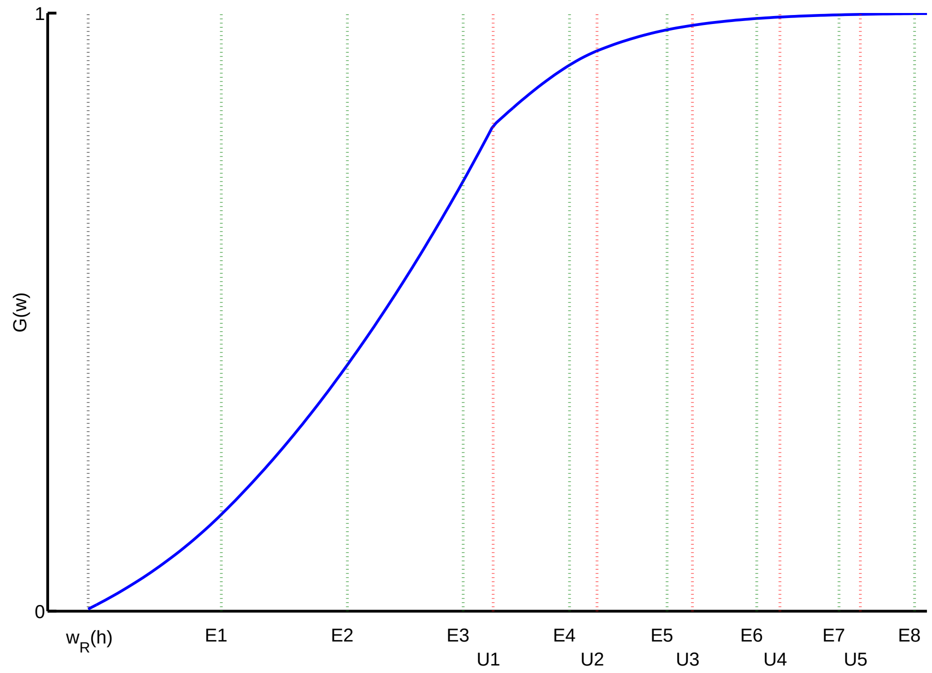
$$O(w) = (1-u) \int_{w_R(h)}^{w} \left(\delta + (1-\delta) \sum_{j=1}^{\infty} \chi_E(j) \left(1 - F_x^j(w)\right) \right) g(x)\, dx. \tag{4.17}$$

Equating inflow and outflow and rewriting the result gives the following integral equation:

$$\frac{u}{1-u} \sum_{j=1}^{\infty} \chi_U(j) F_h^j(w) = \delta G(w) + (1-\delta) \int_{w_R(h)}^{w} \sum_{j=1}^{\infty} \chi_E(j) \left(1 - F_x^j(w)\right) g(x)\, dx. \tag{4.18}$$

Solving this equation gives the expression for the earnings distribution $G(w)$ and taking the first derivative of this expression yields the density $g(w)$. Note that the support of each $F_x(w)$ does not correspond to $\mathscr{G}$. Therefore, the functional form of $G(w)$ varies across subintervals on its support.

In order to derive these subintervals, I define two sets of cutoff points. The first set consists of the maximum wages $\hat{w}_{U,n}$, $n \in \mathbb{N} \setminus \{0\}$ a worker can earn in her n^{th} job after unemployment , as derived in section 4.3.3. The second set follows from considering a worker that is employed at the lowest wage in the economy, i.e. $w = w_R^+ \equiv \lim_{\varepsilon \downarrow 0} w_R(h) + \varepsilon$. She can never earn more than $\overline{w}\left(w_R^+\right)$ in her next job and never more than $\overline{w}\left(\overline{w}\left(w_R^+\right)\right)$ in the job after that. Let $\hat{w}_{E,n}$ denote the maximum wage this worker can earn in her n^{th} job. Hence,



Note that $\hat{w}_{E,n} < \hat{w}_{U,1}$ for $n \leq \hat{n}$ (here $\hat{n} = 3$). After $\hat{w}_{E,\hat{n}}$, the cutoff points formed by $\hat{w}_{U,i}$ and $\hat{w}_{E,\hat{n}+i}$ alternate.

Figure 4.1: Illustration of cutoff points.

$$\hat{w}_{E,n} \equiv \overline{w}^n \left(w_R^+\right) = y - e^{-\alpha_E \psi_E n} \left(y - w_R^+\right) \text{ for } n \in \mathbb{N} \backslash \{0\}.$$

Next, combine both sets of cutoff points and let $\hat{w}_n$ be the n^{th} order statistic of the new set, i.e. the n^{th} smallest value. Note that $\hat{w}_{U,1} > \hat{w}_{E,1}$, implying that $\hat{w}_1 = \hat{w}_{E,1}$. Let $\hat{w}_{E,\hat{n}}$ be the largest $\hat{w}_{E,n}$ smaller than $\hat{w}_{U,1}$. Then it is straightforward to show that the cutoff points $\hat{w}_{U,n}$ and $\hat{w}_{E,n}$ alternate from $\hat{w}_{U,1}$ onwards. Hence

$$\begin{aligned} &\{\hat{w}_0, \hat{w}_1, \hat{w}_2, \ldots, \hat{w}_{\hat{n}}, \hat{w}_{\hat{n}+1}, \hat{w}_{\hat{n}+2}, \hat{w}_{\hat{n}+3}, \ldots\} \\ &= \left\{w_R(h), \hat{w}_{E,1}, \hat{w}_{E,2}, \ldots, \hat{w}_{E,\hat{n}}, \hat{w}_{U,1}, \hat{w}_{E,\hat{n}+1}, \hat{w}_{U,2}, \ldots\right\}. \end{aligned}$$

Figure 4.1 illustrates this by showing the cutoff points for arbitrary parameter choices.

In appendix 4.A.3, I show that the functional form of $G(w)$ and $g(w)$ is different on each interval $(\hat{w}_{n-1}, \hat{w}_n]$. Since $\lim_{n\to\infty} \hat{w}_n = y - k$, there exist infinitely many of such intervals. Hence, I partition the earnings distribution and density as follows

$$\{G(w), g(w)\} = \begin{cases} \{G_1(w), g_1(w)\} & w \in (w_R(h), \hat{w}_1] \\ \{G_n(w), g_n(w)\} & w \in (\hat{w}_{n-1}, \hat{w}_n], \, n \in \mathbb{N} \setminus \{0,1\}. \end{cases}$$

First, I obtain a closed-form expression for $G_1(w)$. Then, I show that the elements of the wage distribution satisfy a recursive structure: knowledge of $G_{n-1}(w)$ or $G_{n-2}(w)$ is sufficient to derive $G_n(w)$. Taking derivatives yields expressions for $g_1(w)$, $g_2(w)$, $g_3(w)$, et cetera. Hence, the entire earnings density can be characterized by the initial element $G_1(w)$ and a recursive equation. This is summarized in proposition 4.5.

Proposition 4.5 *In market equilibrium, the earnings distribution is characterized by the following recursive system*

$$\begin{cases} G_1(w) &=& \delta \Psi_U \left(\left(\frac{\pi_{w_R(h)}}{\pi_w} \right)^{\Delta_E} - 1 \right) \\ G_n(w) &=& C_n - \delta \Psi_U + (1-\delta) \left(G_{n-1}(\underline{w}(w)) - \pi_w^{-\Delta_E} \int \pi_w^{\Delta_E} dG_{n-1}(\underline{w}(w)) \right) \\ & & \textit{if } n \in \{2, \ldots, \hat{n}+1\} \\ G_n(w) &=& C_n + \delta + (1-\delta) \left(G_{n-2}(\underline{w}(w)) - \pi_w^{-\Delta_E} \int \pi_w^{\Delta_E} dG_{n-2}(\underline{w}(w)) \right) \\ & & \textit{if } n \in \{\hat{n}+2, \ldots\}, \end{cases}$$

where $\Delta_E = \frac{1}{1-(1-\delta)e^{-\alpha_E \psi_E}}$, $\Psi_U = \frac{e^{-\alpha_U \psi_U}}{1-e^{-\alpha_U \psi_U}}$, $\underline{w}(w) = y - e^{\alpha_E \psi_E}(y-w)$, *and* C_n *is determined by* $G_n(\hat{w}_{n-1}) = G_{n-1}(\hat{w}_{n-1})$.

Proof. See appendix 4.A.3. ■

The complexity of $G_n(w)$ increases rapidly in n, which impedes derivation of analytical expressions. However, it is straightforward to see that together they create a non-monotonic wage density. The intuition is as follows. In order to be employed at a low wage, a worker must have gotten a low wage offer after an unemployment spell and have remained there since that moment. On the other hand, in order to earn a really high salary, the worker must have experienced many consecutive job-to-job transitions without a job destruction shock in between. The probability of both events is relatively small and therefore the equilibrium fractions of workers earning these wages are small. Intermediate wage levels are much more common, also because there are several ways in which one can obtain such a salary. Some workers get this wage directly after unemployment, while others experience a couple of job-to-job transitions before finding a job paying this wage.

4.4.2 Simulation

Since analytical expressions for $g_n(w)$ become very complicated, I use simulation to present some features of the equilibrium. Numerical calculation of $g(w)$ is straightforward, since the model has the Markov property: the wage of a worker in the next period solely depends on her current wage and not directly on her past wages. Hence, a numerical approximation of $g(w)$ can be obtained by choosing a sufficiently dense grid of wages[10] and by calculating the normalized left eigenvector of the transition matrix associated with eigenvalue 1. In order to do this, I normalize the output y to 1 and set household production h equal to 0.4. Furthermore I fix the number of vacancies per unemployed (i.e. the labor market tightness) to 1. I assume that the length of a period equals 1 month and set the interest rate equal to 5% per year, which corresponds to $r = 0.004$. In line with the data used by Shimer (2007), I set the job destruction rate δ equal to 0.0173. Further, I assume that $\alpha_U = \alpha_E = 2$.

The equilibrium values of the parameters are given in table 4.1. Firms are indifferent between unemployed and employed workers and offer the job with probability 0.304 to an unemployed if they have both types of applicants. The reservation wage of an unemployed worker is equal to 0.728. About 7% of the population is unemployed, while the entry costs for firms equal 2.786. The job offer arrival rate for unemployed is almost 5 times as higher as for employed, reflecting the fact that unemployed would be more attractive in case competition for both types is the same. Figure 4.2 shows the earnings distribution for three different values of the job destruction rate, i.e. $\delta \in \{0.0133, 0.0173, 0.0213\}$. The discontinuity in each of the graphs reflects the ladder structure of the distribution: it

[10] I use a grid with 500 points of support between the reservation wage w_R and the productivity y.

Parameter	Value
λ	0.304
$w_R(h)$	0.728
k	2.786
u	0.070
$\alpha_U \psi_U$	0.263
$\alpha_E \psi_E$	0.056

Table 4.1: Equilibrium parameter values

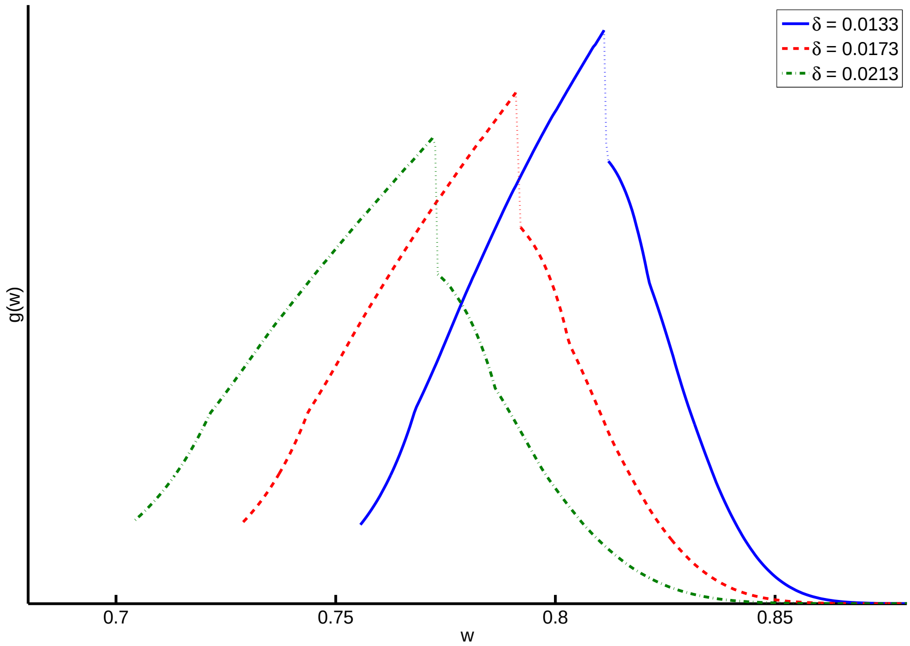


Figure 4.2: Earnings distribution

occurs at the maximum wage that workers can get in their first job after unemployment.

The model presented here is also a useful tool for policy analysis. There is a large literature on the effect of active labor market programs or policy reforms on the job finding rate of unemployed (see Heckman et al., 1999). However, many of those studies do not take equilibrium effects into account. In reality, such equilibrium effects are clearly important. Any policy that affects outcomes for unemployed changes the size and/or composition of the pool of employed workers and is therefore likely to have an effect on job-to-job transitions as well. The microfoundation for the matching technology in my model makes it possible to study how a change in the exogenous parameters influences the job offer

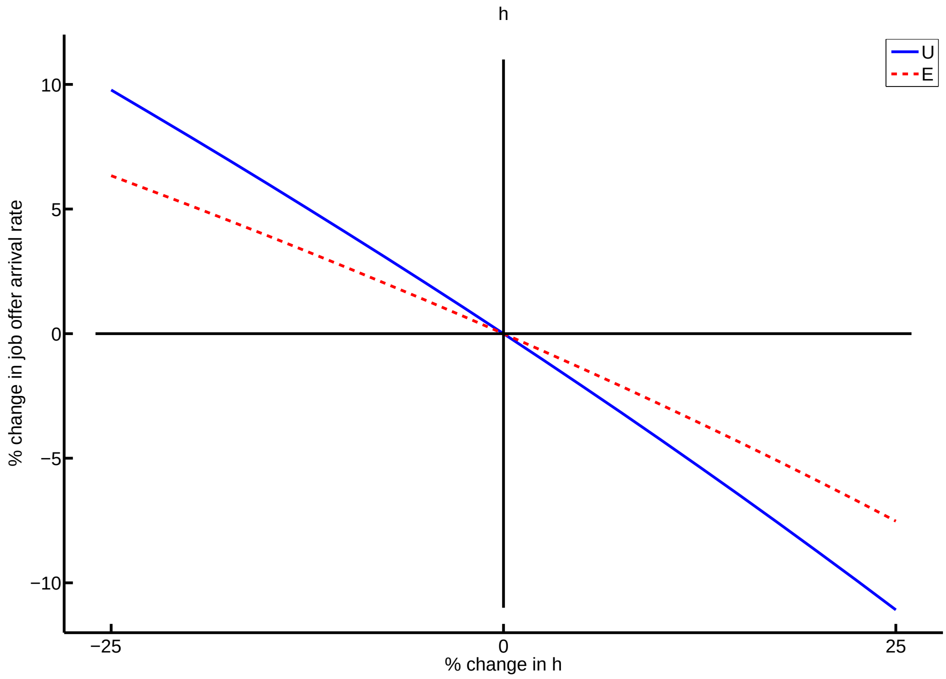


Figure 4.3: Effect of a change in the home production h on the job offer arrival rates

arrival rates of both unemployed and employed. To illustrate this, I simulate changes in the home production h and the job destruction rate δ. Figures 4.3 and 4.4 show the results.

Not surprisingly, there exists a negative relationship between h and the job offer arrival rates. An increase in h raises the reservation wage $w_R(h)$ of the unemployed, which makes it more expensive to hire these workers. As a result the wages of employed become higher as well. Both effects reduce the profits of firms, implying some firms will leave the market and the job offer arrival rates will drop. I find that the result is stronger for workers without a job than for employed workers, which can be explained by the fact that firms will update their value of λ. The higher reservation wage of the unemployed is a direct effect of the increase in h, while the higher wages of the employed are an indirect and smaller effect. Hence, firms will choose more often for an on-the-job searcher now. This substitution aggravates the effect on the unemployed workers.

For the job destruction rate I find opposite effects. The job offer arrival rates are increasing in δ and the elasticity is larger for employed workers. When jobs are destroyed more frequently, the reservation wage of the unemployed goes down. As a result the wages

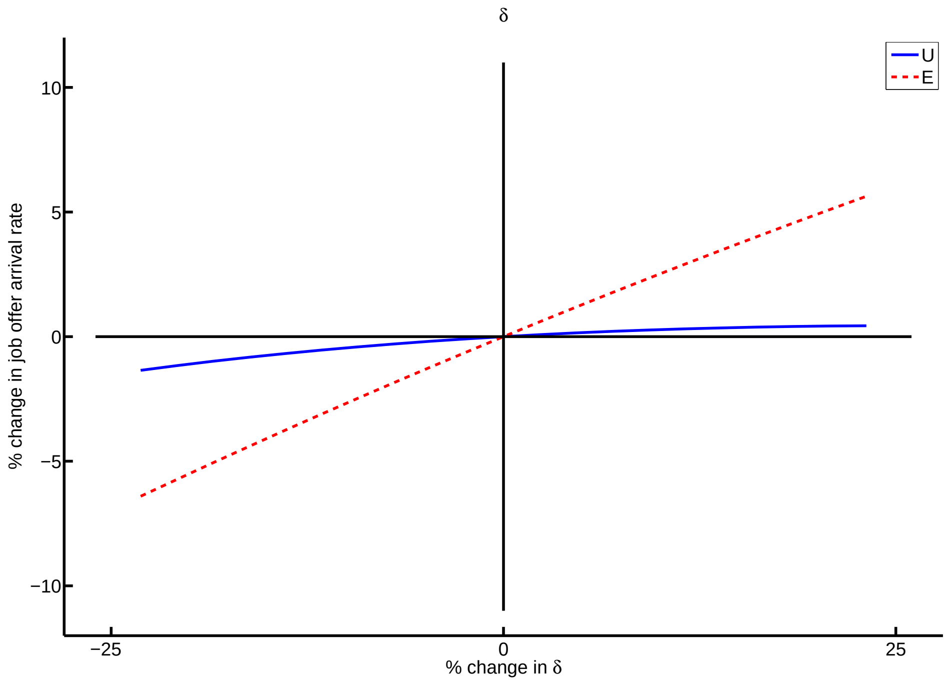


Figure 4.4: Effect of a change in the job destruction rate δ on the job offer arrival rates

earned by employed workers decrease as well, which stimulates entry. Hence, both types of workers are more likely to receive a job offer. However, the increase in δ also increases unemployment, implying that individuals without a job now have more competitors than before, if a firm decides to offer the job to an unemployed worker. This reduces the effect on the job offer arrival rate of unemployed.

4.5 Conclusions

I have presented an equilibrium search model of the labor market in which workers can send multiple applications simultaneously and can search for a better job while employed. Unlike most of the literature, I allow firms to condition their wage offers on the current wage of their job candidates. Wage dispersion is maintained because firms take into account that they might have to compete with other firms for the same worker. I obtain an earnings density that has continuous support, a unique interior mode, and a long right tail, even in a market with identical firms and workers. This feature of the model is relevant for empirical work. The model can easily be extended to include heterogeneity on the firm and/or worker side and can therefore be used to answer an important empirical question: what are the relative contributions of search and coordination frictions, on-the-job search, and productivity differences to wage dispersion?

The model can also be used to improve the evaluation of active labor market programs. I argue that policies affecting the job finding rates of unemployed can be expected to have an effect on job-to-job transitions as well. The framework presented here takes such equilibrium effects into account. For example, the model predicts that lower unemployment benefits do not only increase the job finding rates of the unemployed but also of the employed. Ignoring such effects could potentially lead to wrong conclusions about the desirability of certain policy reforms.

4.A Proofs

4.A.1 Proof of Lemma 4.2

The Bellman equations for V_U and $V_E(x)$ are given by equations (4.2) and (4.3). Both satisfy Blackwell's (1965) sufficient conditions for a contraction mapping, implying that a unique solution for V_U and $V_E(x)$ exists. Conjecture that the solution for $V_E(x)$ is a linear function of x, i.e. $V_E(x) = \beta_0 + \beta_1 (x - w_R(h))$ for some unknown constants β_0 and β_1. Substitute this into the Bellman equation given by (4.3) and (4.4), and use reservation wage property (4.11) to get

$$\begin{aligned} V_E(x) \quad = \quad & x + \frac{\beta_0}{1+r} + \frac{(1-\delta)\beta_1}{1+r} \times \\ & \left(\sum_{j=1}^{\infty} \chi_E(j|\alpha_E) \int_{w \in \mathscr{F}_x} w dF_x^j(w) + \chi_E(0|\alpha_E) x - w_R(h) \right) \end{aligned} \tag{4.19}$$

By substituting equations (4.1) and (4.9), I obtain

$$\begin{aligned}\sum_{j=1}^{\infty}\chi_E\left(j|\alpha_E\right)\int_x^{\overline{w}(x)} wdF_x^j\left(w\right) &= \int_x^{\overline{w}(x)} wd\sum_{j=1}^{\infty} e^{-\alpha_E\psi_E}\frac{\left(\alpha_E\psi_E F_x\left(w\right)\right)^j}{j!}\\ &= \int_x^{\overline{w}(x)} wd\left(e^{-\alpha_E\psi_E\left(1-F_x(w)\right)}-e^{-\alpha_E\psi_E}\right)\\ &= e^{-\alpha_E\psi_E}\int_x^{\overline{w}(x)} wd\frac{\pi_x}{\pi_w}.\end{aligned}$$

Partial integration gives

$$\begin{aligned}\int_x^{\overline{w}(x)} wd\frac{\pi_x}{\pi_w} &= \left[w\frac{\pi_x}{\pi_w}\right]_x^{\overline{w}(x)}-\int_x^{\overline{w}(x)}\frac{\pi_x}{\pi_w}dw\\ &= \overline{w}\left(x\right)\frac{\pi_x}{\pi_{\overline{w}(x)}}-x+\pi_x\left[\log\left(\pi_w\right)\right]_x^{\overline{w}(x)}\\ &= \left(e^{\alpha_E\psi_E}-1\right)y-\alpha_E\psi_E\left(y-x\right)\end{aligned}$$

Hence

$$\sum_{j=1}^{\infty}\chi_E\left(j|\alpha_E\right)\int_x^{\overline{w}(x)} wdF_x^j\left(w\right)=\left(1-e^{-\alpha_E\psi_E}\right)y-\alpha_E\psi_E e^{-\alpha_E\psi_E}\left(y-x\right).$$

This result can be used to rewrite (4.19) as follows

$$\begin{aligned}V_E\left(x\right) &= x+\frac{\beta_0}{1+r}+\frac{\left(1-\delta\right)\beta_1}{1+r}\left(y-w_R\left(h\right)-\Upsilon_E\left(y-x\right)\right). \qquad (4.20)\\ &= \left[w_R\left(h\right)+\frac{\beta_0}{1+r}+\frac{\left(1-\delta\right)\beta_1}{1+r}\left(1-\Upsilon_E\right)\left(y-w_R\left(h\right)\right)\right]\\ &\quad+\left[1+\frac{\left(1-\delta\right)\beta_1}{1+r}\Upsilon_E\right]\left(x-w_R\left(h\right)\right),\end{aligned}$$

where

$$\Upsilon_s=e^{-\alpha_s\psi_s}\left(1+\alpha_s\psi_s\right).$$

This confirms the linear structure of $V_E\left(x\right)$. The coefficients β_0 and β_1 can be found by solving the system

$$\begin{aligned}\beta_0 &= w_R(h) + \frac{\beta_0}{1+r} + \frac{(1-\delta)\beta_1}{1+r}(1-\Upsilon_E)(y - w_R(h)) \\ \beta_1 &= 1 + \frac{(1-\delta)\beta_1}{1+r}\Upsilon_E.\end{aligned}$$

This yields

$$\beta_0 = \frac{1+r}{r} \cdot \frac{(r+\delta)w_R(h) + (1-\delta)(1-\Upsilon_E)y}{1+r-(1-\delta)\Upsilon_E} \tag{4.21}$$

and

$$\beta_1 = \frac{1+r}{1+r-(1-\delta)\Upsilon_E}. \tag{4.22}$$

In the same way in which I derived equation (4.20), one can obtain the following expression for V_U

$$V_U = h + \frac{\beta_0}{1+r} + \frac{\beta_1}{1+r}(1-\Upsilon_U)(y - w_R(h)).$$

Substituting equations (4.21) and (4.22) completes the proof.

4.A.2 Proof of Proposition 4.3

Use equations (4.12), (4.13), and the reservation wage property $V_U = V_E(w_R(h))$ to obtain

$$h + \frac{\left(\frac{\delta}{r} + \Upsilon_U\right) w_R(h) + \left(\frac{1-\delta}{r}(1-\Upsilon_E) + 1 - \Upsilon_U\right) y}{1+r-(1-\delta)\Upsilon_E} = (1+r)\frac{\frac{r+\delta}{r} w_R(h) + \frac{1-\delta}{r}(1-\Upsilon_E)y}{1+r-(1-\delta)\Upsilon_E}$$

Solving this expression for $w_R(h)$ yields

$$w_R(h) = \frac{(1+r-(1-\delta)\Upsilon_E)h + (\delta(1-\Upsilon_E) + \Upsilon_E - \Upsilon_U)y}{1+r+\delta-\Upsilon_U}.$$

4.A.3 Proof of Proposition 4.5

Recall that I define the following cutoff points:

$$
\begin{aligned}
&\{\hat{w}_0, \hat{w}_1, \hat{w}_2, \ldots, \hat{w}_{\hat{n}}, \hat{w}_{\hat{n}+1}, \hat{w}_{\hat{n}+2}, \hat{w}_{\hat{n}+3}, \ldots\} \\
&= \left\{w_R(h), \hat{w}_{E,1}, \hat{w}_{E,2}, \ldots, \hat{w}_{E,\hat{n}}, \hat{w}_{U,1}, \hat{w}_{E,\hat{n}+1}, \hat{w}_{U,2}, \ldots\right\}.
\end{aligned}
$$

The functional form of $G(w)$ and $g(w)$ is different on each interval $(\hat{w}_{n-1}, \hat{w}_n]$ and therefore I partition the earnings distribution and density as follows

$$
\{G(w), g(w)\} = \begin{cases} \{G_1(w), g_1(w)\} & w \in (h, \hat{w}_1] \\ \{G_n(w), g_n(w)\} & w \in (\hat{w}_{n-1}, \hat{w}_n], \, n \in \mathbb{N} \backslash \{0,1\}. \end{cases}
$$

I derive the earnings distribution in a recursive way. First, I obtain a closed-form expression for $G_1(w)$. After that, I derive a recursive relationship between $G_n(w)$ and $G_{n-1}(w)$ and/or $G_{n-2}(w)$. Throughout the proof I simplify notation by using $\Delta_E = \frac{1}{1-(1-\delta)e^{-\alpha_E \psi_E}}$ and $\Psi_s = \frac{e^{-\alpha_s \psi_s}}{1-e^{-\alpha_s \psi_s}}$.

4.A.3.1 Derivation of $g_1(w)$

I derive an expression for $g_1(w)$ by considering $\mathscr{L}(w)$, the set of workers earning less than w, for values of w in the interval $(w_R(h), \hat{w}_1]$. First, recall that

$$
\sum_{j=1}^{\infty} \chi_s(j|\alpha_s) F_x^j(w) \quad = \quad e^{-\alpha_s \psi_s}\left(e^{\alpha_s \psi_s F_x(w)} - 1\right).
$$

Substituting equation (4.9) gives

$$
\sum_{j=1}^{a_s} \chi_s(j|a_s) F_x^j(w) = \begin{cases} 0 & w \leq w_R(x) \\ e^{-\alpha_s \psi_s}\left(\frac{\pi_x}{\pi_w} - 1\right) & w \in (w_R(x), \overline{w}(x)] \\ 1 - e^{-\alpha_s \psi_s} & w > \overline{w}(x). \end{cases} \tag{4.23}
$$

If I substitute this into the steady state equation (4.18) and multiply both the left and the right hand side with π_w, I get

$$\frac{u}{1-u}e^{-\alpha_U\psi_U}\left(\pi_h-\pi_w\right)=\pi_w G_1\left(w\right)-\left(1-\delta\right)e^{-\alpha_E\psi_E}\int_{w_R(h)}^{w}\pi_x g_1\left(x\right)dx.$$

I turn this integral equation into a differential equation by taking the first derivative with respect to w. Simplifying the resulting expression by substituting $\frac{u}{1-u}=\frac{\delta}{1-e^{-\alpha_U\psi_U}}$ yields

$$g_1\left(w\right)-\frac{\Delta_E}{\pi_w}G_1\left(w\right)=\frac{\delta\Psi_U\Delta_E}{\pi_w}.$$

This is a first-order non-homogeneous linear differential equation. Hence, the solution for $G_1\left(w\right)$ is given by

$$G_1\left(w\right) = \exp\left(\Delta_E\int\frac{1}{\pi_w}dw\right)\left(\int\exp\left(-\Delta_E\int\frac{1}{\pi_w}dw\right)\frac{\delta\Psi_U\Delta_E}{\pi_w}dw+C_1\right),$$

where C_1 is a constant. Solving the integrals and simplifying the resulting expression gives

$$G_1\left(w\right) = C_1\pi_w^{-\Delta_E}-\delta\Psi_U,$$

The value of the constant C_1 follows from the condition $G_1\left(w_R\left(h\right)\right)=0$. This implies

$$C_1=\delta\Psi_U\pi_{w_R(h)}^{\Delta_E}.$$

Hence,

$$G_1\left(w\right)=\delta\Psi_U\left(\left(\frac{\pi_{w_R(h)}}{\pi_w}\right)^{\Delta_E}-1\right). \tag{4.24}$$

4.A.3.2 Derivation of $g_2\left(w\right),\ldots,g_{\hat{n}+1}\left(w\right)$

Next, consider $\mathscr{X}\left(w\right)$ for $w\in\left(\hat{w}_{n-1},\hat{w}_n\right]$, $n\in\left\{2,3,\ldots\hat{n}+1\right\}$. In these intervals the inflow still depends on w, as in the derivation of $g_1\left(w\right)$. However not all workers can leave $\mathscr{X}\left(w\right)$ anymore by a job-to-job transition. The workers earning low wages will stay in $\mathscr{X}\left(w\right)$

even if they experience the largest possible wage increase when moving to a new job. Let $\underline{w}(w)$ denote the minimum wage that a worker must earn in order to be able to earn $w > \hat{w}_1$ in his next job. By inverting equation (4.10), I obtain

$$\underline{w}(w) = y - k - e^{\alpha_E \psi_E}(y - k - w). \tag{4.25}$$

Note that $\underline{w}(w)$ is strictly increasing in w and that $\underline{w}(\hat{w}_{n-1}) = \hat{w}_{n-2}$, which confirms that workers earning less than $\hat{w}_{n-2}$ cannot leave $\mathscr{X}(w)$ via a job-to-job transition. By using equation (4.25), I can let the integral in the right hand side of (4.18) start at $\underline{w}(w)$. Substituting the relevant cases of equation (4.23) then gives

$$\frac{u}{1-u}e^{-\alpha_U \psi_U}\left(\frac{\pi_{w_R(h)}}{\pi_w} - 1\right) = G(w) - (1-\delta)\left(G(\underline{w}(w)) + e^{-\alpha_E \psi_E}\int_{\underline{w}(w)}^{w}\frac{\pi_x}{\pi_w}g(x)\,dx\right).$$

Note that $G(w) = G_n(w)$, $G(\underline{w}(w)) = G_{n-1}(\underline{w}(w))$, and that $\int_{\underline{w}(w)}^{w}\frac{\pi_x}{\pi_w}g(x)\,dx$ can be rewritten as

$$\int_{\underline{w}(w)}^{w}\frac{\pi_x}{\pi_w}g(x)\,dx = \int_{\underline{w}(w)}^{\hat{w}_{n-1}}\frac{\pi_x}{\pi_w}g_{n-1}(x)\,dx + \int_{\hat{w}_{n-1}}^{w}\frac{\pi_x}{\pi_w}g_n(x)\,dx.$$

Again, I create a differential equation by multiplying both the left hand side and the right hand side with π_w and taking the first derivative with respect to w. This results in the following expression:

$$g_n(w) - \frac{\Delta_E}{\pi_w}G_n(w) = \frac{\Delta_E}{\pi_w}\left(\delta\Psi_U - (1-\delta)\,G_{n-1}(\underline{w}(w))\right).$$

Like above, this is a first-order non-homogeneous linear differential equation. The solution is equal to

$$G_n(w) = C_n - \delta\Psi_U - (1-\delta)\,\Delta_E\pi_w^{-\Delta_E}\int \pi_w^{\Delta_E - 1}G_{n-1}(\underline{w}(w))\,dw.$$

where the constant C_n follows from the condition $G_n(\hat{w}_{n-1}) = G_{n-1}(\hat{w}_{n-1})$. Finally, integration by parts gives

$$G_n(w) = C_n - \delta\Psi_U + (1-\delta)\left(G_{n-1}(\underline{w}(w)) - \pi_w^{-\Delta_E}\int \pi_w^{\Delta_E} dG_{n-1}(\underline{w}(w))\right).$$

4.A.3.3 Derivation of $g_{\hat{n}+2},\ldots$

Next, consider $\mathscr{X}(w)$ for $w \in (\hat{w}_{n-1}, \hat{w}_n]$, $n \in \{\hat{n}+2, \hat{n}+3, \ldots\}$. In these intervals the inflow no longer depends on w, since unemployed workers finding a job are always hired at a wage below $\hat{w}_n$. Hence, inflow equals

$$I(w) = u\left(1 - e^{-\alpha_U \psi_U}\right).$$

Outflow still depends on w, but again the integral in the right hand side of (4.18) starts at $\underline{w}(w)$. Hence, the earnings distribution is implied by

$$\frac{u}{1-u}\left(1-e^{-\alpha_U\psi_U}\right) = G(w) - (1-\delta)G(\underline{w}(w)) - (1-\delta)e^{-\alpha_E\psi_E}\int_{\underline{w}(w)}^{w}\frac{\pi_x}{\pi_w}g(x)\,dx.$$

Note that now both $\hat{w}_{U,n}$ and $\hat{w}_{E,n}$ contribute to the cutoff points, implying that there are twice as many intervals as for $w < \hat{w}_{U,1}$. More specifically, there are two cutoff points between any w and $\underline{w}(w)$. Hence, $G(w) = G_n(w)$, $G(\underline{w}(w)) = G_{n-2}(\underline{w}(w))$, and $\int_{\underline{w}(w)}^{w}\frac{\pi_x}{\pi_w}g(x)\,dx$ can be rewritten as

$$\int_{\underline{w}(w)}^{w}\frac{\pi_x}{\pi_w}g(x)\,dx = \int_{\underline{w}(w)}^{\hat{w}_{n-2}}\frac{\pi_x}{\pi_w}g_{n-2}(x)\,dx + \int_{\hat{w}_{n-2}}^{\hat{w}_{n-1}}\frac{\pi_x}{\pi_w}g_{n-1}(x)\,dx + \int_{\hat{w}_{n-1}}^{w}\frac{\pi_x}{\pi_w}g_n(x)\,dx.$$

Creating a differential equation by multiplying both the left hand side and the right hand side with π_w, and taking the first derivative with respect to w, yields:

$$g_n(w) - \frac{\Delta_E}{\pi_w}G_n(w) = -\frac{\Delta_E}{\pi_w}\left(\delta + (1-\delta)G_{n-2}(\underline{w}(w))\right).$$

Again, this is a first-order non-homogeneous linear differential equation. The solution is equal to

$$G_n(w) = C_n + \delta - (1-\delta)\Delta_E \pi_w^{-\Delta_E} \int \pi_w^{\Delta_E - 1} G_{n-2}(\underline{w}(w))\, dw,$$

where the constant C_n follows from the condition $G_n(\hat{w}_{n-1}) = G_{n-1}(\hat{w}_{n-1})$. Finally, integration by parts gives

$$G_n(w) = C_n + \delta + (1-\delta)\left(G_{n-2}(\underline{w}(w)) - \pi_w^{-\Delta_E} \int \pi_w^{\Delta_E} dG_{n-2}(\underline{w}(w)) \right). \tag{4.26}$$

5 Conclusion

5.1 Summary

This thesis focuses on the analysis of equilibrium simultaneous search models of the labor market. As described in chapter 1, allowing workers to observe several employment opportunities at the same moment, has proven to provide a successful explanation for the fact that seemingly identical workers can earn very different wages. The essays in this thesis add to the existing literature by analyzing various extensions to the standard setup.

In the standard model, identical workers search for identical jobs by simultaneously applying to several vacancies. Because the labor market is large and agents cannot coordinate their actions, frictions arise. Multiple workers might apply to the same vacancy and/or multiple firms might offer the job to the same worker. It is well known that if some, but not all, workers observe several employment opportunities simultaneously, then firms face a trade-off between the wage offer and the hiring probability. Firms that offer low wages will only hire workers that do not have better offers, but make a high profit per hired worker. On the other hand, firms offering high wages attract workers more easily, but make a lower profit per hired worker. In equilibrium these effects offset, and all firms make the same profit.

The extensions considered in this thesis improve the understanding of this type of models by relaxing the assumptions of the standard setup in three different ways. The first essay addresses heterogeneity of firms. Contrary to what is often assumed, not all firms are equally productive in reality. Some firms have newer technologies which allows them to produce a higher output per unit of labor than other firms. This provides workers with a trade-off. If applying is costly, they have to choose whether they want to apply to the highly productive firms, which can pay higher wages, but where also more competition from other workers can be expected. Alternatively, they can apply to less productive firms, where the opposite holds. The market equilibrium for this setup is derived and compared to a social planner's solution.

The second essay takes the sector in which workers search as given, but focuses on the question whether workers send the right number of applications. It endogenizes the workers' choice of search intensity by presenting an equilibrium search model in which sending multiple applications is more costly but can also result in a higher wage, since the worker accepts the best job offer if he gets more than one. The workers' job search intensities, the firms' entry and the wage distribution are all simultaneously determined in market equilibrium. The model is estimated using wage and aggregate labor force data, after which efficiency is considered, again by comparing the market equilibrium to a social planner's solution.

The last essay in this thesis studies the effect of on-the-job search in an equilibrium model of simultaneous search. An important novelty compared to existing on-the-job search models is that workers can communicate their current wage to the firm offering them a new job. Firms can choose whether they want to offer the job to an unemployed or an employed worker and they make wage offers conditional on the applicant's current wage. Unemployed workers are cheaper since they have lower outside options, but the competition from other firms for them is higher than for an employed worker. This provides firms with a trade-off.

The next section describes in greater detail in which way the three chapters in this thesis contribute to the literature. The contributions of the first and the third essay are mainly of a theoretical nature, while the second paper also adds to the methodological and empirical literature. Further, the section describes the policy implications of the findings in this thesis.

5.2 Contributions to Literature

First of all, this thesis contributes to the discussion whether the market equilibrium in the labor market is efficient from a social point of view. Both chapter 2 and 3 find that this is not the case. In the first essay workers fail to diversify their applications over the high and the low productivity sector, while in the second essay a fraction of the workers does not send the socially desired number of applications.

These inefficiency results are interesting from a theoretical viewpoint. For example, the first essay shows that, for many parameter values, the inefficiency continues to exist even when workers can observe the wage offers by the firms before they send their applications. Hence, the inefficiency of the market equilibrium is not driven by the random search assumption, but has a more fundamental cause. Workers try to maximize the productivity-weighted probability to get more than one job offer, while the social planner wants to spread applications over the sectors in order to reduce the coordination frictions. This implies that in the discussion on (in)efficiency of directed search models it is important to carefully consider heterogeneity among workers and/or firms.

A second contribution of this thesis concerns the shape of the wage distribution. Seminal papers explaining wage dispersion, like Burdett and Judd (1983) and Burdett and Mortensen (1998), generate a wage distribution that has a strictly upward sloping density. This is at odds with the data, which typically shows a density with a unique interior mode and a long and fat right tail. The standard way to obtain a better fit to the data is by allowing for worker and/or firm heterogeneity. Chapter 3 and 4 show that such ex ante heterogeneity is not required. In chapter 3 all workers are ex ante identical. Only ex post they differ in their search costs. As a result some workers send few applications, while others send many. This leads to the desired shape of the wage distribution.

Chapter 4 imposes an even stricter form of homogeneity. The only source of variation across workers is the randomness that is inherent to the matching process. Some workers are lucky and get high wage offers, while others are unfortunate and earn less. Since prospective employers observe the current wages of their applicants and base their wage offers on that information, the effect of a good or bad wage offer persists until the next unemployment spell. The wage distribution has a ladder property: high wage levels cannot be reached directly from unemployment, but require several job-to-job transitions. As a result, again a density with a unique interior mode is obtained.

Having models that can explain the shape of the wage distribution without allowing for heterogeneity in productivity is relevant for empirical work like considering the relative contributions of search and coordination frictions, on-the-job search, productivity differences, and measurement error to wage dispersion. Past literature has typically found that productivity differences are a large component of the variation in wages. This result is not surprising given that a good fit to the wage data was not possible without such differences. Using the models presented in this thesis might however lead to different conclusions.

All three essays add to the literature by extending the standard urn-ball matching function with multiple applications as presented by Albrecht et al. (2004). In the first essay workers are given an additional choice. They have to decide to which sectors they want to send their applications. It turns out that in most cases workers are indifferent between sending two applications to the high productivity sector and two applications to the low productivity sector. The second essay generalizes the standard matching function by endogenizing search intensity and by allowing for variation in the number of applications sent by the workers. Further, it provides an estimate of the resulting matching technology. In the third essay, the number of applications sent by the workers is determined by a draw from an exogenously given distribution. The novelty in this essay is the extra choice that firms have. They observe the employment state of all their applicants and decide whether they want to hire an unemployed or an employed worker.

Finally, the second essay describes how, using wage data, one can estimate the search cost distribution of workers, the implied matching probabilities, the productivity of a match, and the flow value of non-labor market time. Further, it shows how these estimates can be used to derive the socially optimal firm entry rates and distribution of job search intensities. As such the model can be used to analyze the effects of active labor market programs that try to increase the search intensity of certain groups of workers. The model is estimated for the Dutch labor market using data from wage records of firms. The level of inefficiency is quantified at approximately 12% of total output.

5.3 Policy Implications

The results found in this thesis have important policy implications. First of all, the inefficiency result of the first two essays implies that government intervention in the labor market could be beneficial. The first essay shows that the socially optimal outcome is not achieved because workers fail to diversify their applications across the high and the low productivity sector. Hence, governments could increase welfare if they can change the fraction of workers mixing between both sectors. This is however not straightforward, since most of the instruments that governments have, like taxation, will make one of both sectors more attractive. This will then only increase the fraction of workers sending both applications to this sector without changing the fraction of workers diversifying their applications. In any case, governments should try to ensure that excessive numbers of applications are not sent to any particular sector, as such congestion creates coordination frictions that reduce social welfare.

A similar conclusion is drawn from chapter 3. Using structural estimation it is shown that several inefficiencies coexist in the labor market. Too many workers do not search for a job, but at the same time a small fraction of the workers sends out too many applications. Moreover, too many vacancies are created for a given level of workers' search intensity. This essay argues that a marginal increase of the minimum wage and/or unemployment benefits conditional on actively searching for a job could reduce all three sources of inefficiency and therefore improve welfare. Both instruments increase the expected payoff of the first application and therefore increase participation. At the same time, they discourage workers from sending many applications by compressing the wage distribution a bit. They also reduce firms' profits by increasing the equilibrium wages, which lowers entry of vacancies.

Finally, all three essays are equilibrium models of the labor market. Hence, they are suitable for analyzing the equilibrium effects of active labor market programs or policy reforms. In the evaluation of these programs it is quite common to focus on the effect on the treated, but the essays in this thesis show that the spillovers on other agents in the market are potentially very large. For example, any program that changes the search decision of a fraction of the workers, has an effect on the wage distribution, the number of firms entering the market, the search decision of other workers, et cetera. Therefore, it is crucial to take the equilibrium effects into account for a proper evaluation of public policy.

5.4 Directions for Further Research

The essays in this thesis relax the assumptions of the standard simultaneous search model in a couple of ways. However, many other research projects could be pursued. For example, in models of simultaneous search the role of firms, besides making a wage offer, is often quite limited. Chapter 4 tries to improve on this by letting firms decide whether they want to hire an unemployed or an employed job candidate, but in reality the choice firms face is more complex. For example, workers differ in their productivity and this creates a second choice that has to be made by the firms. Although a firm would prefer to hire the most productive worker, it realizes that the competition from other firms for this worker is very large. If interview resources and/or time are scarce, this might induce the firm to choose another worker, who is slightly less productive, but easier to hire. At the same time however, workers with a very low productivity are only attractive candidates if no better workers show up, implying that in equilibrium endogenous segmentation of the labor market could arise. Incorporating productivity differences among workers in this way would also greatly improve the estimation of simultaneous search models, since it makes it feasible to consider the labor market as one large market with heterogeneous agents rather than a collection of submarkets, each with homogeneous agents.

Another interesting direction of extension would be to relax the assumption that each firm has one vacancy. In reality many firms have multiple vacancies simultaneously and assuming that an independent hiring process for each vacancy exists seems questionable. If workers observe the wages and the number of vacancies at each firm, firms with more vacancies can offer lower wages since they provide the worker with a larger probability to be hired. It is well known that if all firms have one vacancy and all workers send two applications, a wage distribution with two points of support arises. Each worker sends her first application to a firm offering the low wage and the second to a firm offering the high wage. An interesting question would therefore be how heterogeneity in the number of vacancies changes this result: do all workers send both their applications to the same type of firms, like in the first essay in this thesis, or do they diversify over different firm types?

Samenvatting (Summary in Dutch)

De geschiedenis van economisch modelleren voert terug tot Cournot (1838), die pionierswerk verrichtte met het gebruik van calculus voor het analyseren van economische markten, zoals een monopolie, duopolie en markten met volledige mededinging. Zijn boek, dat ook het concept van een vraagcurve introduceerde, inspireerde later Marshall (1890) om een stijgende aanbodcurve en een dalende vraagcurve te gebruiken als instrumenten voor de bepaling van de marktprijs. Dit model van vraag en aanbod werd de hoeksteen voor de volgende generaties economen.

In de loop der tijd begonnen economen zich echter te realiseren dat dit eenvoudige model enkele belangrijke tekortkomingen vertoont. In de eerste plaats verklaart het niet hoe vraag en aanbod met elkaar in contact komen en hoe de evenwichtsprijs wordt bereikt. Walras (1874) introduceerde een veilingmeester die een prijs voorstelt, vraag en aanbod registreert, en vervolgens de prijs verlaagt / verhoogt indien er sprake is van een aanbod- dan wel vraagoverschot, totdat de prijs bereikt wordt die vraag en aanbod met elkaar in overeenstemming brengt. In de meeste markten bestaat er echter geen centrale persoon of instantie die de rol van veilingmeester kan spelen. Spelers in een markt zijn vaak niet op de hoogte van de acties van andere spelers en coördinatie is vaak onmogelijk vanwege de omvang van de markt. Een tweede tekortkoming van het eenvoudige, competitieve model is dat het leidt tot de *wet van één prijs*: een homogeen product wordt altijd verkocht tegen dezelfde, unieke evenwichtsprijs. Dit is duidelijk in tegenspraak met wat men in werkelijkheid waarneemt: verschillende bedrijven vragen vaak heel verschillende prijzen voor hetzelfde product en vergelijkbare werknemers verdienen vaak zeer verschillende lonen.

In het begin van de jaren zestig probeerden verschillende auteurs tegemoet te komen aan deze tekortkomingen door modellen te ontwikkelen waarin kopers/werknemers een aantal prijzen/lonen konden observeren uit een vaste, exogene verdeling. In een artikel geschreven door Karlin (1962) ontvangen verkopers van een goed sequentieel biedingen en moeten zij besluiten of zij het huidige bod accepteren of verwerpen en hopen op een beter bod in de volgende periode. Rond hetzelfde moment analyseerde Stigler (1961, 1962) modellen waarin kopers/werknemers verschillende prijzen/lonen tegelijkertijd kunnen observeren en de meeste gunstige hiervan accepteren. Hij schreef:

> "Prijzen veranderen met variërende frequentie in elke markt, en tenzij een markt volledig gecentraliseerd is, is niemand op enig moment op de hoogte van alle prijzen die door verkopers (of kopers) vastgesteld zijn. Een koper (of verkoper) die de meest gunstige prijs te weten wil komen, moet verschillende verkopers (of kopers) aan een grondig onderzoek onderwerpen - een fenomeen dat ik zal betitelen als *zoeken*." (Stigler, 1961)

Sinds dat moment heeft zich een omvangrijke literatuur met zoekmodellen ontwikkeld, die stuk voor stuk een explicietere beschrijving geven van het contact tussen vraag en aanbod en/of het bestaan van prijs-/loonverschillen. Dit proefschrift ("Essays over simultane zoekevenwichten") draagt bij aan die literatuur met drie artikelen over *simultaan zoekgedrag*, oftewel situaties waarin werknemers op zoek naar een baan gelijktijdig meerdere sollicitaties versturen. Het eerste en het derde essay zijn voornamelijk theoretische bijdragen aan de literatuur. Het tweede essay bevat ook een belangrijk methodologisch en empirisch gedeelte.

Het eerste essay bestudeert het effect van heterogene productiviteit van bedrijven in het geval van simultaan zoekgedrag. In een markt met volledige mededinging zullen de productiefste bedrijven uiteindelijk alle werknemers in dienst hebben en zullen de minder productieve bedrijven weggeconcurreerd worden. In een wereld met fricties kunnen zowel bedrijven met een hoge productiviteit als bedrijven met een lage productiviteit overleven. In zo'n geval is het de vraag of er een optimale allocatie van werknemers over beide typen bedrijven optreedt. Het essay analyseert dit voor een economie met twee typen bedrijven (bedrijven met een hoge productiviteit en bedrijven met een lage productiviteit) en waarin werknemers twee sollicitaties kunnen versturen. Bedrijven die om de diensten van dezelfde werknemer strijden kunnen hun loonbod zo vaak verhogen als zij wensen. Er wordt aangetoond dat er een uniek symmetrisch evenwicht bestaat waarin werknemers randomiseren tussen het sturen van beide sollicitaties naar de sector met de hoge productiviteit en het sturen van beide naar de sector met de lage productiviteit. De welvaart zou echter gemaximaliseerd worden wanneer de werknemers hun sollicitaties zouden spreiden over de sectoren, omdat op die manier meer vacatures in de productiefste sector vervuld kunnen worden met minder sollicitaties (en dus minder coördinatiefricties). In de markt wordt het loon van een werknemer echter bepaald door hoeveel het bedrijf met het één-na-hoogste bereid is te betalen. Dit weerhoudt de werknemer ervan om in beide sectoren te solliciteren. Het essay toont aan dat de evenwichtsuitkomsten hetzelfde zijn ongeacht of werknemers wel of niet alle aangeboden lonen kunnen observeren voordat zij solliciteren. Dit betekent dat de resultaten niet gedreven worden door de aanname dat zij hun sollicitaties naar willekeurige bedrijven sturen, wat in veel andere artikelen het geval is. Verder wordt een uitbreiding van het model besproken waarin bedrijven vrijelijk tot de markt kunnen toetreden. Dit blijkt tot een tweede soort inefficiëntie te leiden. Ten slotte beschrijft het hoofdstuk de effecten van een toename van het aantal sollicitaties per werknemer en laat het zien dat de resultaten eenvoudig kunnen worden gegeneraliseerd naar een groter aantal typen bedrijven.

Het tweede essay beschouwt de sector waarin de werknemers solliciteren als gegeven, maar concentreert zich op de vraag of zij het sociaal wenselijke aantal sollicitaties versturen. In veel landen bestaan er arbeidsmarktprogramma's die als doel hebben om bepaalde groepen (werkloze) werknemers harder te laten zoeken. Werknemers die meer sollicitaties versturen profiteren hier over het algemeen van in de zin dat zij sneller een baan vinden. De verandering in hun gedrag genereert echter externaliteiten voor de andere spelers in de markt. De hogere zoekintensiteit maakt het voor andere werknemers lastiger om een baan te vinden, maar eenvoudiger voor bedrijven om een vacature te vervullen.

Ook de loonverdeling zal beïnvloed worden. Voor een grondige evaluatie van de arbeidsmarktprogramma's is het daarom noodzakelijk om een model te hebben dat rekening houdt met al deze effecten. Dit essay presenteert zo'n model, waarin de loonverdeling, de zoekintensiteit, en de toetreding van bedrijven tot de markt simultaan bepaald worden in het marktevenwicht, met de zoekkosten van werknemers als exogeen gegeven fundamentele variabelen. Gebruikmakend van loongegevens wordt vervolgens een structurele schatting verkregen van de zoekkostenverdeling, de daarvan afgeleide kansen om een baan/werknemer te vinden, de productiviteit van een vervulde vacature, en de waarde van de tijd die doorgebracht wordt buiten de arbeidsmarkt; deze schattingen worden vervolgens gebruikt om te berekenen wat sociaal gezien het optimale aantal vacatures en de optimale verdeling van zoekintensiteiten is. Er worden drie vormen van inefficiëntie gevonden. Het blijkt dat, vanuit een sociaal oogpunt, (i) te weinig individuen participeren op de arbeidsmarkt, (ii) dat sommige werklozen te veel sollicitaties versturen, en (iii) dat gegeven de zoekintensiteit van de werknemers, te veel bedrijven toetreden tot de markt. De lage arbeidsparticipatie weerspiegelt een standaard *hold-up* probleem en het te grote aantal sollicitaties is het gevolg van coördinatiefricties en het feit dat werknemers trachten een zo groot mogelijk deel van de productie te bemachtigen. Een vrij grote welvaartswinst (ongeveer 12%) kan bereikt worden door deze inefficiënties te corrigeren. Het essay betoogt dat de welvaart verhoogd kan worden door een kleine verhoging van het minimumloon of door werkloosheidsuitkeringen die vereisen dat werklozen minstens één sollicitatie per periode versturen. De reden hiervoor is dat het de arbeidsmarktparticipatie stimuleert zonder een overmatige aantal sollicitaties te belonen.

Het laatste essay in dit proefschrift beschrijft het evenwicht in een zoekmodel voor de arbeidsmarkt, waarin werknemers kunnen blijven zoeken naar een betere baan terwijl zij in dienst zijn van een bedrijf. Een belangrijke vernieuwing ten opzichte van bestaande modellen is dat werknemers hun huidige loon kunnen rapporteren aan het bedrijf dat hen een nieuwe baan aanbiedt. De bedrijven doen een bindend loonbod gebaseerd op deze informatie. Aangezien werknemers meerdere sollicitaties simultaan kunnen versturen, blijft er sprake van loonverschillen in het evenwicht, omdat bedrijven rekening houden met het feit dat ze mogelijk met een ander bedrijf om de diensten van dezelfde werknemer strijden. Het essay laat zien dat in het evenwicht de kansdichtheid van de lonen positieve waarden aanneemt op een continu interval en dat het een unieke, inwendige modus heeft, zelfs in een markt met volledig identieke werknemers en bedrijven. Hoge lonen kunnen niet direct vanuit werkloosheid bereikt worden, maar vereisen een opeenvolging van verschillende banen zonder periode van werkloosheid ertussenin. De snelheid waarmee werknemers de 'loonladder' beklimmen is stochastisch. Met behulp van calibratie van het model wordt aangetoond dat veranderingen in de werkloosheidsuitkeringen een groter effect hebben op de frequentie waarmee werklozen een baan aangeboden krijgen dan op de frequentie voor werkenden. Het omgekeerde geldt voor veranderingen in de baanvernietigingskans.

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