An Empirical Equilibrium Search Model of the Labour Market

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Summary.
In structural empirical models of labour market search, the distribution of wage offers is usually assumed to be exogenous. Because in setting their wages profit-maximizing firms should consider the reservation wages of job seekers, this assumption is hard to justify. In this paper we estimate a structural equilibrium search model in which the wage offer distribution is endogenous. The model takes account of the behaviour of unemployed and employed individuals as well as firm behaviour. The equilibrium distributions of wage offers and earnings are non-degenerate. The model is estimated using panel data on unemployed and employed individuals. We distinguish between separate segments of the labour market. The results are used to estimate the degree of monopsony power of firms. Further, the effect of changes in the legal minimum wage are examined.

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1. Introduction

Eckstein and Wolpin's (1990) empirical analysis of the Albrecht and Axell (1984) equilibrium search model is an important step forward in the structural analysis of labour market search. For the first time in an empirical study labour market search is modelled as the outcome of optimal choices by both workers and employers. This improves on the structural partial models of job search in which workers choose an optimal strategy given the decisions of firms, but the distribution of wage offers is assumed to be invariant and is treated as exogenous in the empirical analysis. If workers make a labour supply decision given a wage offer, and firms set wages for vacancies, then current empirical models of job search are based on the assumptions that (i) in setting wages firms do not consider the strategy of workers and (ii) the resulting wage offers are dispersed.¹

The optimal strategy of the workers usually has the reservation wage property. If employers know this, then in equilibrium their wage offers must equal the reservation wage of some (group of) worker(s). In particular, if all job searchers have a common reservation wage, then the equilibrium wage offer distribution is degenerate. The assumption of a dispersed wage offer distribution therefore requires a dispersed distribution of reservation wages. If the wage offer distribution depends on the distribution of reservation wages, then parameter changes that affect the reservation wages of job searchers also affect the wage offer distribution that they face.

Hence, the assumption that the wage offer distribution is exogenous to the job search model is hard to justify. This affects both the key assumption in this model, i.e. the assumption that job seekers face a dispersed wage offer distribution, and the comparative-static analysis of parameter changes. For instance, an important question as the effect of unemployment benefits on job search by the unemployed cannot be answered satisfactorily without allowing for the possibility that employers respond to changes in the behaviour of job seekers. In an equilibrium search model the wage offer distribution is endogenous. It results from optimal wage setting by firms that take account of the responses by job seekers and other firms. The fact that the wage offer distribution is determined by the model is convenient, since this distribution is essentially unobservable. If job seekers use a reservation wage strategy,

then the distribution of accepted wages is the wage offer distribution truncated at the reservation wage. It is well-known that one cannot recover the complete wage offer distribution from this truncated distribution of accepted wages (Flinn and Heckman (1982)).

The Albrecht–Axell model is not the only equilibrium search model that is amenable to estimation. Our empirical analysis of equilibrium search is based on a model proposed by Burdett and Mortensen (1993). There are some striking differences between these two models, and it should be stressed from the outset that the empirical models of Eckstein and Wolpin (1990) and of this paper are not nested. In the Albrecht–Axell model all job seekers are unemployed. Hence, to obtain a dispersed wage offer distribution it is required that the unemployed are heterogeneous. Albrecht and Axell (1984) and Eckstein and Wolpin (1990) assume that there are several types of job searchers differing in their value of non-market time. Because the number of types is finite, the number of distinct reservation wages and hence the number of points of support of the wage offer distribution is also finite. This is clearly an unattractive feature of the model. In the Albrecht–Axell model it is also assumed that firms are heterogeneous with respect to their labour productivity. This allows for an equilibrium in which low productivity firms and high productivity firms have different profit-maximizing wage offers, so the resulting wage offer distribution is non-degenerate.

A second difference between the two models (related to the first) concerns job durations. In the Albrecht–Axell model, jobs, once accepted, are held forever. In particular, the model does not allow for job-to-job transitions. This is clearly counterfactual. Moreover, job-to-job transitions are the most important source of wage growth (Topel and Ward (1993)), and this points at the importance of wage setting for maintaining the workforce of a firm. The possibility of on-the-job search changes the optimal search strategy of unemployed job seekers. Furthermore, since the reservation wage of an employed job seeker is equal to his current wage, allowing for on-the-job search extends the range of reservation wages considerably. Hence, the possibility of search while employed aids in generating a dispersed equilibrium wage offer distribution.

Indeed, Burdett and Mortensen (1993) show that if workers continuously search for a better-paying job (and face a risk of becoming unemployed during that quest) then the equilibrium wage offer distribution is dispersed, even if all workers and firms are identical. In the latter case they obtain explicit solutions for the equilibrium wage offer and earnings distributions.

The availability of an explicit solution is an advantage in the empirical
analysis of the model. On the other hand, the solution has some unattractive features. In particular, the wage offer and earnings distribution have increasing densities. This implication is at odds with all the evidence on the shape of the income distribution, which is closely related to the earnings distribution. However, it should be stressed that the explicit solution refers to a homogeneous population of workers and firms. Allowing for observed and unobserved population heterogeneity makes the model more realistic and more able to give an acceptable fit to the data. The heterogeneity in our empirical version of the Burdett–Mortensen model is different from the heterogeneity in the Albrecht–Axell model. We consider a labour market that consists of a large number of segments. Every segment is a labour market of its own, and all workers and firms in a particular segment are identical. The segments differ according to the age, the educational level and the occupational level of the workers/jobs. Besides these observed differences we allow for unobserved differences in the productivity of the jobs or other characteristics of the segment. Eckstein and Wolpin (1990), in their empirical analysis of the Albrecht–Axell model, consider a single labour market with unobserved differences in the value of leisure between workers and unobserved differences in productivity between firms (they do not allow for observed differences between workers and firms). So our treatment of population heterogeneity allows for between-market heterogeneity while Eckstein and Wolpin (1990) allow for within-market heterogeneity. As we will show, this distinction has important consequences as it makes the effect of a change in the minimum wage qualitatively different.

We estimate the model by Maximum Likelihood, using panel data on unemployed and employed individuals. For most individuals in the data, multiple durations (like unemployment durations and job durations) are observed. In particular, for some respondents, we observe consecutive job durations and corresponding wages. The estimation results are used to estimate the degree of monopsony power of firms and the effects of changes in the legal minimum wage.

The outline of the paper is as follows. Section 2 presents the model. We argue that it is consistent with a number of stylized facts. In Section 3 we discuss the data used to estimate the model. In Section 4 we derive the likelihood function. Section 5 contains the results. We also discuss some implications of the estimates. Conclusions and some suggestions for further research are in Section 6.
2. The equilibrium search model

In this section we present the equilibrium search model as developed by Burdett and Mortensen (1993) (see also Mortensen (1990)). First, we consider a labour market with homogeneous workers and firms. Later we shall indicate how we take account of heterogeneity of workers and firms.

We make the following assumptions:

A1. There are continua of workers and firms with measures \(m\) and \(1\), respectively.
A2. Workers receive job offers at rate \(\lambda_0\) if unemployed and \(\lambda_1\) if employed. A job offer is an i.i.d. drawing from a wage offer distribution with c.d.f. \(F(w)\). An offer has to be accepted or rejected upon arrival. During tenure of a job the wage is constant.
A3. Job-worker matches break up at rate \(\delta\). If this happens the worker becomes unemployed. The utility flow of being unemployed is \(b\).
A4. Firms have a linear production function and the marginal (=average) revenue product is \(p\). A firm pays all its workers the same wage \(w\).
A5. Workers maximize their expected wealth, and firms maximize their steady-state profits.
A6. The firms cannot set their wage below the mandatory minimum wage \(w_L\).

Under these assumptions the supply side of the model is equivalent to the standard job search model with on-the-job search (see e.g. Mortensen (1986)). Thus, the optimal strategy of an unemployed individual has the reservation wage property. Under zero discounting, the reservation wage \(r\) can be shown to be

\[
(2.1) \quad r = b + (\lambda_0 - \lambda_1) \int_{r}^{\infty} \frac{F(w)}{\delta + \lambda_1 F(w)} \, dw \quad \quad \text{with} \quad F = 1 - F
\]

Further, an employed individual accepts a wage offer if and only if it exceeds his current wage. So a worker is continuously searching for a better paying job, but this effort may be frustrated by a spell of unemployment. The worker never quits a job to search for a better paying one while unemployed.

It is important to distinguish between the distribution of wages offered to job seekers, which is the wage offer distribution \(F\), and the distribution of wages received by workers who are currently employed. The latter distribution is referred to as the earnings distribution, and we denote this
distribution by $G$. Concentrate for the moment on the employed workers who receive a wage $w$ or less. There are $G(w)(m-u)$ such workers, where $u$ is the number of unemployed workers. The flow of workers to jobs with a wage that exceeds $w$ is equal to $\lambda_1 F(w) G(w)(m-u)$ and the flow of workers into unemployment is $6G(w)(m-u)$. In a steady-state equilibrium this outflow must be balanced by an inflow from unemployment. This inflow equals $\lambda_0(F(w)-F(r))u$, where it is obvious that $F(r)=0$, because firms offering a wage below $r$ never attract any worker and therefore cannot survive. Hence, in a steady state we have the following relation between the earnings distribution and the wage offer distribution,

$$G(w) = \frac{F(w)}{\delta + \lambda_1 F(w)} \cdot \frac{\lambda_0 u}{(m-u)}$$

The steady-state unemployment rate $u/m$ follows by imposing $w=\infty$ in (2.2), or, equivalently, by equating flows into and out of unemployment,

$$\frac{u}{m} = \frac{\delta}{\delta + \lambda_0}$$

This can be used to simplify (2.2) to

$$G(w) = \frac{\delta . F(w)}{\delta + \lambda_1 F(w)}$$

The flow of revenue $p$ generated by employing a worker must satisfy $b < p < \infty$, i.e. there must be a positive gain from trade. A match between a worker and a firm has a net revenue flow of $p-b$. At the prevailing wage $w$, the firm receives the $p-w$ of this flow, and the worker receives $w-b$. Recall that it is assumed that the wage paid by the firm is posted prior to the moment at which it contacts searching individuals, and that there is no bargaining over the wage.

To derive the equilibrium distribution of wage offers over firms, we have to be more specific on the behaviour of the firms. The steady-state level of production is determined by the size of the steady-state workforce $l$ that is available to the firm. The latter number is determined by the wage $w$ set by the firm, by the reservation wage $r$ set by the unemployed individuals, and by the distribution $F$ of wages set by the other firms competing for the same workers (see Burdett and Mortensen (1993) for the general expression of $l(w;r,F)$). We assume that each firm chooses $w$ to maximize its steady-state profit flow $\pi$, which equals $(p-w)\xi(w;r,F)$, given $r$ and $F$ and subject to the
restriction that \( w \) exceeds the mandatory minimum wage \( w_L \). Hence, the firm does not react to random fluctuations in its workforce\(^2\). For firms to have a positive level of employment, we need \( w_L < p \).

A non-cooperative steady-state equilibrium solution consists of a reservation wage \( r \) and a wage offer distribution \( F \) such that (i) \( r \) satisfies (2.1) given \( F \), and (ii) every \( w \) in the support of \( F \) maximizes the steady-state profit flow \( (p-w)\ell(w;r,F) \). Burdett and Mortensen (1993) prove that there is a unique equilibrium, and they give a complete characterization of it.

First, the equilibrium \( F \) is absolutely continuous, and therefore not degenerate. Suppose there were a mass of firms offering a wage \( w \). Then, by offering a wage slightly higher than \( w \), each of these firms could increase its labour force significantly, attracting workers that currently earn \( w \), while suffering only a second order loss of profit per worker \( p-w \). Hence, profits \( \pi \) would increase. The assumption that on-the-job search is possible is crucial for this result. It is not possible to have an equilibrium \( F \) for which the lower bound of the support exceeds both \( r \) and \( w_L \), or an equilibrium \( F \) with gaps in its support, because then firms offering \( w \) (firms at the upper boundary of the gap) can increase their profits by offering a wage equal to \( \max(w_L,r) \) (by offering a wage equal to the lower boundary of the gap).

We conclude that \( F \) and \( G \) have probability density functions \( f \) and \( g \) with support equal to \([w,\bar{w}]\), with

\[
\bar{w} = \max(\omega_L, r)
\]

and \( \bar{w} \) the upper bound of the support of \( F \) and \( G \) with \( \bar{w} < p \). The measure of individuals earning a wage \( w \) equals \( g(w)(m-u)\,dw \), and the measure of firms offering a wage \( w \) equals \( f(w)\,dw \). Consequently, \( \ell(w;r,F) \) equals

\[
(2.5) \quad \ell(w;r,F) = \frac{g(w)\,dw}{f(w)\,dw} (m-u) = \frac{m \lambda_0 \delta(\delta + \lambda_1)}{(\delta + \lambda_2)(\delta + \lambda_1)F(w)}
\]

on \([w,\bar{w}]\).

In the steady state a firm offering \( w \) has a positive workforce \( \ell(w;r,F) \) equal to \( m \lambda_0 \delta/((\delta + \lambda_2)(\delta + \lambda_1)) \). Of course, the employees, who were all previously unemployed, leave for the first job they locate at another firm. The steady-state profit rate of the firms paying \( w \) is \( \pi = (p-w)\ell(w;r,F) \). In equilibrium all higher paying firms have the same profit rate. Substitution of (2.5) gives the equilibrium wage offer distribution.

\(^2\) The model becomes intractable if this is relaxed, see Wernerfelt (1988).
with corresponding density

\[
 f(w) = \frac{\delta + \lambda_1}{2\lambda_1 \sqrt{p-w} \sqrt{p-w}^2} \quad \text{on } [w, \bar{w}]
\]

It follows from (2.6) and (2.1) that

\[
 r = \frac{(\delta + \lambda_1)^2 b + (\lambda_0 - \lambda_1) \lambda_1 p + (\lambda_0 - \lambda_1) w \lambda}{(\delta + \lambda_0)(\delta + \lambda_1)} \quad \text{if } r < \underline{w}
\]

\[
 = \frac{(\delta + \lambda_1)^2 b + (\lambda_0 - \lambda_1) \lambda_1 p}{(\delta + \lambda_1)^2 + (\lambda_0 - \lambda_1) \lambda_1} \quad \text{if } r \geq \underline{w}
\]

\[
 \bar{w} = \frac{\delta}{\delta + \lambda_1} \bar{w} + \left[ 1 - \left( \frac{\delta}{\delta + \lambda_1} \right)^2 \right] p
\]

If \( r \geq \underline{w} \), then \( r \) and \( \bar{w} \) are weighted averages of \( b \) and \( p \). Note that \( r \) is smaller than \( b \) if \( \lambda_1 \) is larger than \( \lambda_0 \). If the arrival rate of job offers is larger when employed, unemployed workers accept job offers that are below \( b \) in order to engage in the more rewarding search on the job.

The function \( \ell(w; r, F) \) increases in \( w \), so there is a positive relation between the size of the firm and the wage it offers. A large (small) wage implies that the turnover of workers at the firm is small (large). Hence, in terms of total profits of a firm, there is a trade-off between the profit per worker and the steady-state number of workers at the firm. In this respect, there is a strong similarity to "turnover costs" efficiency wage models (see e.g. Stiglitz (1985) and Weiss (1991)). The maximum wage offer is strictly smaller than the productivity level \( p \), which Burdett and Mortensen (1993) call the competitive equilibrium wage, i.e. the single equilibrium wage in the absence of search frictions. The search and wage-setting game has a monopsonistic equilibrium.

From (2.4), the equilibrium earnings density is

\[
 g(w) = \frac{\delta \sqrt{p-w}}{2\lambda_1} \cdot \frac{1}{(p-w)^{3/2}} \quad \text{on } [\underline{w}, \bar{w}]
\]

Note that both \( f \) and \( g \) are increasing densities. The earnings distribution is
related to the income distribution, and there is abundant empirical evidence that the income distribution does not have an increasing density. However, as noted before, the increasing densities are derived for a homogeneous labour market with identical workers and firms. To show that allowing for heterogeneous workers and/or firms indeed improves the fit to the observed wage offer and earnings distributions we consider the following transformation of $w$

\begin{equation}
(2.11) \quad y = \frac{\bar{w} - w}{p - \bar{w}}
\end{equation}

so that the excess wage $w - \bar{w}$ satisfies

\begin{equation}
(2.12) \quad w - \bar{w} = (1-y)(p - \bar{w})
\end{equation}

The density of $y$ is for the wage offer distribution

\begin{equation}
(2.13) \quad f_y(y) = \frac{1}{2(1-\eta)^{3/2}} \eta^{3/2} \quad \eta^2 \leq y \leq 1
\end{equation}

and for the earnings distribution

\begin{equation}
(2.14) \quad g_y(y) = \frac{\eta}{2(1-\eta)} y^{-1/2} \quad \eta^2 \leq y \leq 1
\end{equation}

with $\eta = \delta/(\delta + \lambda_1)$. Equation (2.12) describes the wage determination in the Burdett–Mortensen model. The excess wage $w - \bar{w}$ is a fraction of the excess productivity $p - \bar{w}$. This fraction is a random variable with a distribution that depends on $\lambda_1/\delta$, the expected number of wage offers during a spell of employment, i.e. a spell that starts with the acceptance of a job from unemployment and ends with a layoff. This number is a measure of the speed at which the worker climbs the job (and wage) ladder with $y=1$ corresponding to the bottom, $w = \bar{w}$, and $y = \eta$ to the top, $w = \bar{w}$, of this ladder. From (2.12) it follows that the moments of $w - \bar{w}$ in either the wage offer or the earnings distribution are the product of $(p - \bar{w})^n$ and an expression that only depends on $\eta$. By choosing an appropriate distribution of the productivity $p$, the moments of any observed wage offer or earnings distribution can be matched. Hence, we expect that an acceptable fit to the data will depend on allowance for sufficient heterogeneity in $p$. 

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From the expressions above it is straightforward to derive the distributions of the sojourn times in different states. The duration of unemployment has an exponential distribution with parameter $\lambda_0$. The duration of a job that pays a wage $w$ has an exponential distribution with parameter $\delta + \lambda_1 F(w)$. Exit from this job into unemployment occurs with probability $\delta/(\delta + \lambda_1 F(w))$ and exit into another job with probability $\lambda_1 F(w)/(\delta + \lambda_1 F(w))$. These distributions, as well as $G(w)$ and $F(w)$ itself, will be used in Section 4 to construct the likelihood function of the model. Note that if $r < w_0$, then $r$ (and therefore $b$) enters none of the distributions mentioned here.

Despite its apparent restrictiveness the model is consistent with results from previous empirical studies on unemployment durations and job durations in The Netherlands in the eighties. For example, the model implication that all jobs are acceptable to the unemployed is consistent with the finding from studies with partial job search models that the acceptance probability of unemployed workers is close to 1 (see e.g. Van den Berg (1990b); Devine and Kiefer (1990) survey the evidence for the US). The model also implies that a change of the benefit level does not affect unemployment. When $b$ increases, then, as in partial job search models, the unemployed individual's reservation wage increases. However, in the present model employers modify their wage offers in response to this, and the net result is that the exit rate out of unemployment does not change (as long as $b < p$). This is consistent with the findings of many structural and reduced-form empirical studies on unemployment duration based on data from The Netherlands (see a survey in Van den Berg (1990b)).

In empirical search models it is often found that the reservation wage is smaller than the benefits level (see e.g. Narendranathan and Nickell (1985), Van den Berg (1990a) and Van den Berg (1990b)). This is usually attributed to the existence of a non-pecuniary disutility of being unemployed. The present model generates $r < b$ if the job offer arrival rate is larger in employment than in unemployment. Finally, studies based on the panel data used in this paper confirm that there is an inverse relation between the wage in a job and the exit rate from a job, holding other factors constant (see Lindeboom and Theeuwes (1991) and Van den Berg (1992))$^3$.

$^3$Kiefer and Neumann (1991) and Machin and Manning (1991) discuss the consistency of the model with some additional stylized facts.
3. The data


In the OSA panel a random sample of households in The Netherlands is followed over time. The study concentrates on individuals who are between 15 and 61 years of age, and who are not full-time students. Therefore only households with at least one person in this category are included. All individuals (and in all cases the head of the household) in this category are interviewed. The first wave consists of 4020 individuals (in 2132 households). In 1992, 1384 (34%) of these individuals are still in the panel. In 1986, 1988, and 1990, refreshment samples were drawn, so that in 1990 the sample size was 4438 individuals.

In the OSA panel an effort is made to collect extensive information on the labour market histories of the individuals. From these labour market histories we obtain the sequence of labour market states occupied by the individuals and the sojourn times in these states. A number of variables that give a more detailed description of the various positions is also recorded, notably income and occupation. Part of the information is retrospective. For example, in the first wave (in 1985) an attempt was made to reconstruct the labour market histories from January 1, 1980 until the date of interview (in 1985). The following labour market positions are distinguished: employment (job-to-job changes are recorded), self-employment, unemployment, and "not in the labour force" (military service, full-time education, and other activities not related to the labour market). In addition to these labour market histories, a number of time-constant individual characteristics are recorded, and an attempt was made to keep track of changes in time-varying characteristics as family composition, marital status, and level of education.

In this paper we restrict attention to respondents who were participating in the panel as of the first wave, but not necessarily until and including the fourth wave. Individuals who were self-employed for some period during the time span covered by the survey are omitted, since it is likely that the behaviour of such individuals, at least in a certain period, deviates substantially from the behaviour that the model intends to describe. For similar reasons, we do not use information on respondents who are observed to be working in a part-time job or who are observed to be a nonparticipant for
some period. An alternative approach would be to extend the model to include a state of nonparticipation, and allow for transitions to and from this state. Van den Berg and Ridder (1993) develop such a model. It turns out that the main features of the model of Section 2 are insensitive to the inclusion of such a state. Moreover, transitions to and from nonparticipation are rare in the data. Therefore, using information on such transitions in an extended model context would, except for a number of imprecisely estimated nuisance parameters, probably not result in any gains. The restrictions reduce the number of labour market states to two: unemployment and full-time employment.

The indicated selection results in a sample of 1949 individuals, of which 217 (1732) were unemployed (employed) at the date of the first interview. In our sample, 34% participates in all four waves of the panel, while 33% only participates in the first wave.

Income changes at transitions before the date of the first interview (April 1985) are only recorded to lie in one of a few broad intervals, so income in states occupied before this date is reported inaccurately relative to income in later states, for which we observe income levels. Moreover, using the spells ending before the date of the first interview generates computational problems in the estimation, as will become clear in the next section. To avoid these problems we do not use these spells, so the first spell used is the one that is ongoing at the date of the first interview. We basically use at most one additional spell besides this first spell (see the next section for details).

After this selection we end up with 366 unemployment durations, of which 30% is right-censored, and 2941 job durations, of which 42% is right-censored. Note that the right-censoring point is not fixed across spells. The number of observed wages is 1951.

So far, $b$ has been interpreted somewhat vaguely as the full opportunity cost of employment. In the sequel we take $b$ to equal the benefit level. At the end of Section 4 we discuss how we assign numerical values to $b$.

4. Empirical implementation

4.1. The likelihood function

We estimate the model of Section 2 by Maximum Likelihood, using the panel data described in Section 3. In this section we derive the individual contributions to the likelihood function. Because the expressions depend critically on the
labour market state occupied at the date of the first interview, we derive the contributions separately for individuals who are unemployed and individuals who are employed at that date. The results of Section 2 imply that the appropriate statistical model for labour market histories can be embedded in a three-state stationary Continuous-time Markov Chain (CTMC). The three states are unemployment and two employment states to allow for job-to-job transitions.

The first job or unemployment spell we use is the spell that is ongoing at the date of the first interview. In the sequel we invoke arguments analogous to those in Ridder (1984) to derive the exact distribution of such spells. First, consider an individual who is unemployed at the date of the first interview. If the labour market history CTMC is in equilibrium, the probability of being unemployed at a randomly chosen date equals \( \delta / (\delta + \lambda_0) \). Conditional on the individual being unemployed at the date of the first interview, the elapsed unemployment duration \( t_{0b} \) and the residual unemployment duration \( t_{0f} \) are i.i.d. and have an exponential distribution with parameter \( \lambda_0 \). Let \( d_{0b} (d_{0f}) \) denote a dummy that is one if it is only known that the elapsed (residual) duration exceeds a certain value (i.e. is right-censored), and zero otherwise. The likelihood contribution of the events until and including the moment of exit out of unemployment or censoring is

\[
L_0 = \frac{\delta}{\delta + \lambda_0} \cdot \lambda_0^{1-d_{0b}+1-d_{0f}} \exp(-\lambda_0(t_{0b}+t_{0f}))
\]

All events occurring after exit out of unemployment are independent of the events up to exit. Consequently, their probability can be derived separately. The first relevant event is the realization of the wage \( w \) in the accepted job. This is a random draw from the wage offer distribution \( F(w) \). Conditional on \( w \), the job duration \( t_x \) has an exponential distribution with parameter \( \delta + \lambda_1 F(w) \). Exit into unemployment occurs with probability \( \delta / (\delta + \lambda_1 F(w)) \) and exit into another job with probability \( \lambda_2 F(w) / (\delta + \lambda_1 F(w)) \).

We assume that wages are measured with error. Hartog and Van Ophem (1991) provide evidence for the presence of measurement errors in the wage data in the OSA panel. This is particularly serious, because as shown in Van den Berg and Ridder (1993), the dependence of the support of \( F(w) \) on the parameters of the model implies that the ML estimates of the parameters are sensitive to measurement errors in the wage data. Similar problems arise in the structural estimation of partial job search models (see e.g. Wolpin (1987)). To deal with these problems, we assume that the observed wage \( \bar{w} \) equals the true wage \( w \) times an error term \( \epsilon \) which is i.i.d. across job spells and across
individuals, and which is independent of all other random variables in the model. Note that allowing for measurement errors also prevents the log likelihood to equal $-\infty$ if a transition from a job to a job with a lower wage is observed\(^4\). We assume that $\varepsilon$ has a log-normal distribution with mean 1 and $\text{var}(\log \varepsilon) = \sigma^2$.

Let $d_1$ denote a variable being one if $\tilde{w}$ is unobserved and zero otherwise. If $d_{0f}=1$ or $d_1=1$ then the individual is not followed any further, so in that case (4.1) gives the total individual likelihood contribution. Let $d_2=1$ if $t_1$ is right-censored and $d_2=0$ otherwise, and let $d_3=1$ if the destination following exit out of the job is unknown and $d_3=0$ otherwise. Finally, let $d_4=1$ if the destination is another job and $d_4=0$ if the destination is unemployment.

If $d_{0f}=0$ and $d_4=0$ and if the wages are measured without error, then the individual likelihood contribution $L_1$ of the events between entry into employment and exit out of the first job equals

$$L_1 = f(w) \cdot \exp (- (\delta + \lambda_1 F(w)) t_1) \cdot (\delta + \lambda_1 F(w)) d_2 (1 - d_2) \cdot (\lambda_1 F(w)) d_4 (1 - d_4) (1 - d_2)$$

with $w < \bar{w}, \tilde{w}$ (see Section 2 for the equations for $w$, $\bar{w}$, $f(w)$ and $F(w)$).

Now let $\varepsilon$ be a random drawing from the density $h(\varepsilon)$. If $d_{0f}=0$ and $d_1=0$ then

$$\tilde{w}/\bar{w}$$

$$L_1 = \int f(\tilde{w}/\varepsilon) \cdot \exp (- (\delta + \lambda_1 F(\tilde{w}/\varepsilon)) t_1) \cdot (\delta + \lambda_1 F(\tilde{w}/\varepsilon)) d_2 (1 - d_2)$$

$$\cdot (\lambda_1 F(\tilde{w}/\varepsilon)) d_4 (1 - d_2) (1 - d_3) (1 - d_2) \frac{1}{\varepsilon} h(\varepsilon) \, d\varepsilon$$

with $\tilde{w} < 0, \infty$.

To summarize, the total individual likelihood contribution for a respondent who is unemployed at the date of the first interview equals

$$L_0 \cdot L_1 (1 - d_{0f}) (1 - d_1)$$

The integral in equation (4.2) has to be evaluated numerically.

It is possible to use information on events occurring after completion of

\(^4\)In our sample, 11% of the observed job-to-job transitions result in a decrease of the observed wage.
However, note that the wage in a second job has to exceed the wage in the first one, so, conditional on the observed wages, the range of possible values of the second error term depends on the value of the first one. Because of this, the joint density of observed wages in consecutive job spells contains a multidimensional integral that is computationally demanding. For only 23 of the respondents who are unemployed at the first interview we observe more than one transition. Because of this, we have decided not to use information on events after exit from the first job after unemployment.

Now consider an individual who is employed at the date of the first interview. Under the assumptions made above, the probability of being employed at a randomly chosen date equals \( \lambda_0/(\delta+\lambda_0) \). Given that the individual is employed at the date of the interview, his wage \( w_1 \) at that date is a random draw from the distribution \( G(w) \) of paid wages. As before, we take \( \tilde{w}_1 = w_1 \cdot e_1 \). Let \( d_4 = 1 \) if \( \tilde{w}_1 \) is unobserved, and \( d_4 = 0 \) otherwise. If \( d_4 = 1 \) then the likelihood contribution is set to \( \lambda_0/(\delta+\lambda_0) \); otherwise it is constructed as follows.

Conditional on being employed in a job with a wage \( w_1 \) at the date of the first interview, the elapsed job duration \( t_{1b} \) and the residual job duration \( t_{1f} \) are i.i.d. and have an exponential distribution with parameter \( \delta+\lambda_1 F(w_1) \). Exit into unemployment occurs with probability \( \delta/(\delta+\lambda_1 F(w_1)) \) and exit into another job with probability \( \lambda_1 F(w_1)/(\delta+\lambda_1 F(w_1)) \). Let \( d_{6b} \) (\( d_{6f} \)) denote a variable being one if it is only known that the elapsed (residual) duration exceeds a certain value (i.e. is right-censored), and zero otherwise. Further, let \( d_7 = 1 \) if the destination following exit out of the job is unknown and \( d_7 = 0 \) otherwise, and let \( d_8 = 1 \) if the destination is another job and \( d_8 = 0 \) if it is unemployment.

Suppose that the destination is unemployment. The unemployment duration \( t_0 \) has an exponential distribution with parameter \( \lambda_0 \). We define \( d_9 = 1 \) if \( t_0 \) is right-censored, and \( d_9 = 0 \) otherwise. If the destination is another job, then the wage \( w_2 \) in the new job is a random draw from the wage offer distribution truncated from below at \( w_1 \), that is, from \( F(w)/F(w_1) \). Again, we set \( \tilde{w}_2 = w_2 \cdot e_2 \). The duration \( t_2 \) of the new job has an exponential distribution with parameter \( \delta+\lambda_2 F(w_2) \). Exit into unemployment occurs with probability \( \delta/(\delta+\lambda_2 F(w_2)) \) and exit into another job with probability \( \lambda_2 F(w_2)/(\delta+\lambda_2 F(w_2)) \). We define dummy variables \( d_{10}, d_{11}, d_{12} \) and \( d_{13} \) that indicate whether \( \tilde{w}_2 \) is unobserved, whether \( t_2 \) is right-censored, whether the destination state is unemployment, and whether the destination state is another job.

\[ ^5 \text{In the data, } d_5 = 0 \text{ for 92\% of the 1732 individuals employed at the date of the first interview.} \]
For computational reasons, we do not use information on events that occur after the completion of a second spell. As a consequence, the dimension of the (numerical) integral in the likelihood contribution is at most two. Further reduction of the dimension of the integral by deleting information on the job after a job-to-job transition would cause the loss of valuable information on the parameters of the job-to-job transition rate.

If $d_5=0$, and if there is no measurement error, then the individual likelihood contribution equals

$$
L = \frac{\lambda_0}{\delta + \lambda_0} \cdot g(w_1) \cdot (\delta + \lambda_1 F(w_1))^{1-d_6} \cdot \exp\left(-\left((\delta + \lambda_2 F(w_1)) \cdot (t_{15} + t_{12})\right) \cdot (\delta + \lambda_0 (1-d_5) \cdot \exp(-\lambda_0 t_0))^{(1-d_5)(1-d_7)(1-d_6)}
$$

(4.3)

$$
\cdot \left[ \lambda_2 F(w_1) \cdot \left[ \frac{f(w_2)}{F(w_1)} \cdot (\delta + \lambda_3 F(w_2))^{d_{12}} (1-d_{11}) \cdot \exp(-((\delta + \lambda_2 F(w_2)) \cdot t_2) \cdot \delta^{(1-d_{11})(1-d_{12})(1-d_{11})})
\cdot (\lambda_3 F(w_2))^{d_{12}} (1-d_{12})(1-d_{11}) \right] (1-d_{19}) \right] (1-d_{19}) \right]
$$

with $w_1 < w_2$ and $w_2 < w_3$. Now let $e_1$ and $e_2$ be independent random drawings from the density $h(e)$. If $d_5=0$ then $L$ can be rewritten as in (4.2), with a bivariate integral over $e_1$ (ranging from $\tilde{w}_1/\tilde{w}$ to $\tilde{w}_1/w$) and $e_2$ (ranging from $\tilde{w}_1/\tilde{w}$ to $\tilde{w}_2/\tilde{w}_1$) and with $0 < \tilde{w}_1, \tilde{w}_2 < \infty$. This bivariate integral must be computed numerically. Apart from the area of integration, the integral factorizes in two one-dimensional integrals. This can be exploited to increase the speed of the numerical calculation.

There are some simple checks on the specification of the model. Suppose that the estimate of the standard deviation of $e$ is relatively large. Then a large fraction of the variation in the wages cannot be explained by the model. Consequently, one may conclude that the model is not adequate. Further, by re-estimating the model using only subsets of the observed endogenous variables, various parts of the specification can be tested in a natural way.

Mortensen (1990), Kiefer and Neumann (1991) and Van den Berg and Ridder (1993) extensively discuss the identification of the model of Section 2. Data on unemployment durations, job durations, destination states following exit out of a job and accepted wage offers suffice to identify the model parameters. Note that we observe more endogenous variables than this.

Because the measurement error in the wages makes the support of the
distributions of observed wages independent of the parameters, the ML estimation of the model is standard. If there are no measurement errors, the support of $F$ and $G$ depends on unknown parameters, so ML estimators have non-standard properties. Kiefer and Neumann (1991) suggest to use order statistics to estimate the bounds of the support. The parameters in these bounds can then be estimated from these superconsistent estimates. We do not follow this suggestion because of the sensitivity of the resulting estimates to outliers and measurement errors, and because this method cannot deal easily with (un)observed population heterogeneity.

4.2. Heterogeneity

We introduce heterogeneity by assuming that there are separate labour markets (or segments of the labour market) for different types of individuals and firms. For example, there may be separate markets for individuals with different educational backgrounds. Furthermore, for each level of education, there may be separate markets for different age categories. To deal with this type of heterogeneity, the model can be estimated separately for each labour market, using only those individuals that belong to the labour market at hand. However, the number of individuals per market may be very small. In the empirical analysis below we distinguish separate labour markets by level of education, age, and occupation level (defined by the complexity of the job). For each of these three variables we distinguish four levels, so that we have 64 segments. Six of these do not contain any respondents. For only 34 segments we observe more than 10 individuals.

Instead of estimating separate models for the 58 (64 minus 6) segments, we assume that the deep structural parameters in the model ($p$, $\lambda_0$, $\lambda_1$, and $\delta$) vary over the different labour markets in a fairly regular way that can be captured by simple parametric functions. Then we can estimate the parameters using the data on all markets simultaneously. In other words, we estimate the models for all markets simultaneously. Let $x$ be the vector of age, education, and occupation dummies (and a constant, so $x$ contains 10 elements). We assume that $p$, $\lambda_0$, $\lambda_1$, and $\delta$ are log-linear functions of $x$.

---

6 We distinguish between the following levels of education: (1) at most lower secondary education, (2) medium or higher secondary education, (3) higher vocational training, (4) university. The occupation levels refer to a categorization based on a five-digit occupation characterization.
\begin{align}
\lambda_0 &= \exp(\beta_2'x) \\
\delta &= \exp(\beta_4'x)
\end{align}

The only restriction in comparison to separate estimation for each market is that in \( \lambda_0, \lambda_1, \) and \( \delta \) there are no interactions between age and educational and occupational level. In the estimation we merge the two highest levels of education and the two highest occupation levels in \( \lambda_1 \) and \( \delta \) (so \( \beta_3 \) and \( \beta_4 \) contain 8 free parameters), because of computational problems encountered when estimating the unrestricted version.\footnote{For the highest education and occupation categories, which are the smallest categories, transitions from employment to unemployment are rarely observed. Moreover, as will be shown below, the information in the wage data does not contribute much to the estimation of the arrival rate parameters.}

Variables like gender or non-wage income do not define separate labour markets, and hence cannot be included in \( x \). Allowing structural parameters like \( \lambda_1 \) to depend on a range of personal characteristics that do not characterize different segments of the labour market introduces heterogeneity within a labour market, and the result is a different model with a different equilibrium wage offer distribution. So, not every parameterization of our model makes sense. We replicated the empirical analysis below using four type of industry dummies instead of the four occupation dummies, but the estimates of the corresponding parameters turned out to be insignificant.

The variables in \( x \) divide the labour market into segments on the basis of observed characteristics. However, this segmentation may not be sufficiently detailed. For example, within each category defined by age and levels of education and occupation, there may be separate segments. They can be distinguished by unobserved heterogeneity variables \( v \) affecting the structural parameters. It is important to include this into the model. In particular, as noted in Section 2, allowing for sufficient heterogeneity in the productivity level \( p \) improves the fit to the wage data. We therefore replace the specification for \( p \) in (4.4) by

\begin{align}
(4.5) \quad p &= v \exp(\beta_1'x)
\end{align}

We assume that \( v \) has a discrete distribution with three unknown points of support. The family of discrete distributions is attractive for reasons of flexibility as well as for computational reasons. We experimented with additional points of support, but during the ML procedure these sometimes converged to existing ones while at other times they caused the \( p \) for certain
values of $x$ and $v$ to converge to $w_L$. We denote the three points of support by $v_1<v_2<v_3$, and the corresponding probabilities by $\exp(q_i)/(\sum \exp(q_i))$, with normalization $q_3=0$. The parameter in $\beta_1$ corresponding to the constant in $x$ is normalized to zero.

The distribution of $v$ does not depend on $x$, so that (4.5) effectively increases the number of segments by a factor equal to the number of points of support of $v$. Hence, each segment as defined by $x$ consists of three subsegments. As a consequence, $F$ and $G$ (conditional on $x$) are mixtures of $F$ and $G$ in the model without heterogeneity (see Eckstein and Wolpin (1992) for a similar approach in an equilibrium matching model). The likelihood is simply obtained by integrating the likelihood derived in Subsection 4.1 over the discrete distribution of $v$. In Section 5 we also examine whether the other structural parameters depend on $v$. Those extensions turn out to be empirically uninteresting.

For each segment, the benefits level $b$ is taken to be the corresponding predicted value of a regression of the observed log unemployment benefits on the dummy variables defining the segments. Alternatively, one could treat $b$ as an unknown parameter with a parameterization analogous to (4.4). From the information in the survey, the mandatory minimum wage $w_L$ can be calculated for each respondent. The resulting values vary slightly within segments. In the empirical analysis, we take the sample average per segment. In Section 5 we argue that (within bounds) the way in which $b$ and $w_L$ are calculated does not substantially alter the main results.

5. Results

5.1. Parameter estimates and their implications

We estimated the model by ML, using the BHHH algorithm with analytical derivatives. The univariate and bivariate integrals in the likelihood are evaluated numerically using Gauss-Legendre quadrature. The parameter estimates for the model with only observed heterogeneity are in Table 1. Time and money are measured in months and Dutch Guilders, respectively. For the education, age, and occupation level dummies, the reference categories are level of education 1, age 16-22, and occupation level 1, respectively.

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\[ This \text{ can be interpreted as suggesting that in reality the location of the mass points of } v \text{ varies over the segments (i.e. is dependent on } x), \text{ contrary to what is assumed. } \]
Given the parameter estimates, we can calculate estimates of \( p \), \( \lambda_0 \), \( \lambda_1 \), \( \delta \), \( r \), \( \bar{w} \), and the mean of \( F(w) \) for every labour market segment. The sample averages of these estimates are listed in Table 2. Because the effect of age is particularly pronounced, we also list averages per age category. Generally, the values of these averages seem to be in accordance with intuition. All three rates (\( \lambda_0 \), \( \lambda_1 \), \( \delta \)) are monotonically decreasing in age. For every age category \( \delta < \lambda_0 < \lambda_1 \). Because \( \delta \) is much smaller than \( \lambda_1 \), \( \bar{w} \) is very close to \( p \). If \( \delta/\lambda_1 \) tends to zero, i.e. if the number of offers during a spell of employment is very large, workers climb the wage ladder with high speed, and the resulting competition between firms causes \( F(w) \) and \( G(w) \) to tend to a degenerate distribution at \( p \).

Because in most segments \( \lambda_0 \) is smaller than \( \lambda_1 \), most unemployed individuals have a reservation wage \( r < b \). In other words, most unemployed individuals are willing to accept a job with a wage equal to or even below the benefits level. As noted in Section 2, this is in line with previous empirical results based on partial job search models. Further, in most segments \( r < w_L \), so in general the lowest wage offer \( w \) equals the mandatory minimum wage. As a result, in most segments there holds that \( r < b < w_L = w < \bar{w} < p \), although given the age category there are also segments with a different ordering.

The equilibrium wage offer distribution implies that employers have monopsony power. The measure of monopsony power \( \mu \) in Table 2 is defined for each segment as

\[
\mu = \frac{p - E_F(w)}{E_F(w)} \quad \text{with} \quad E_F(w) = p - (p-w) \cdot \frac{(\delta^2 + \delta \lambda_1 + \lambda_1^2)}{(\delta + \lambda_1)^2}
\]

In traditional monopsony models of the labour market, \( (p-w)/w \) is used as a measure of the monopsony power of the firm. In the present context, wages are dispersed, so (5.1) merely gives an indication of the average monopsony power of firms. It should be noted that in traditional models as well as in the present model, \( w/(p-w) \) equals the elasticity of the steady state workforce of the firm with respect to the wage offered by the firm (Machin and Manning (1991)). So, \( \mu \) can also be interpreted as the relative increase in \( w \) needed for a 1% increase in the workforce of the firm, evaluated at \( w = E_F(w) \).

Monopsony power is not very strong in any segment. This can be explained by the fact that in general search frictions in employment are small, so individuals move relatively fast to jobs with high wages. Still, firms offer wages that are on average 13% below the competitive wage.

Table 2 also gives the steady-state levels of unemployment \( u/m \). Except for
the age category 16-22 these are close to the national statistics for the mid-80s. For young individuals the assumption of a constant inflow rate into employment prior to the date of the first interview (which follows from the equilibrium CTMC assumption in Subsection 4.1) may be severely violated because these individuals left school not very long before that interview. In that case \( \delta \) is underestimated from the employment durations ongoing at the date of the first interview, and \( v/m \) is overestimated.

The results can be used to calculate more statistics than those presented in Table 2. For example, \( ((\delta + \lambda_1)/\delta)^2 \) equals the ratio \( \ell(\bar{w})/\ell(w) \) of the workforces of the largest and smallest firms in a segment.

We performed some simple sensitivity checks on the specification of the model. First, we deleted the information on the accepted wages after a job-to-job transition. In the notation of Section 4, this means that we set \( d_{10} = 1 \) for all individuals, so information on behaviour after such transitions is not used. If the distribution of accepted wages from a job differs from the distribution of accepted wages from unemployment, we expect changes in the estimates. However, the estimates (not reported) do not differ much from those in Tables 1 and 2. The latter conclusion also holds for estimation results obtained for the model in which wage measurement errors are additive (so \( \tilde{w} = w + \varepsilon \)) and normally distributed. We also estimated the model using observed unemployment durations only. This amounts to estimating a reduced-form duration model with the duration having an exponential distribution with parameter \( \exp(\beta x) \). It turns out that the parameter estimates are virtually equal to those for \( \lambda_0 \) in Table 1.

The estimate of \( \sigma^2 \) implies that in 95% (5%) of all cases the observed wage is within (outside of) a range of 41% of the true wage. Table 3 decomposes the total variation in observable wage offers into three components due to (i) the wage measurement error variation, (ii) the observed between-market variation, and (iii) the within-market variation due to the matching process. These components are computed by successive conditioning of \( \text{var}(\tilde{w}) \) on \( \varepsilon \) and \( x \), using the estimated \( F \) and \( \sigma \) and the empirical distribution of \( x \) to calculate the values of the components. In formula,

\[
\text{var}(\tilde{w}) = [E_x(E_F(w|x))]^2 \text{var}_x(\varepsilon) + E_\varepsilon(\varepsilon^2) \text{var}_x(E_F(w|x)) + \\
E_\varepsilon(\varepsilon^2)E_x(\text{var}_F(w|x))
\]

Because \( \tilde{w} \) is a nonlinear function of \( \varepsilon \) and \( x \), the decomposition is dependent on the order of the successive conditioning. However, the results are not
sensitive to this order.

The model itself explains only 19% of the variation in observable wage offers. This is a consequence of the fact that the parameter estimates imply that \( F(w) \) is concentrated near \( p \). Recall that this in turn is a consequence of the fact that the estimate of \( \lambda_1 / \delta \) is quite large. In the data, transitions from one job to another are much more frequently observed than transitions from a job to unemployment. Apparently, the information in the duration data dominates the information in the wage data (Eckstein and Wolpin (1990) arrive at the same conclusion in the estimation of a wholly different equilibrium search model).

Even though we allow for differences across segments as defined by observables \( x \), a large fraction of the variation in observable wage offers is "explained" by measurement errors. However, as noted below equation (2.12), allowing for sufficient unobserved heterogeneity in \( p \) improves the fit to the wage data. In Table 4 we report estimation results for the model with unobserved heterogeneity in \( p \). Except for the the estimate of \( \sigma^2 \), the estimates do not differ much from Table 1. The estimated distribution of \( v \) is skewed to the right. As a consequence, the fractions of workers affiliated to segments associated with \( v=v_1 \), \( v=v_2 \) and \( v=v_3 \) are 81%, 16% and 3%, respectively. The segments associated with \( v_2 \) (\( v_3 \)) have values of \( p \) that are 36% (95%) higher than that of the segments associated with \( v_1 \).

Table 5 shows that the characteristics of the equilibrium (averaged over the distribution of \( v \)) are almost identical to those in Table 2, with the main exception of \( \lambda_1 \) which is smaller than in Table 2. As a result, \( \lambda_1 \) is fairly close to \( \lambda_0 \), so search in employment is about as effective as in unemployment. This in turn implies that \( r \) and \( b \) generally do not differ much from each other. Because \( b \) and \( w_L \) are the same as in the model without unobserved heterogeneity, this means that again \( \mu = w_L \) for most segments.

The monopsony power averaged over the distribution of \( v \) is the same as in Table 2. Segments with \( v=v_1 \) have a value of \( \mu \) which is generally smaller than the value of \( \mu \) for segments with \( v=v_2 \) or \( v=v_3 \).

The inclusion of unobserved heterogeneity improves the fit of the model substantially. When going from the model without unobserved heterogeneity to the model in which \( v \) has a discrete distribution with at most 2 points of support (which amounts to adding 2 parameters), the log likelihood increases with 105 points (from -26425 to -26320). When increasing the maximum number of points of support from 2 to 3 (which again amounts to adding 2 parameters), the log likelihood increases further with 14 points. In the latter model, the estimate of \( \sigma^2 \) is about half of that in the model without unobserved
heterogeneity. As a result, in 95% (5%) of all cases the observed wage is within (outside of) a range of 29% of the true wage.

The estimates allow us to decompose the total variation in observable wage offers into four components due to (i) the wage measurement error variation, (ii) the observed between-market variation, (iii) the between-market variation due to unobserved heterogeneity in \( p \), and (iv) the within-market variation due to the matching process (see Table 3). The approach is similar to that for the model without unobserved heterogeneity (see equation (5.2) above). In formula,

\[
\text{var}(\tilde{w}) = \left[ \text{E}_x \text{E}_v \text{E}_f(w|x,v) \right]^2 \text{var}_\varepsilon(\varepsilon) + \text{E}_\varepsilon(\varepsilon^2) \text{var}_x(\text{E}_v \text{E}_f(w|x,v)) \\
+ \text{E}_\varepsilon(\varepsilon^2) \text{E}_x(\text{var}_v(\text{E}_f(w|x,v))) + \text{E}_\varepsilon(\varepsilon^2) \text{E}_v(\text{var}_f(w|x,v))
\]

(5.3)

Clearly, the variation due to \( \varepsilon \) is much smaller than in the model without unobserved heterogeneity. The between-market variation due to unobserved heterogeneity in \( p \) constitutes a substantial part of the wage variation. In conclusion, the model with unobserved heterogeneity in \( p \) seems to be a more satisfactory model.

We also estimated model versions with unobserved heterogeneity in the arrival rates \( \lambda_0, \lambda_1 \) and \( \delta \). Negative duration dependence of an observed exit rate can be explained by the presence of unobserved heterogeneity, so allowing for the latter may improve the fit to the duration data. Prior to the structural estimation we estimated simple reduced-form duration models for the transition rates from unemployment to employment and vice versa and from one job to other jobs. Only for the transition rate from unemployment to employment did we find evidence for negative duration dependence. This suggests that there is unobserved heterogeneity in \( \lambda_0 \) but not in \( \lambda_1 \) and \( \delta \). Because of the complicated way in which most parameters affect \( F \), the analysis of relations between the duration spent in a state and the accepted wage after leaving that state does not give insights into sources of heterogeneity.

In the structural estimation we allowed for between-market unobserved heterogeneity in the arrival rates, and we took specifications analogous to (4.5) with discrete distributions for the unobserved heterogeneity terms. The results are in accordance to the reduced-form results mentioned in the previous paragraph. We found evidence for unobserved heterogeneity in \( \lambda_0 \) with two points of support (the log likelihood in the model without unobserved heterogeneity).

\[9\] The fact that job durations do not display negative duration dependence is in line with previous studies based on Dutch data (Lindeboom and Theeuwes (1991), Van den Berg (1992)).
heterogeneity in \( p \) equals -26415). The other estimates are virtually identical to those in Table 1. As can be expected, this extension does not improve the fit to the wage data. The unobserved heterogeneity in \( \lambda_0 \) can explain only 2% of the variation in observable wage offers. The ML estimates of the unobserved heterogeneity distributions for \( \lambda_1 \) and \( \delta \) turned out to be degenerate.

The model does not allow for an unobserved component in \( b \), let alone for unobserved heterogeneity in \( b \). However, it is questionable whether such a generalization would improve the fit to the wage data. The value of \( b \) affects \( F \) and \( G \) by way of the lower bound of their support. Given that the duration information dominates the wage information, and that the resulting estimate of \( \lambda_1/\delta \) is very large, most probability mass of \( F \) and \( G \) is concentrated close to \( p \), and there is virtually no mass in the left tail of these distributions. Consequently, changing the value of \( b \) (within bounds) for some or all individuals will not affect the fit (note that therefore small biases in the values of \( b \) and \( \omega_L \) will not affect the fit either). For similar reasons, it is not clear whether allowing for within-market heterogeneity can improve the fit to the wage data very much. On the other hand, as will be argued below, such generalizations may generate different policy implications.

The model of Eckstein and Wolpin (1990) can be estimated with duration data only. This provides a natural specification test. For our model such an approach is not natural because in our model the exit rate out of the job depends on the wage which is observed with error.

5.2. Effect of a change in the minimum wage

It is interesting to examine the effect of an increase of the legal minimum wage on equilibrium unemployment in the present context. As mentioned in Burdett and Mortensen (1993), the imposition of a minimum wage \( \omega_L \) does not affect equilibrium unemployment as long as \( \omega_L < p \). A minimum wage exceeding the reservation wage \( r \) merely redistributes part of the rent of the match from the firm to the worker, or, in other words, it decreases the monopsony power of the firm. In a segmented labour market consisting of segments with different productivity levels, the imposition of a minimum wage \( \omega_L \) exceeding the productivity level \( p \) of a particular segment makes all firms in that segment unprofitable. All individuals associated with segments for which \( p < \omega_L \) are permanently unemployed (or, perhaps more accurately, are nonparticipant on the
labour market). So, in a segmented labour market the minimum wage causes a trade-off between monopsony power and unemployment.

It turns out that a 25% increase of the legal minimum wage makes 7 segments unprofitable on a total of 174 segments (the data contain 58 different types of individuals in terms of their x value, and for each x value there are 3 possible v values). These segments together contain 16% of all individuals (see Table 6). So, a 25% increase of the minimum wage makes 16% of the workers permanently unemployed. Most of the individuals affected by the 25% increase of \( w_L \) are between 22 and 30 years of age. Not surprisingly, this is also the age group for which the corresponding labour market segments display the weakest monopsony power \( \mu \) of the firms.

Note that because the number of segments is finite, the size of the effect does not vary smoothly with the size of the change of \( w_L \). The size of the effect depends on the distribution of the levels of \( p \), and a more smooth specification of this distribution should give a more accurate estimate of the effect of small changes of \( w_L \). Now suppose that the over-all shape of the distribution of \( p \) above \( w_L \) can be extrapolated to values of \( p \) just below \( w_L \). Then our results suggest that unemployment of individuals aged in their twenties can be reduced in the long run if their minimum wage is reduced. (Such reasoning is similar to that applied in structural empirical analyses of partial job search models when addressing the effects of downward shifts in \( b \).)

In a labour market with considerable observed and unobserved heterogeneity in \( p \), a uniform minimum wage can easily cause unemployment, and this can only be avoided by making the minimum wage dependent on \( p \). Imposing a minimum wage can make firms unprofitable in the Albrecht-Axell model as well. However, in the single labour market of that model, workers can shift from low-productivity to high-productivity firms. In the heterogeneous Burdett-Mortensen model such shifts cannot occur, so that the two models represent polar cases. A synthesis of these models would improve our insight into the effect of the minimum wage on unemployment.

A change of the benefits level \( b \) has the same qualitative implications for unemployment as a change of \( w_L \). If the new value of \( b \) exceeds the productivity level \( p \) of a particular segment, then all individuals in that segment become permanently unemployed. It turns out that even a 25% increase of the benefits levels in all segments keeps \( b \) well below the corresponding values of \( p \). So,

\[ \text{(It should be noted that in tables 2 and 4 for every segment the estimated value of } p \text{ exceeds } w_L. \]
moderate changes of $b$ do not affect unemployment. This is in line with the results cited in Section 2.

6. Conclusion

In this paper we have estimated the Burdett–Mortensen equilibrium search model of the labour market. In this model, the distributions of wage offers and earnings are endogenous and non-degenerate. We allowed for observed and unobserved heterogeneity across different segments of the labour market. It turned out that the possibility of on-the-job search has a distinctive effect on the equilibrium wage offer and earnings distributions. Because the job offer arrival rate while employed is much larger than the layoff rate, workers can climb the wage ladder with high speed, and as a result most wages within a particular segment are concentrated close to the marginal revenue product. As a consequence, the homogeneous model explains about 20% of the variation in observable wage offers. Observed and unobserved heterogeneity in productivity levels across segments turned out to be the other main determinant of wages.

The results were used to examine the effects of changes in the mandatory minimum wage on unemployment. Because for most individuals aged in their twenties the productivity level is close to the present mandatory minimum wage, changing the latter has a large impact on the level of unemployment for those individuals.

The model can explain most of the observed wage variation as due to either the process of labour market search or to heterogeneity in the parameters, in particular in $p$. Hence, we are more optimistic than Eckstein and Wolpin who conclude that in their model the wage variation is mainly "explained" by measurement error. Of course, a synthesis of the two models is desirable. Mortensen (1990) discusses such a synthesis. A potential problem is that uniqueness of equilibrium in the synthesis has not been proven. As argued in Section 5, empirical analysis of such a synthesis may increase our understanding of the effect of policy interventions as a change in the mandatory minimum wage.

Finally, it is clear the demand side of the model is not very realistic. In particular, the model assumes a stationary environment, and thus it does not allow for external shocks that lead to the creation or destruction of jobs at firms. Although incorporation of this would make the model very complicated, it seems important to direct future research to this as well.
Table 1. Estimates for the equilibrium search model: observed heterogeneity (standard errors).

<table>
<thead>
<tr>
<th>parameter</th>
<th>$p$</th>
<th>$\lambda_0$</th>
<th>$\lambda_1$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>7.17 (0.03)</td>
<td>-2.94 (0.14)</td>
<td>-2.23 (0.47)</td>
<td>-4.24 (0.14)</td>
</tr>
<tr>
<td>education level 2</td>
<td>0.06 (0.02)</td>
<td>0.31 (0.11)</td>
<td>0.34 (0.27)</td>
<td>-0.02 (0.11)</td>
</tr>
<tr>
<td>education level 3</td>
<td>0.16 (0.02)</td>
<td>0.46 (0.16)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>education level 4</td>
<td>0.27 (0.03)</td>
<td>0.37 (0.22)</td>
<td>0.46 (0.32)</td>
<td>0.22 (0.15)</td>
</tr>
<tr>
<td>age category 23–29</td>
<td>0.24 (0.03)</td>
<td>-0.41 (0.16)</td>
<td>-0.26 (0.46)</td>
<td>-0.80 (0.14)</td>
</tr>
<tr>
<td>age category 30–38</td>
<td>0.39 (0.03)</td>
<td>-0.95 (0.17)</td>
<td>-0.96 (0.46)</td>
<td>-1.56 (0.15)</td>
</tr>
<tr>
<td>age category 39–70</td>
<td>0.47 (0.03)</td>
<td>-1.53 (0.16)</td>
<td>-1.67 (0.45)</td>
<td>-2.13 (0.14)</td>
</tr>
<tr>
<td>occupation level 2</td>
<td>0.06 (0.02)</td>
<td>0.05 (0.13)</td>
<td>-0.44 (0.34)</td>
<td>-0.02 (0.11)</td>
</tr>
<tr>
<td>occupation level 3</td>
<td>0.14 (0.02)</td>
<td>-0.16 (0.18)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>occupation level 4</td>
<td>0.30 (0.02)</td>
<td>0.25 (0.20)</td>
<td>-0.61 (0.35)</td>
<td>0.04 (0.15)</td>
</tr>
</tbody>
</table>

$\sigma^2$                  | 0.0447 (0.0013) |

log likelihood = -26425
Table 2. Characteristics of the equilibrium: observed heterogeneity.

<table>
<thead>
<tr>
<th>age category:</th>
<th>16–22</th>
<th>23–29</th>
<th>30–38</th>
<th>39–70</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p ) (productivity)</td>
<td>1435</td>
<td>1941</td>
<td>2353</td>
<td>2532</td>
<td>2208</td>
</tr>
<tr>
<td>( \lambda_0 ) (arrival rate in unemployment)</td>
<td>0.065</td>
<td>0.047</td>
<td>0.029</td>
<td>0.016</td>
<td>0.033</td>
</tr>
<tr>
<td>( \lambda_1 ) (arrival in employment)</td>
<td>0.095</td>
<td>0.075</td>
<td>0.037</td>
<td>0.018</td>
<td>0.047</td>
</tr>
<tr>
<td>( \delta ) (separation rate)</td>
<td>0.014</td>
<td>0.007</td>
<td>0.003</td>
<td>0.002</td>
<td>0.005</td>
</tr>
<tr>
<td>( r ) (reservation wage)</td>
<td>607</td>
<td>756</td>
<td>1038</td>
<td>1226</td>
<td>982</td>
</tr>
<tr>
<td>( \bar{w} ) (highest wage)</td>
<td>1426</td>
<td>1937</td>
<td>2345</td>
<td>2521</td>
<td>2200</td>
</tr>
<tr>
<td>( E F(w) ) (mean wage offer)</td>
<td>1267</td>
<td>1761</td>
<td>2023</td>
<td>2153</td>
<td>1917</td>
</tr>
<tr>
<td>( w ) (lowest wage)</td>
<td>999</td>
<td>1449</td>
<td>1450</td>
<td>1506</td>
<td>1420</td>
</tr>
<tr>
<td>( b ) (benefits level)</td>
<td>807</td>
<td>1120</td>
<td>1248</td>
<td>1320</td>
<td>1192</td>
</tr>
<tr>
<td>( \mu ) (monopsony power)</td>
<td>0.13</td>
<td>0.10</td>
<td>0.16</td>
<td>0.17</td>
<td>0.14</td>
</tr>
<tr>
<td>( u/m ) (unemployment)</td>
<td>0.18</td>
<td>0.13</td>
<td>0.10</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td># observations</td>
<td>212</td>
<td>494</td>
<td>595</td>
<td>648</td>
<td>1949</td>
</tr>
</tbody>
</table>
Table 3. Decomposition of total variation in observable wage offers.

<table>
<thead>
<tr>
<th>model:</th>
<th>observed heterogeneity</th>
<th>observed and unobserved heterogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td>due to $\varepsilon$</td>
<td>44%</td>
<td>23%</td>
</tr>
<tr>
<td>due to $x$</td>
<td>37%</td>
<td>33%</td>
</tr>
<tr>
<td>due to $v$ in $p$</td>
<td>-</td>
<td>21%</td>
</tr>
<tr>
<td>due to $w$</td>
<td>19%</td>
<td>22%</td>
</tr>
</tbody>
</table>

Tables 4 and 5: see next pages.

Table 6. The percentage of individuals becoming unemployed when the minimum wage is increased.

<table>
<thead>
<tr>
<th>model:</th>
<th>observed heterogeneity</th>
<th>observed and unobserved heterogeneity</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% increase of $w_L$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>0%</td>
<td>3%</td>
</tr>
<tr>
<td>age category 16-22</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>age category 23-29</td>
<td>0%</td>
<td>10%</td>
</tr>
<tr>
<td>age category 30-38</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>age category 39-70</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>25% increase of $w_L$:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>11%</td>
<td>16%</td>
</tr>
<tr>
<td>age category 16-22</td>
<td>0%</td>
<td>15%</td>
</tr>
<tr>
<td>age category 23-29</td>
<td>42%</td>
<td>56%</td>
</tr>
<tr>
<td>age category 30-38</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>age category 39-70</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>
Table 4. Estimates equilibrium search model: observed and unobserved heterogeneity (standard errors).

<table>
<thead>
<tr>
<th>parameter</th>
<th>( p )</th>
<th>( \lambda_0 )</th>
<th>( \lambda_1 )</th>
<th>( \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>0 (-)</td>
<td>-2.92 (0.13)</td>
<td>-2.46 (0.37)</td>
<td>-4.21 (0.14)</td>
</tr>
<tr>
<td>education level 2</td>
<td>0.06 (0.02)</td>
<td>0.31 (0.10)</td>
<td>0.28 (0.22)</td>
<td>-0.02 (0.11)</td>
</tr>
<tr>
<td>education level 3</td>
<td>0.14 (0.02)</td>
<td>0.51 (0.15)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>education level 4</td>
<td>0.24 (0.03)</td>
<td>0.53 (0.20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>age category 23–29</td>
<td>0.25 (0.03)</td>
<td>-0.40 (0.16)</td>
<td>-0.40 (0.37)</td>
<td>-0.78 (0.14)</td>
</tr>
<tr>
<td>age category 30–38</td>
<td>0.42 (0.03)</td>
<td>-0.95 (0.16)</td>
<td>-1.20 (0.36)</td>
<td>-1.52 (0.15)</td>
</tr>
<tr>
<td>age category 39–70</td>
<td>0.47 (0.03)</td>
<td>-1.53 (0.15)</td>
<td>-1.75 (0.36)</td>
<td>-2.11 (0.14)</td>
</tr>
<tr>
<td>occupation level 2</td>
<td>0.06 (0.02)</td>
<td>0.05 (0.13)</td>
<td>-0.46 (0.26)</td>
<td>-0.00 (0.11)</td>
</tr>
<tr>
<td>occupation level 3</td>
<td>0.11 (0.02)</td>
<td>-0.19 (0.18)</td>
<td>-0.27 (0.29)</td>
<td>-0.01 (0.15)</td>
</tr>
<tr>
<td>occupation level 4</td>
<td>0.26 (0.02)</td>
<td>0.13 (0.18)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \log v_1 )</td>
<td>7.10 (0.03)</td>
<td></td>
<td>3.35 (0.30)</td>
<td></td>
</tr>
<tr>
<td>( \log v_2 )</td>
<td>7.41 (0.04)</td>
<td></td>
<td>1.72 (0.31)</td>
<td></td>
</tr>
<tr>
<td>( \log v_3 )</td>
<td>7.77 (0.05)</td>
<td></td>
<td>0 (-)</td>
<td></td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>0.0220 (0.0013)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( \log \) likelihood = -26306
Table 5. Characteristics of the equilibrium: observed and unobserved heterogeneity.

<table>
<thead>
<tr>
<th>age category:</th>
<th>16–22</th>
<th>23–29</th>
<th>30–38</th>
<th>39–70</th>
<th>average</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p ) (productivity)</td>
<td>1446</td>
<td>1951</td>
<td>2392</td>
<td>2509</td>
<td>2216</td>
</tr>
<tr>
<td>( \lambda_0 ) (arrival rate in unemployment)</td>
<td>0.066</td>
<td>0.048</td>
<td>0.029</td>
<td>0.016</td>
<td>0.033</td>
</tr>
<tr>
<td>( \lambda_1 ) (arrival in employment)</td>
<td>0.075</td>
<td>0.055</td>
<td>0.025</td>
<td>0.015</td>
<td>0.035</td>
</tr>
<tr>
<td>( \delta ) (separation rate)</td>
<td>0.015</td>
<td>0.007</td>
<td>0.003</td>
<td>0.002</td>
<td>0.005</td>
</tr>
<tr>
<td>( r ) (reservation wage)</td>
<td>743</td>
<td>1022</td>
<td>1352</td>
<td>1400</td>
<td>1218</td>
</tr>
<tr>
<td>( \bar{w} ) (highest wage)</td>
<td>1433</td>
<td>1944</td>
<td>2379</td>
<td>2495</td>
<td>2204</td>
</tr>
<tr>
<td>( E_f(w) ) (mean wage offer)</td>
<td>1268</td>
<td>1762</td>
<td>2046</td>
<td>2130</td>
<td>1917</td>
</tr>
<tr>
<td>( w ) (lowest wage)</td>
<td>999</td>
<td>1450</td>
<td>1475</td>
<td>1502</td>
<td>1426</td>
</tr>
<tr>
<td>( b ) (benefits level)</td>
<td>807</td>
<td>1120</td>
<td>1248</td>
<td>1320</td>
<td>1192</td>
</tr>
<tr>
<td>( \mu ) (monopsony power)</td>
<td>0.13</td>
<td>0.10</td>
<td>0.16</td>
<td>0.17</td>
<td>0.14</td>
</tr>
<tr>
<td>( u/m ) (unemployment)</td>
<td>0.19</td>
<td>0.13</td>
<td>0.11</td>
<td>0.11</td>
<td>0.12</td>
</tr>
<tr>
<td># observations</td>
<td>212</td>
<td>494</td>
<td>595</td>
<td>648</td>
<td>1949</td>
</tr>
</tbody>
</table>