Estimating an Equilibrium Search Model from Wage Data

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Abstract

We propose a simple estimation method for parameters of the Burdett-Mortensen equilibrium search model. The method only uses wage data. It also applies if the parameters of the model vary within the population. As we show, such variation is essential for an acceptable fit to the observed wage distribution. We apply the method to wage data from a Dutch panel study.

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1. Introduction

The equilibrium search model of Burdett and Mortensen (see e.g. Burdett and Mortensen (1993)) is beginning to have an impact on empirical work. The first applications are in Van den Berg and Ridder (1992) and Kiefer and Neumann (1992). In both papers individual labor market histories are used to estimate the simplest version of the Burdett-Mortensen (BM) model. The estimation method is maximum likelihood.

The BM model gives explicit solutions for the equilibrium distributions of wages paid and wages offered. The distribution of wages paid or earnings refers to a cross-section of employees at a particular moment, and the distribution of wages offered refers to the wage offers that (un)employed job seekers receive. In a steady state the earnings density is obtained by multiplying the density of wage offers by the number of workers employed at a particular wage.

In the sequel we concentrate on the earnings and wage offer distributions as implied by the BM model. There are a number of reasons for a closer inspection of these distributions. Firstly, the shape of the equilibrium densities seems to preclude a good fit to data. In figure 1 we have plotted the distribution of wages in the homogeneous BM model for a priori plausible parameter values, and the observed distribution of wages paid (note that the scales differ). It is clear that the homogeneous BM model, i.e. the BM model in which all workers and employers are assumed to be identical, cannot describe the observed earnings distribution. Secondly, the expressions for the wage and earnings densities make estimation of their parameters difficult. The support of the distributions depends on the parameters of interest, and maximum likelihood estimation of the parameters is not standard.

As noted by Van den Berg and Ridder (1993), the fit can be improved by allowing for heterogeneity of the parameters that describe workers and employers, and indeed Van den Berg and Ridder (1992) find that in particular allowance for heterogeneity of the productivity of workers improves the fit substantially. The problems with maximum likelihood disappear if we assume that the wages are measured with error, and this is what we do in Van den Berg and Ridder (1992).

In this paper we show that allowance for heterogeneity in the productivity of workers is the key to obtaining a good fit to the observed earnings distribution. Moreover, we can recover the productivity distribution from the earnings and wage offer distribution. Estimation of some other parameters of the BM model
is possible, but it turns out that the wage distributions are not very informative. It would require truly large samples to obtain accurate estimates of these other parameters. Because these parameters relate to transition intensities between labor market states, they can be estimated more accurately from panel data. The estimates of the parameters of the productivity distribution and of the other parameters are obtained by the method of moments. We use the estimates to decompose the earnings variance into a component due to the heterogeneity of the productivities and a component corresponding to the wage dispersion induced by the BM model. It turns out that the contribution of the productivity heterogeneity dominates.

In section 2 we introduce the basic BM model. The estimation method is described in section 3. Section 4 contains the estimates and some implications of the estimates.

2. The Homogeneous Equilibrium Search Model

We make the following assumptions:

A1. There are continua of workers and firms with measures \( m \) and \( 1 \), respectively.

A2. Workers receive job offers at rate \( \lambda_0 \) if unemployed and \( \lambda_1 \) if employed. A job offer is an i.i.d. draw from a wage offer distribution with c.d.f. \( F(w) \). An offer has to be accepted or rejected upon arrival.

A3. Job-worker matches break up at rate \( \delta \). If this happens the worker becomes unemployed. The level of unemployment benefits is \( b \).

A4. Firms have a linear production function and the average value product is \( p \). A firm pays all its workers the same wage \( w \).

A5. Workers maximize their expected wealth, and firms maximize their steady-state profits.

A6. The firms can not set their wage below the mandatory minimum wage \( w_L \).
Under these assumptions the optimal strategy of the unemployed is a reservation strategy with reservation wage $r$. The optimal strategy of the employed is to quit their current job with wage $w$ for a better paying one. So the employed are assumed to climb a job ladder, on the way increasing their wage. However, there is a chance that an employed worker becomes unemployed, and in that case he has to start again at the bottom of the ladder. Contrary to conventional job search models for the unemployed job search continues after acceptance of a wage offer. Hence, we refer to the strategy of the workers as repeated search.

Each firm chooses a wage rate $w$, that maximizes its steady-state profit flow $(p-w)l(w)$ with $l(w)$ the workforce of a firm that sets its wage at $w$. The firm sets its wage knowing the wages set by other firms. Hence, it knows that setting a low wage means high profits per worker, but also high turnover and a small workforce, and the opposite holds if it sets a high wage. Because workers continue to search after acceptance of a job, there can not be a positive fraction of firms that pay a particular wage. Increasing the wage infinitesimally would increase the labor force much more than it would decrease the profit per worker, thereby increasing the total profit, so that the original wage was not profit maximizing. Because firms that set $w$ equal to the reservation wage $r$ of the unemployed have a nonzero workforce, and firms that offer a lower wage cannot survive, we have that $r$ is the lower bound of the support of the wage offer distribution. Firms that offer $r$ make positive profits, and as a consequence the profit rate of any firm is equal to the profit rate of the firms that offer $r$. Because setting the wage at $p$ would imply zero profits, the upper bound of the support is strictly smaller than $p$. As argued before there cannot be a positive fraction of firms that offer this maximal wage $w_\star$. There cannot be gaps in the support of the wage offer distribution, because firms at the upper boundary of the gap can increase their profits by offering a wage at the lower boundary of the gap.

We conclude that there exists a wage offer density with support $[r, w_\star]$. The explicit solution for the wage offer density follows from the condition that all firms on the support have the same profit rate. Until now we have assumed that the mandatory minimum wage $w_L$ is smaller than $r$. If it exceeds $r$, then the lowest paying firm is the one that offers $w_L$, and the equilibrium profit rate is the profit rate of this firm. Note that in both cases the unemployed accept the first job that is offered to them.
The equilibrium wage offer density is

\[
(2.1) \quad f(w) = \frac{\delta + \lambda_1}{2\lambda_1(p - w_0)^{1/2}} \frac{1}{(p - w)^{1/2}}, \quad w_0 \leq w \leq w_1
\]

and the equilibrium earnings density is

\[
(2.2) \quad g(w) = \frac{\delta(p - w_0)^{1/2}}{2\lambda_1} \frac{1}{(p - w)^{3/2}}, \quad w_0 \leq w \leq w_1
\]

with

\[
(2.3) \quad w_0 = \max (w_L, r)
\]

\[
(2.4) \quad r = \frac{(\delta + \lambda_1)^2 b + (\lambda_0 - \lambda_1)\lambda_0 p + (\lambda_0 - \lambda_1) w_L}{(\delta + \lambda_0)(\delta + \lambda_1)} \quad \text{if } w_L \geq r
\]

\[
= \frac{(\delta + \lambda_1)^2 + (\lambda_0 - \lambda_1)\lambda_0}{(\delta + \lambda_1)^2 + (\lambda_0 - \lambda_1)\lambda_1} \quad \text{if } w_L < r
\]

and

\[
(2.5) \quad w_1 = \left(\frac{\delta}{\delta + \lambda_1}\right)^2 w_0 + (1 - \left(\frac{\delta}{\delta + \lambda_1}\right)^2) p
\]

The upper bound of the support is a weighted average of the lowest wage and \(p\), and hence strictly smaller than \(p\). Note also that the densities depend on \(\lambda_0\), \(\lambda_1\) and \(\delta\) only through \(\lambda_0/\delta\) and \(\lambda_1/\delta\).

In the homogeneous BM model all workers are equally productive and have the same arrival and lay-off rates. Moreover, the assumption that a firm
employs identical workers is not realistic. We should think of a firm in the BM model as a department within a firm, that is responsible for its own hiring and firing decisions. It is more realistic to assume that workers have different productivities and arrival/lay-off rates. Indeed, we shall assume that this heterogeneity can be described by a distribution of $p$ and $\lambda_0/\delta$, $\lambda_1/\delta$ over the workers. We also assume that for every triplet $(p, \lambda_0/\delta, \lambda_1/\delta)$ there is a separate market, so that the equilibrium distributions are mixtures over these triplets. Next, we study the estimation of the parameters of such mixtures of homogeneous BM models. We refer to this model as the Mixture BM (MBM) model.

3. Estimation of the Parameters of the Mixture Burdett-Mortensen Model

We assume that we have a random sample of observations from the wage offer distribution with density (2.1) and from the earnings distribution with density (2.2). Observations from the wage offer distribution are obtained by recording the wages accepted by unemployed job seekers. We obtain observations from the earnings distribution by recording the wages of a cross-section of employees.

In the homogeneous BM model maximum likelihood estimation (MLE) of the parameters is not standard. To see this note that the likelihood function based on either (2.1) or (2.2) becomes infinite, if we estimate $p$ by the sample maximum. Because the sample maximum converges to $w_1 < p$, this estimator of $p$ is not consistent. The reason for this anomalous behaviour of the MLE is that the support of the wage offer and earning distributions depends on the parameters of interest. As noted by Kiefer and Neumann (1992) this feature can be exploited to obtain consistent estimators of the parameters. They propose to use the sample minimum and maximum to estimate $w_0$ and $w_1$. Substitution of these estimators in (2.4) and (2.5) gives us two equations that can be solved for $p$ and $b$ if we have estimates of $\lambda_0$, $\lambda_1$ and $\delta$. Estimates of these parameters can be obtained from panel data that record labor market transitions. Indeed, Kiefer and Neumann derive the likelihood function for panel data. After substitution for $p$ and $b$ in (2.1) and (2.2) this function only depends on $\eta_0$, $\lambda_1$ and $\delta$, and is a standard likelihood. This method has two drawbacks. Because it is based on the sample minimum and maximum, the estimates are sensitive to measurement errors in the wages. Irrespective of the availability of panel data, the procedure also breaks down in the MBM model, because the lower and upper bounds of the support of the distributions are not given by (2.3)-(2.5) if the parameters follow some distribution. For instance, if the distribution of $p$ does not have an
upper bound, so that \( w_i \) is infinite. As argued below, allowing for heterogeneity in \( p \) is essential for a good fit to wage data.

An alternative procedure starts from the following transformation of \( w \)

\[
y = \frac{p - w}{p - w_0}
\]

(3.1)

so that the excess wage \( w-w_0 \) satisfies

\[
w-w_0 = (1-y)(p-w_0)
\]

(3.2)

The density of \( y \) is for the wage offer distribution

\[
f_y(y) = \frac{1}{2(1-\eta)}y^{-1/2}, \quad \eta^2 \leq y \leq 1
\]

(3.3)

and for the earnings distribution

\[
g_y(y) = \frac{\eta}{2(1-\eta)}y^{-3/2}, \quad \eta^2 \leq y \leq 1
\]

(3.4)

with

\[
\eta = \frac{\delta}{\delta + \lambda_1}
\]

(3.5)

Equation (3.2) describes the wage determination in the BM model. The excess wage \( w-w_0 \) is a fraction of the excess productivity \( p-w_0 \). This fraction is a random variable with a distribution that depends on the expected number of wage offers during a spell of employment, i.e. a spell that starts with the
acceptance of a job from unemployment and ends with a lay-off. This number given by \( \lambda / \delta \) is a measure of the speed at which the worker climbs the job (and wage) ladder.

From these expressions we find the moments of the wage offer and earnings distributions. We obtain

\[
E_f(w-w_0)^n = (p-w_0)^n \frac{1}{2(1-\eta)} \sum_{k=0}^{n} \binom{n}{k} (-1)^k \frac{1}{k+1} (1-\eta^{2k+1})
\]

(3.6)

\[
E_g(w-w_0)^n = (p-w_0)^n \frac{\eta}{2(1-\eta)} \sum_{k=0}^{n} \binom{n}{k} (-1)^k \frac{1}{k-1} (1-\eta^{2k-1})
\]

(3.7)

In particular

\[
E_f(w-w_0) = (p-w_0)(1-\frac{1}{3}(\eta^2+\eta+1))
\]

(3.8)

\[
E_g(w-w_0) = (p-w_0)(1-\eta)
\]

(3.9)

\[
V_g(w-w_0) = (p-w_0)^2 \frac{1}{3} \eta(1-\eta)^2
\]

(3.10)

We can use (3.8) and (3.9) to estimate the parameters of the homogeneous BM model, if we replace the population means on left-hand sides of these equations by sample means and if \( w_0 = w_L \). If we make assumptions B1 and B2 below then we can use the same approach in the MBM model.
B1. The productivity $p$ varies in the population, but $\eta$ is constant in the population.

B2. The lower bound $w_0$ is equal to the mandatory minimum wage $w_L$.

Assumption B1 seems arbitrary. However, the estimation results in Van den Berg and Ridder (1992) indicate that there is not much variation in $\lambda / \delta$, but that the fit of the model improves dramatically if we allow for variation in $p$. Because $r$ depends on the transition parameters and $p$, we require in assumption B2 that the inequality holds uniformly in these parameters. Again the estimation results in Van den Berg and Ridder (1992) indicate that this holds for workers younger than 39. For these workers wage offers are more frequent when employed, so that their reservation wage is lower than the benefit level $b$ which in turn is lower than $w_L$.

Under assumptions B1 and B2 (3.8) and (3.9) are linear in $p$. Hence, they also hold for the MBM model, if we interpret $p$ as the mean in the population, $p_m$.

Hence, we can solve for $\eta$ and $p_m$.

\[
\eta = 3 \frac{E_f (w-w_0)}{E_g (w-w_0)} - 2
\]

\[
p_m = w_0 + \frac{(E_g (w-w_0))^2}{3(E_g (w-w_0) - E_f (w-w_0))}
\]
If there is heterogeneity in $p$, then an obvious generalization of (3.10) can be solved for the variance of $p$

$$V(p) = \frac{V_k(w-w_o) - \frac{1}{3}(p_m-w_o)^2\eta(1-\eta)^2}{(1-\eta)^2(1+\frac{1}{3}\eta)}$$

Note that we did not make any assumption on the distribution of $p$. Under assumptions B1 and B2 we can write the wage excess as the product of two random variables: $1-y$ that indicates the position of the worker on the job ladder and $p-w_o$, the excess productivity of the selected worker. If we write the first factor on the right-hand side of (3.7) as

$$E(p-p_m)^n + (p_m-w_o)^n$$

we see that we can match the moments of the observed earnings distribution by choosing appropriate values for the moments of $p$. Hence, the fit to observed wage data depends on the choice of the distribution of $p$, and (3.7) and (3.14) guide us in our choice of this distribution.

Because $0 \leq \eta \leq 1$, we obtain from (3.11)

$$\frac{3}{2}E_j(w-w_o) \geq E_k(w-w_o) \geq E_j(w-w_o)$$

Hence, the MBM model implies a rather tight restriction on the mean offer and the mean earnings. In particular, the upper bound can be violated. It can be shown that allowing for variation in $\eta$ increases the constant in the upper bound to at most 1.71.

In the next section we shall use (3.11)-(3.13) to estimate the parameters of the MBM model.
4. Estimates and Implications

We use data on wage offers and earnings from the OSA Panel Survey. This panel started in 1985. Since then three more waves have become available (in 1986, 1988, and 1990). We need only a fraction of the information. Specifically, we use the wage at the moment of the first wave of the individuals that were employed at that time, and accepted wages of unemployed workers who found a job in the period 1985-1990. All wages are in Dutch guilders per month.

For the whole sample and the subsample that excludes individuals over 39 with a higher vocational or university education we find

\begin{align*}
\text{All individuals} & \\
\text{Average excess earnings} &= 691 \quad \text{Average excess accepted wage} = 307 \\
N &= 1905 \quad N = 44
\end{align*}

\begin{align*}
\text{Excluding workers over 39 with higher education} & \\
\text{Average excess earnings} &= 539 \quad \text{Average excess accepted wage} = 303 \\
N &= 1301 \quad N = 28
\end{align*}

First, one is struck by the small number of unemployed who found a job in the period of the survey. It is likely that this small group is a highly selective sample. This causes a problem if the sample is selective with respect to \( p \). Secondly, in both samples the restriction (3.15) is violated. The average excess earnings are much too large relative to the average excess accepted wage. This is also true if we allow for heterogeneity in \( \eta \).

An obvious explanation is that the sample of the re-employed is selective. If we compare the re-employed with the employed, then the re-employed are younger, lower educated, and have lower-level jobs. If we regress for the employed the earnings at the date of the first wave on a number of characteristics we obtain the following estimates
<table>
<thead>
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<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
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</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1126.7</td>
<td>50.2</td>
</tr>
<tr>
<td><strong>Education</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Lower</td>
<td>124.8</td>
<td>31.9</td>
</tr>
<tr>
<td>Intermed.</td>
<td>339.3</td>
<td>43.7</td>
</tr>
<tr>
<td>Higher</td>
<td>696.9</td>
<td>68.8</td>
</tr>
<tr>
<td><strong>Age</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16-22</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>23-29</td>
<td>378.9</td>
<td>49.9</td>
</tr>
<tr>
<td>30-38</td>
<td>704.2</td>
<td>49.0</td>
</tr>
<tr>
<td>39-</td>
<td>944.8</td>
<td>49.1</td>
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<tr>
<td><strong>Job level</strong></td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>87.5</td>
<td>35.4</td>
</tr>
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<td>s</td>
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<td></td>
</tr>
<tr>
<td>R</td>
<td>.46</td>
<td></td>
</tr>
</tbody>
</table>

Given these estimates and the characteristics of the re-employed, it is not surprising that the average excess earnings of this selective group of re-employed workers is overestimated by the corresponding average of the employed.

We use the regression results to predict the average excess earnings for the re-employed in the subsample defined above. We impose the latter restriction to ensure that the minimum wage exceeds the reservation wage. We obtain

Predicted average excess earnings = 442

Note that we only correct for observed differences between the re-employed and other employees. If there are unobserved differences, then the estimate of the average excess earnings may be biased. This estimate implies
The variance of the estimator of $\lambda_i/\delta$ is obtained by the delta-method. The reported standard error is very large, because it is based on a very small sample (28 observations), and because the variances of the earnings and wage offer distributions are large. For instance, if the sample size were $N=5000$, then the standard error would still be 17.0. Hence the large standard error reflects the fact that the wage data are not very informative on $\lambda_i/\delta$. The estimate for $\lambda_i/\delta$ can be compared with the estimate 7.62 obtained for this subsample by Van den Berg and Ridder (1992).

Because the estimation error in $\eta$ is large, we give estimates of the other parameters based on (3.12) and (3.13), and estimates based on the same expressions but with $\eta=.12$ which is obtained from the transition data.

Estimates as in (3.12) and (3.13)

$$\hat{p}_m = 1822.5 \ (260.7) \quad \text{(V(p))}^{1/2} = 120.0 \quad \text{Fraction due to } p: .78$$

Estimates based on $\eta=.12$

$$\hat{p}_m = 1856.0 \ (147.1) \quad \text{(V(p))}^{1/2} = 105.3 \quad \text{Fraction due to } p: .53$$

The standard error of the estimator (3.12) is obtained by the delta-method. The fraction due to $p$ is the fraction of the earnings variance due to the heterogeneity in $p$. For both estimates the heterogeneity is the main source of variation.

These results show that, although it is possible to estimate $\lambda_i/\delta$ from cross-section data on wages, the estimate is imprecise. The average productivity can be estimated more precisely. Hence, we clearly need panel data. With panel data the cross-section moment conditions of the present paper can be combined with the likelihood for the transition data. Such a procedure is attractive,
because it avoids the use of sample extremes and also does not require measurement error to deal with the dependence of the support of the wage distributions on the parameters of interest.

References

Figure 1. Equilibrium earnings distribution ($\lambda/\delta = 7.62$, $w_L = 1401$, $p = 2073$: averages of parameter estimates in Van den Berg and Ridder (1992)) and observed distribution of earnings.