Summary

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When Bayesian methods are applied to nonparametric models, the choice of the prior appears to be of crucial importance for the asymptotic behaviour of the posterior. In this thesis this is worked out for various models; we focus on the convergence rate of the posterior for different choices of the prior. In the first chapter we introduce some definitions and theorems from the literature.

The second chapter is about density estimation on the unit interval using mixtures of beta-densities. Petrone’s ([7],[6]) construction has various appealing properties, but the convergence rate is suboptimal for the class of $\alpha$-Lipschitz densities; see Ghosal [1]. We study the reason for this. For $\alpha \in (0, 1]$ we then construct a prior that leads to the optimal (posterior) convergence rate. We obtain a procedure that is adaptive in $\alpha$, and that can be generalized to higher dimensions.

In chapter 3 we study convergence rates of Bayesian density estimators based on finite location-scale mixtures of a kernel with exponential tails. It is assumed that the underlying density is either twice continuously differentiable with exponential tails, or is a finite mixture itself. Regarding the priors on the weights, locations, and number of components, we provide general conditions under which the posterior converges at a near optimal rate. Examples of priors which satisfy these conditions include Dirichlet and Polya tree priors for the weights, and Poisson processes for the locations. Some of the results can be extended to mixtures of symmetric stable kernels.

In chapter 4 we look at various nonparametric Bayesian estimation problems within the information theoretic framework proposed by Zhang ([8],[9]). In particular, we show that his results can be extended to misspecified models. We find results similar to those in Kleijn and der Vaart [5], who generalized the ‘test-functions based approach’ of Ghosal, Ghosh and van der Vaart [2]). We give an alternative proof to Kleijn and van der Vaart’s ([5]) asymptotic results for misspecified random-design regression models, which allows for an easy extension to fixed-design models.

In chapter 5 we consider a special case of regression, where the prior exhibits a certain spatial structure. We study anisotropic Gaussische Markov Random Fields (GMRF’s). For infinite lattices these are easy to define, but for finite lattices problems may occur at the boundaries. Using Gershgorin’s disc theorem, it can be seen that the covariance matrix will often become singular. In order to obtain a non-singular covariance matrix, we propose a small modification of the Markov structure at the boundaries. This modification can also be made for multivariate GMRF’s. Finally, we describe an implementation of an MCMC-algorithm, which is applied to a data set from weed-ecology (Heijting [3],[4]).
References


