Mathematical Formalization and the Analysis of Cantillon Effects

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1. Introduction

The 1970s saw the rise of two research programmes, namely the Austrian School and New Classical Economics.² Both programmes have given considerable attention to monetary explanations of business cycles. The Austrian revival aims at elaborating the ideas of Ludwig von Mises and Friedrich von Hayek. In the early 1930s the latter's business cycle theory was one of the main explanations of the business cycle, but it was virtually swept aside by the Keynesian revolution. It emphasized distributional effects in transition (disequilibrium) periods, which arise because of the non-neutrality of money. New Classical Economics can be regarded as a strong version of Monetarism. It draws on the ideas of Friedman (1968) and Phelps (1967), in the sense that it holds that in a situation of complete information (i.e. in the 'long run') money is neutral. Business cycles result from imperfect knowledge and imperfect foresight of economic agents.

The respective Austrian and New Classical explanations of business cycles closely resemble each other.³ Both deal with rational economic agents who make decisions in a flexible-price environment and who must deal with imperfect knowledge and imperfect foresight. Furthermore, both stress the importance of economy-wide interdependencies. This has led some economists to conclude that New Classics elaborate on the work of Mises and particularly Hayek, so that the Austrian revival and New Classical Economics are variations on a theme.⁴ Butos (1986) opined that this conclusion is correct though misleading, because it neglects some important methodological differences. New Classical methodology prescribes that economic theories should be formulated as mathematical models. Lucas (1980) defined a theory as "... an explicit set of in-

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² We shall use the Lakatosian term 'research programme' in a loose, indicative way, i.e., without specifying a hard core, protective belt, and positive and negative heuristic. Nor shall we appraise the growth of knowledge in both 'programmes'.

³ We shall limit our analysis to monetary explanations of business cycles.

⁴ For instance, see Kantor (1979), Colander and Guthrie (1980), Laidler (1982) and Schefd (1986).
structions for building a parallel or analogue system - a mechanical, imitation economy" (p. 272). He opined that scientific analysis presupposes mathematical formalization. In contrast, Hayek increasingly recognized the limitations of comparative-static general-equilibrium analysis. This also applies to the mathematical versions of such analysis, so that he was somewhat sceptical about the usefulness of mathematical formalization. In general, Austrians criticize mathematical formalization on at least three accounts. Firstly, they argue that in principle mathematical techniques cannot successfully deal with key features of economic reality, because mathematical models cannot adequately incorporate the role of entrepreneurship and Knightean uncertainty in market economies. Secondly, Austrians criticize the mechanistic interpretation of mathematical models. According to Mises (1953), such an interpretation does not leave any room for future

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5 Page numbers refer to the 1981 reprint of Lucas (1980).

6 Lucas (1980, p. 272) identified advances in mathematical problem-solving techniques as one of the main sources of scientific progress. As Butos (1986, p. 337n10) observed, this is reflected in Lucas and Sargent's work on the Rational Expectations Hypothesis. In the 1960s they had already been aware of Muth's (1961) REH, but mathematical difficulties prevented them from building general-equilibrium models which included the hypothesis. Lucas's elimination of Knightean uncertainty also reflects the view that 'mathematical techniques' constrain scientific analysis.

7 This is not to say, of course, that in his view economy-wide interdependencies are not important. On the contrary, he continued to stress these interdependencies.

8 Hayek (1941 (1950), p. viii) acknowledged that mathematical formalization may be useful in the analysis of "... a greater number of variables than can conveniently be dealt with in ordinary language", but he added that "... the power of mathematical tools - and most certainly of those which I could command - also has its limits [because] ... the problems with which we have to deal here are so complex that I soon found that, in order to make them amenable to exact mathematical treatment and at the same time to keep this treatment on a plane where I could even attempt it, I had to introduce much more drastic simplifications than seemed compatible with the object" (p. ix). In other words, Hayek argued that he could only have formalized his pure theory of capital and hence his version of the ABCT at the cost of oversimplification. However, unlike Mises and his followers, he does not exclude the use of mathematical formalizations per se.

9 Austrians argue that mathematical models can only incorporate 'risk' (with the help of probability calculus) and not 'uncertainty'. Given the importance of the latter for entrepreneurship, mathematical models cannot deal with the crucial role of entrepreneurship in any market economy. This means that they abstract from a feature of economic reality which Austrians consider of great significance. Lucas (1976, p. 224) downplayed this line of argument by arguing that fundamental uncertainty does not play an important role in the explanation of business cycles, because business cycle theorists (including the Austrians) regard these cycles as repeated instances of essentially similar events. Their recurrence means that risk will be much more important than fundamental uncertainty. He even opined that "[i]n cases of uncertainty, economic reasoning will be of no value" (p. 224). However, this conclusion appears to go too far because it eliminates entrepreneurship from the domain of economics.
decisions which may alter the structure (and not merely the parameter values) of the economy. This, in turn, is inconsistent with the (Austrian) view that man is 'free' to choose. Thirdly, Austrians argue that the restrictions imposed on economic theory in the process of mathematization are inappropriate, in the sense that they exclude relevant aspects of economic reality from analysis. This is not to say that they reject abstraction and simplification. After all, all theorizing implies the introduction of simplifying assumptions. Instead, the Austrian position holds that mathematical models oversimplify social reality.

Austrians and New Classicals thus differ in their views about the appropriateness of mathematical formalization. Butos (1986, p. 337) even concluded that "... new classical economists limit their conceptualization by the techniques available, while for Hayek [and other Austrians] the conceptualization of a problem points to the limitations of the available techniques." These limitations force New Classicals to introduce additional simplifying assumptions. In contrast, Austrians reject this introduction on the grounds that they unduely restrict the scope of economic analysis. Butos's claim, however, does not make clear how the difference in methodological prescription brings about differences in content between the respective business cycle theories. That is, it does not explicate which additional simplifying assumptions are introduced. Our paper attempts to give such an explication. The problem then rises which additional abstractions should be introduced. Hoover (1988) has labelled this problem the Cournot problem, as August Cournot (1927, p. 127) already observed that "... in reality the economic system is a whole of which all the parts are connected and react on each other. ... It seems, therefore, as if, for a complete and rigorous solution of the problems relative to some parts of the economic system, it were indispensable to take the entire system into consideration. But this would surpass the powers of mathematical analysis and of our practical methods of calculation, even if the values of all the constants could be assigned to them numerically." The economy is too complex to be modelled in its entirety, and hence some simplifying assumptions must be introduced in order to render mathematical formalization possible. This introduction of additional simplifying assumptions implies that mathematical models will differ in content from non-mathematically formulated theories. It will be argued that in the case of New Classical and Austrian economics these differences concern distribution effects and, more specifically, what Blaug (1990, p. 21) called Cantillon effects. Our paper will list those New Classical assumptions which ensure the elimination of these effects from analysis, and explains some differences in content between Austrian and New Classical business cycle theory as the result of their differences in opinion on the
appropriateness of mathematical formalization.

The paper is organized as follows. Section 2 provides a short historical discussion of the concept of monetary neutrality, arguing that money can hardly be neutral in the long term, let alone in the short one. Section 3 covers Hayek’s business cycle theory, which can be seen as a complex synthesis of Menger’s structure of production, Wicksell’s distinction between the natural and the market rate of interest, Mises’s monetary theory, Cantillon’s distribution effects and Hayek’s own views on the dispersion of knowledge (and hence imperfect knowledge and expectational errors). Section 4 discusses New Classical monetary business cycle theory in general, and its modelling strategies in particular. New Classicals adopt two such strategies, namely the ‘representative-agent approach’ and the ‘islands approach’, with the latter a sophisticated version of the former. Section 5 gives some conclusions. It lists the simplifying assumptions introduced by the NCE, and discusses the way in which they preclude the analysis of distribution effects.

2. A short history of monetary (non-)neutrality

In the discussion on the neutrality of money two strands can be discerned. First of all, we have the quantity theory proposition that *ceteris paribus* a change in the money supply only influences the price level. Neutrality in this case means that changes in the money supply in a comparative-statics analysis leave relative prices and real magnitudes unaffected. During the adjustment process from one equilibrium state to another money is seen as affecting relative prices and quantities, though. Secondly, attempts have been made to identify the conditions under which money would not affect relative prices and quantities at all. Neutrality in this case means that *monetary equilibrium* is maintained at all times. We shall first discuss neutrality of money in the second sense and then dwell somewhat longer on neutrality in the first sense.

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10 Some writers see this as the *more recent* definition of neutral money, cf. Lutz (1969, p. 112) and Klausinger (1989, p. 177). But it was implied in the quantity theory all along and thus predates the other connotation of neutral money by a couple of centuries.

11 Monetary equilibrium is defined as the situation in which the money supply at any moment in time equals the volume of money demanded. In Dutch discussions of the 1950s the term was also used within the framework of period analysis, in the sense that income earned in one period and available in the next period is equal to income earned in the next period. In other words, national income is constant. This connotation of the term is not applied here. See for references Visser (1971, p. 404).
The idea of neutrality of money in the sense of uninterrupted monetary equilibrium is commonly traced back to Wicksell's *Geldzins und Güterpreise* (1962, p. 102), even if Wicksell did not use the term *neutral money.*

His definition of *neutral interest* was tantamount to neutrality of money in the second sense, though. Wicksell maintained that neutrality implied a stable price level. His Swedish colleague Davidson was quick to point out that in an economy with increasing factor productivity monetary equilibrium or neutral money implied a falling price level (cf. Myrdal (1933, p. 436) and Thomas (1935, pp. 37 - 40)). Hayek refined Wicksell's analysis, first seeing a constant money supply and later a constant money flow per unit of time as the criterion for the neutrality of money, until Koopmans finally showed that no empirical yardstick can be found for monetary equilibrium.

The aim of the monetary equilibrium theorists was to find the conditions under which a monetary economy would function as if money did not exist and relative prices consequently were only determined in the real sector. A monetary economy was thus compared with a barter economy, a barter economy, however, where markets function without frictions. This amounts to a wild goose chase, because if markets function without friction no reason can be found why people would ever decide to use money in the first place. Money that is no more than a veil will be blown away and if it is indeed "... the oil which renders the motion of the wheels [of trade] more smooth and easy" as Hume (1955, p. 33) put it, the wheels of trade will run much faster and further with money than without. In other words, a monetary economy has a different transactions technology than a barter economy. Now the likes of Koopmans realized full well that money is used just because barter exchange is not frictionless and their

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12 The term 'neutral money' appears to have been introduced by L. von Bortkiewicz in 1919, see De Jong (1973, p. 8n) and Patinkin and Steiger (1989, p. 135).

13 Wicksell (1962, p. 102) defines the natural rate of interest as "the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods."

14 The discussion has been admirably summarized by De Jong (1973). See also Visser (1971).

15 Cf. Hayek (1967, p. 31; 1984, p. 159) and Koopmans (1933, p. 228).

16 In a letter to Robertson (reprinted in Fase (1983, pp. 322 - 28)) Koopmans explained that his frame of reference was an *ideal* equilibrium economy where, in his words, no stocks of goods had to be held for mere liquidity purposes, i.e., a frictionless barter economy.
The construct of a frictionless barter economy was only a thought experiment.\textsuperscript{17} Lutz (1969, p. 105) sees this as an attempt to express the idea that, if a general depression is to be avoided in a monetary economy, it should behave like a barter economy because in a barter economy Say's Law applies.\textsuperscript{18} Still, this could not disguise the futility of the whole operation. Leaving on one side the question whether there is much sense in analysing money in a general-equilibrium model where all goods exchange against all goods, the analysis of neutral money in the sense of monetary equilibrium at all times can better remain confined to the model of a monetary economy. There is no need to make a comparison with a barter economy.\textsuperscript{19} Incidentally, if barter trade is not frictionless, even Say's Law would not suffice to prevent depressions. With frictions, in particular imperfect information on other agents' supply and demand functions, some goods offered in the market may fail to find a taker and economic activity declines, even if all excess demands neatly sum up to zero. In a general-equilibrium model with money, the conditions under which monetary equilibrium is maintained can be analyzed with exactly the results which Wicksell, Davidson, Hayek and Koopmans came up with. If monetary equilibrium is maintained at all times, Say's Equality holds true, i.e., there is no excess demand or excess supply on all goods markets combined (if we neglect the markets for other financial assets than money). Still, equilibrium relative prices in goods markets are also dependent on monetary factors, as has been made clear by the discussion on the classical dichotomy (Becker and Baumol (1962) and Patinkin (1965, pp. 75, 175)). If the monetary and real sectors are completely separate, the price level is indeterminate and the quantity theory cannot hold good.

The discussion on the neutrality of money in the monetary equilibrium sense focused on the conditions necessary for maintaining monetary equilibrium

\textsuperscript{17} Koopmans (1933, p. 230) talks about an hypothetical situation that could not exist in reality, a "hypothetische, in der Realität wohl überhaupt nicht denkbare Zustand."

\textsuperscript{18} Hayek (1967, p. 130) defined the problem as "... one of isolating the one-sided effects of money... which will appear when, after the division of the barter transaction into two separate transactions, one of these takes place without the other complementary transaction." This supports Lutz's view.

\textsuperscript{19} Patinkin (1965, p. 75) attempted to "... conceive of a barter economy as the limiting position of a money economy whose nominal quantity of money is made smaller and smaller." He could not but concede that the price level falls in the process, leaving the real quantity of money unchanged, but still maintained, then and also a quarter of a century later (Patinkin and Steiger (1989, p. 138)) that "[t]his drawback notwithstanding, there does not seem to be any other meaningful way of comparing the respective equilibrium positions of a barter and money economy." We would suggest that there is no meaningful way at all to compare these equilibrium positions.
in the face of specific changes in the real sector of the economy. The excess demand functions in the real sector of the economy and with it the demand functions for real cash balances were assumed to change. The quantity-theory comparative-statics approach to the neutrality of money by contrast can be said to study the effects of changes in the money supply with constant excess demand functions in the real sectors of the economy and consequently a constant demand function for real cash balances. The interesting question here is whether, and if so to what extent, developments in the monetary sphere can have lasting effects on relative prices and hence quantities in the real sector.\(^{20}\) The conditions for neutrality in the comparative-statics sense have been shown to be quite strict. First of all, money should be intrinsically valueless paper money or bank money. A change in the volume of full-blooded silver or gold money would not only have an impact on the general price level, but would also change relative prices of goods produced from silver or gold and other goods. Aschheim and Hsieh (1969, pp. 213 - 15) formulate six further conditions for neutrality of money in the comparative-statics sense: (1) price flexibility, (2) absence of money illusion, (3) absence of distribution effects, (4) static price expectations, (5) absence of a combination of internal and external money, or credit money and base money, (6) absence of government debt and open-market policies.\(^{21}\) Aschheim (1973, p. 78) even goes so far as to claim that the conditions for neutrality of money imply, again, the absence of money (as a unit of account). Static price expectations are in his view incompatible with perfectly flexible prices, freedom from money illusion and the absence of uncompensated distribution effects of price level changes.\(^{22}\) The condition of static price expectations may, however, be too strict.\(^{23}\) Conditions (5) and (6) are based on the idea that changes in the money supply change the composition of asset holders' wealth portfolios and with it the rate of interest.\(^{24}\) These conditions could be reformulated in the sense that

\(^{20}\) Few, if any, quantity theorists denied that money is non-neutral in the short period, see Visser (1974, pp. 135 - 36) and Humphrey (1991).

\(^{21}\) Cf. the seminal article by Metzler (1951).

\(^{22}\) Conditions (5) and (6) are not mentioned in Aschheim's 1973 article and condition (3) is reformulated as the absence of *uncompensated* distribution effects. Presumably conditions (5) and (6) are now subsumed under (3).


\(^{24}\) See on this Patinkin (1965, pp. 288 - 94). Patinkin notes that neutrality is retained if government bonds are not seen as net wealth. See also Patinkin (1989).
portfolio composition should not change, so that if the money supply changes, the volume of nominal debt changes proportionally. Likewise, condition (3) could be broadened to include the neutrality of distribution effects, as Aschheim in fact did in his 1973 article. Finally, the long-term inflation rate in the economy should be constant, as both the Mundell-Tobin effect and non-neutral tax effects preclude superneutrality. Neutrality in the comparative-statics sense therefore can only be restricted to stepwise changes in the money supply, not to changes in the growth rate of money. This case could be subsumed under the condition of the neutrality of distribution effects.

The distribution effects call for further comment. As will be more extensively discussed below, Austrian analysis follows Cantillon and emphasizes that money enters into circulation in specific ways, affecting the structure of spending and consequently relative prices, including the rate of interest, and quantities. It follows that money, especially if created through credit but also from an international payments balance, changes demand and supply functions in the real sector. Cantillon (1964, Part 2, Ch. 10) explained that money that enters into circulation through money lenders brings with it a lowering of the rate of interest, while money that enters into circulation through consumptive expenditure by the rich makes the rate of interest rise, because the producers of consumption goods face an increase in demand and have to borrow money in order to increase production. One expression of distribution effects is the phenomenon of forced savings. This follows if the expenditure of entrepreneurs who borrow money to increase production makes prices rise and in that way brings about a reduction in real consumption. Another case of distribution effects is the redistribution of real wealth between debtors and creditors in the case of (unforeseen) price level changes, a redistribution that according to Fisher (1933) and Keynes (1961, p. 264) could seriously worsen a depression, once started. A special kind of distribution effects, which has only recently received serious attention (Bikker (1991a and 1991b), concerns the redistribution of interest income if the rate of interest changes.

25 Post-Keynesian analysis also emphasizes distribution effects, but it tends to play down the impact of money on other economic variables, stressing instead the residual character of money. Nonetheless, money is far from neutral in their analysis, as its creation is dependent on the borrowing needs of all kinds of economic actors and borrowing is influenced by the central bank's interest rate policies. See Arestis (ed.) (1988).

26 The phenomenon of 'forced savings' was already discerned by Ricardo, who considered it relevant for the short run only. Other economists have acknowledged that forced savings could also change the natural rate of interest. For a history of the notion of forced saving see Hayek (1932), Viner (1964, pp. 187 - 97) and Humphrey (1991, pp. 9 - 11).
New developments in growth theory serve to underline the likelihood of non-neutrality of money supply changes. In general-equilibrium models, demand and supply functions usually are well-behaved linear functions and the system is stable or assumed to be stable: after the business cycle has run its course everybody is happily back on his or her largely unchanged demand or supply curve. With nonlinearities, as emphasized by the new growth theory, any temporary expansion in an industry may perpetuate itself because relative prices between goods are changed in the process. It is, for example, quite possible for a country to specialize in the production of a good the production of which was quite accidentally increased during some period thanks to a monetary shock. The new growth economics emphasizes that investment recreates investment opportunities (see FitzGerald Scott (1989, in particular p. 159), Lucas (1988), Romer (1986)), or success breeds success. Distribution effects are then even more likely to result in lasting changes in the real sectors. More generally it is acknowledged that deviations from an equilibrium growth path, through their impact on investments and on technology, if technological growth is endogenous, may change the equilibrium growth path itself (cf. Solow (1988, in particular pp. 311 - 12)). This ties in with chaos theory, which shows that in nonlinear systems a small change in initial conditions will lead to significant changes in the time path of a function (see, e.g., Butler (1990), Kelsey (1990)). The result is hysteresis, or the phenomenon that deviations from an equilibrium path change the equilibrium path itself. New developments in economic theory therefore tend to emphasize the deviation-amplifying tendencies of shocks rather than the stability of any growth path.

The above indicates that recently economists have increasingly recognized the importance of forces that affect the comparative-statics neutrality of money. It will be shown below that Hayek focused on a subset of these forces. His business cycle theory interprets cyclical fluctuations in output and investment in terms of non-neutral monetary injections in a specific market. The ensuing Cantillon effects distort the so-called 'structure of production'.

3. The analysis of Cantillon effects: the Hayekian BCT

Hayek's version of Austrian business cycle theory (ABCT) starts from a notion of intertemporal general equilibrium, which he defined as a series of situations in which economic agents have complete information and perfect foresight about preferences and the means of production at every point of time during the planning period (1928, p. 76). Such an equilibrium is maintained if the agents do not make expectational errors (p. 85). However, in reality agents do not have
complete information, so that they cannot avoid making such errors. In fact, Hayek (1933a, p. 141) argued that "[e]very explanation of economic crises [and, more generally, of business cycles] must include the assumption that entrepreneurs have committed errors." That is, he interpreted business cycles as disequilibrium phenomena. But such cycles will only arise if economic agents (or rather entrepreneurs) make similar expectational errors. According to Hayek (1933a, p. 141), this will be the case if they are misled by following guides and symptoms which as a rule prove reliable. One of these rules (and in capitalist economies presumably the most important one) is the price system. Economic agents base their actions on actual and expected prices. In equilibrium, actual prices reflect the scarcity of consumer goods and production factors, which means that they are in accordance with the preferences of the economic agents. However, in disequilibrium they are distorted, in the sense that they do not accurately reflect these underlying preferences. In general, Austrians interpret business cycles as the result of price distortions.

Hayek concentrated on the price which figures most prominently in investment decisions, namely the interest rate. His theory explains why this rate may be distorted and what the consequences of such distortion are. He thereby adopted Wicksell's distinction between the market and the natural rate of interest. Wicksell (1962) defined the latter as the long-run equilibrium rate of interest, which reflects the time-preferences of the economic agents. The 'market' (or money) rate of interest is determined on the loan market, and is the rate at which the supply of loanable funds equals the demand for loans. This rate need not equal the natural rate, and thus need not reflect the agents' time-preferences. This difference results either because of a change in the natural rate, or because of a change in the market rate. Hayek argued that the process resulting from the former change must be interpreted as a mere adjustment process to a new equilibrium situation, instead of a business cycle. Therefore, we concentrate on a change in the market rate, which (initially) does not affect the

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27 As Haberler (1946, p. 36, note 1) observed, "[t]he concept of a 'natural rate' (and even the term) can be found in earlier English economic writings." For instance, Adam Smith (1976, I.vii, p. 65) already used the term when arguing that "[t]he natural price ... is, as it were, the central price, to which the prices of all commodities are continually gravitating." More importantly, Henry Thornton (1978, pp. 253 - 55) already explicated the so-called indirect mechanism, which holds that an increase in the supply of money is translated in a rise in the general price level through a fall in the market rate of interest and the ensuing increase in investment. He used the idea of an equilibrium or natural rate of interest, although he did not yet attach the adjective 'natural' to it. He did use this term in 1811. However, Wicksell's student Carl Uhr (1962, p. 200) argued that Wicksell had not been directly exposed to Thornton's ideas, and that the influence was merely indirect, namely through Ricardo.
natural rate. Such a change is caused by monetary expansion. As Hayek (1975, pp. 148 - 49) observed, this expansion of the money supply may be brought about by either of three circumstances: (1) changes in the volume of cash, caused by the in- and outflow of gold, (2) changes in the volume of money, as regulated by the monetary authorities (which include the government and the central bank), and (3) the creation of deposits by private banks. As Hayek's emphasis shifted in time, we shall discuss monetary expansion in general, disregarding its source. The ABCT assumes that such an expansion enters the economy on the loan market. The market rate of interest then falls below the natural rate, which raises the present value of all investment projects. As O'Driscoll and Rizzo (1985, p. 205) showed, this rise in present value is a result of three effects: (1) a discount rate effect, (2) cost effects, and (3) derived-demand effects. The first two effects work in the same direction, namely by increasing the present value of all investment projects. The discount rate effect starts from the fact that the market rate of interest is used as a discount rate, so that the present value of the expected future returns of investment projects increases. Moreover, the market rate is also the price to be paid on loans. A fall in this rate reduces the costs of investment projects, and therefore increases their net present value. The discount-rate and cost effects result in general overinvest-

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28 This means that in Hayek's terminology a business cycle is by definition caused by monetary expansion.

29 This process of credit creation can be made clear as follows. Consider a banking system in which private banks must adhere to a fractional reserve ratio. An increase in the volume of deposits will raise the reserve ratio, thus enabling the bank to increase its loans (and vice versa). However, such a simple relationship between loans and deposits does not apply to the banking system as a whole. This system is able to multiply credit, if the initial credit expansion is (partly) deposited at another bank, which in turn increases its loans. This process of 'credit multiplication' can go on indefinitely, as long as the additional credit is not used in a way which leads quickly to the market for consumer goods (as in the latter case the credit is not deposited with another bank). Hayek (1975, p. 160) concluded that in this way the banking system as a whole "... can grant credit to an amount several times greater than the sum originally deposited." The 'credit multiplier' process is an unintended consequence which arises from the fact that it is profitable for private banks to lend as much as possible, given the fractional reserve ratio of the banking system under consideration.

30 Contrary to his forerunner Mises, Hayek (1975, p. 150) considered the first influence empirically less important and the second a rather special case. Mises had criticized the monetary policy of the 1920s, which were highly inflationist, thus explaining cyclical fluctuations as the result of exogenous disturbances. In contrast, Hayek attempted to elaborate an endogenous business cycle theory, thereby concentrating on the third influence. However, in the 1970s he considered the second influence to be the most important. He then proposed to denationalize money, which would render it impossible for the monetary authorities to pursue inflationary policy. This changed his explanation of business cycles from an endogenous into an exogenous one.
ment, which will exist as long as entrepreneurs find it worthwhile to invest (i.e., as long as they expect that the process of monetary expansion continues). The ensuing gap between planned investment and planned savings is filled by forced savings. This phenomenon arises because during the expansion process entrepreneurs compete for the factors of production. As the boom proceeds, these factors become more and more fully employed. The higher rate of investment can then be maintained only if the factors of production are transferred from the consumer to the producer goods industries. Entrepreneurs bid up the prices of these factors. As a consequence, their owners experience a rise in income. They increase their consumer demand. However, the transfer of the factors of production from the consumer to the capital goods industries has prohibited a rise in the supply of the former. Hence the prices of these goods rise. This forces individuals who have not (yet) experienced a rise in income to curb their consumption. Forced savings thus result from redistributions in income. These redistributions presuppose that some individuals will experience a rise in income earlier than others, because their productive activities are more in demand. This can only be the case if their (productive) activities differ from those of others. Furthermore, the non-simultaneity of the rise in their respective incomes also reflects the fact that money does not spread immediately throughout the economy because production is organized 'vertically'. That is, it takes place in successive stages of production and hence is a time-consuming activity (see below). The boom comes to an end when the additional money stops flowing into the economy.

The ABCT thus explains industrial fluctuations as a result of an increase in the money supply, the fall in the market rate of interest, and the ensuing discount-rate and costs effects. However, this fall does not only affect the volume of investment but also its composition. An important characteristic of the ABCT is its emphasis on Cantillon effects, which arise from the fact that the additional money enters the economy at a specific point (namely as investable funds) and

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31 Note that this result not only applies to unanticipated but also to anticipated monetary expansion.

32 Note that when preferences and patterns of expenditure differ between economic agents, the redistributions of income also imply shifts in the relevant general equilibrium, and hence changes in the natural rate of interest. Moreover, the effects of the redistributions depend on the route according to which the additional money spreads throughout the economy. It seems therefore correct to infer that the relevant general equilibrium changes continuously: 'the target is moving'.
only gradually spreads through the economy. The effects of this spread can be discussed in more detail. However, we must first briefly consider the framework in which this discussion takes place. This framework is constituted by the structure of production. Carl Menger (1968, pp. 8 - 10) classified capital goods according to the functions they perform in the process of production. He used their 'remoteness' from consumption as a criterion for classification. Consumer goods are called 'goods of the first order', capital goods which are used in the production of these first-order goods are called second-order goods, capital goods which produce second-order goods are called third-order goods, and so forth. Hence there is a vertical relationship between heterogeneous capital goods. It is possible to distinguish stages of production which can be classified in a manner similar to that of capital goods. Taken together, these stages form a structure of production. This structure is vertical, in the sense that production takes place in successive stages: production in a given stage depends on production in a higher-order stage.

The fall in the market rate of interest does not affect all investment projects to the same degree, nor even in the same manner. That is, it does not only lead to a discount-rate effect and cost effects, but also to derived-demand effects. These effects affect different orders of goods differently. If entrepreneurs mistakenly interpret the fall in the market rate of interest as reflecting a fall in the natural rate, they will conclude that the agents' time-preferences have decreased. This means that they expect current consumption demand to fall and future consumption demand to rise. It will then appear to be more profitable to invest in productive activities which yield consumption output in the more distant future. In other words, the present value of investment projects which involve higher order goods has increased, whereas that of others has decreased. This induces entrepreneurs to reallocate capital from the lower to the higher stages of production. In Austrian terms, the structure of production is 'lengthened' (or has become more 'roundabout'). However, this process of lengthening cannot go on indefinitely, because it will be checked by a shortage of resources.

33 In order to clarify his views Hayek (1969, p. 281) compared a situation of continuous credit creation to a situation in which a viscous liquid, such as honey (read: money), is poured into a vessel (the economy): "... if the stream hits the surface at one point, a little mound will form there from which the additional matter will slowly spread outward." This analogy reflects that there is no such thing as 'helicopter money'. It leaves the question unanswered why the liquid is viscous (i.e. why it takes so long for the additional money to spread throughout the economy).

34 In his criticisms on the Ricardo effect, Wilson (1940, p. 172) wondered whether it would not be possible for the economic system to remain in a new equilibrium at a higher level of activity. In his view, the initial credit expansion would lead to a larger (more capital-intensive) capital
(1939 (1950), p. 11) argued that at the later stages of the boom "... the prices of consumer goods do as a rule rise and real wages fall."\(^{35}\) The rise in consumer prices increases the profit margins on these goods, and hence induces entrepreneurs to reallocate capital from the production of capital goods to that of consumer goods. The fall in real wages makes shorter methods of production more profitable, so that there will be a tendency to use more labour with the existing machinery (by working overtime, double shifts, etc.), and to invest in less expensive, less labour-saving or less durable machinery. Hayek argued that the implied decrease in the demand for capital goods will become so strong as to turn the boom into a recession. This effect is called the *Ricardo effect*.\(^{36}\) During the recession entrepreneurs will correct their investment errors by reallocating their capital from higher to lower order goods.\(^{37}\) In turn, the recession ends either because of an increase in the supply of money, or because of the reverse operation of the Ricardo effect.

The above suggests that the ABCT contains at least two unique features. Firstly, it adopts the Austrian framework of the structure of production. This means that capital goods are heterogeneous, and that they stand in a vertical, time-consuming relationship to each other. Secondly, the ABCT incorporates Cantillon effects, in the sense that an increase in the supply of money leads to redistributions between the owners of different factors of production and between the entrepreneurs in different stages of production. As Foss (1990, p. 6, italics in original) pointed out, "Hayek's argument is ... dependent ... on the time structure. This means that the economy is richer with a larger per capita capital stock that could produce enough income to generate the savings to maintain itself. Then there is no reason why a depression would emerge. Hayek (1933b (1939), p. 180) had already allowed for this possibility, but considered it very unlikely.

\(^{35}\) He defined the real wage rate as the ratio of the money wage rate and the prices of the consumer goods produced by the labour under consideration (p. 8).

\(^{36}\) This effect derives its name from Ricardo's (1978, Ch. I, Section V, p. 26ff.) statement that "... with every rise in the price of labour, new temptations are offered to the use of machinery." It is designed to show that the boom will come to an end even if the market rate of interest were to remain constant. Hayek had already shown in his *Prices and Production* (1931) that the boom would cease in a situation in which the market rate of interest was perfectly flexible.

\(^{37}\) It should be noted that the Ricardo effect assumes that entrepreneurs expect the relatively higher prices of consumer goods and the relatively lower real wages to persist for a period of time that is long enough to make it worthwhile to change their methods of production. Hayek (1939 (1950), pp. 16 - 18) acknowledged this. Moreover, he also addressed the case in which the entrepreneurs expect that the prices of consumer goods continue to rise. This will merely reinforce the effect, due to the continuing fall in real wages.
path of real factor incomes relative to the linear representation of productive activities [i.e. the vertical structure of production] and the lags this implies." It takes time before the mound of money is spread out evenly throughout the economy. The ensuing distortion of the structure of relative prices follows from the fact that economic agents differ from each other. These differences may concern (a) their utility functions, (b) their abilities to gather information (and hence their information sets), (c) their expectations formation functions, and/or (d) their productive activities (i.e. the order of goods which they produce). The latter feature appears to be the most important as it accounts for the malfin-vestments and the ensuing distortion of the structure of production.

As was stated above, Austrians tend to deny the appropriateness and usefulness of mathematical formalization. Not surprisingly, they have never attempted to formalize their views, although in recent years some hybrid forms have arisen. For instance, Thalenhorst and Wenig (1984) offer a mathematical formalization of Hayek's *Prices and Production* (1931), but they assume that the rate of profit is identical for each stage of production (p. 217). This absence of inter-stage profit opportunities means that there are no distortions of the structure of production and hence that there cannot have been malfinestments. As Thalenhorst and Wenig acknowledge, their analysis cannot discuss Hayekian monetary dynamics (p. 215). It should be noted, though, that their mathematical model may serve as a starting point for mathematical analysis. Relaxation of some of its assumptions may render the analysis of Hayekian monetary dynamics possible. Nevertheless, until now Hayek's ABCT has never been formalized as a mathematical model. In contrast, New Classicals prescribe such mathematization, so that they are faced with the 'Cournot problem'. They 'solve' this problem by aggregating economic agents and/or goods.

4. Abstracting from Cantillon effects: New Classical BCT

In the 1970s interest in business cycle theory was revived by the so-called 'rational expectations revolution'. The resulting research programme was called 'New Classical Economics' (NCE).\textsuperscript{38} It started by formalizing Friedman's and Phelps' Natural Rate Hypothesis (NRH), which holds that in the long-run the economy will tend towards its so-called 'natural rate of output' (NRO) or

\textsuperscript{38} The main proponents are Robert Lucas, Jr., Thomas Sargent, Neil Wallace, Robert Barro, Edward Prescott, and Finn Kydland.
equivalently ‘natural rate of unemployment’ (NRU).\textsuperscript{39} We restrict our analysis to the monetary version of New Classical business cycle theory.

New Classical business cycle theory, or equilibrium business cycle theory (EBCT), adopts at least the following four assumptions. Firstly, economic agents are assumed to be price-takers, and a Walrasian tatonnement process ensures continuous market-clearing. Secondly, aggregate real output is formulated as the ‘Lucas supply curve’. Given the agents’ information sets and expectations, the NRH holds. Thirdly, all agents form their expectations according to the Rational Expectations Hypothesis (REH). Fourthly, such expectations formation takes place on the basis of an information set which does not allow for systematic expectational errors. That is, all persistence in deviations from the NRO (or equivalently NRU) must be accounted for by propagation mechanisms.

New Classicals argue that business cycles are caused by exogenous shocks, which are either real or monetary in nature. The shocks are reflected in the prices which agents observe. They are thus confronted with a so-called signal extraction problem, in the sense that they must determine whether a given price change reflects a change in relative prices or in the general price level. The response of economic agents to a given price movement depends crucially on their expectations and hence on their information set. Under the assumption of perfect knowledge, the agents know which part of a price movement is permanent, so that they can infer how to change their actions in order to act ex-post optimally. Monetary changes are presumed not to influence real variables, because rational economic agents do not suffer from money illusion, so that money is assumed to be neutral in the comparative-static sense.\textsuperscript{40} The complete-information equilibrium is called the Natural Rate Equilibrium (NRE). In contrast, in the Rational Expectations Equilibrium (REE) economic agents do not have complete information, so that they are confronted with a problem how to interpret a given price change. The solution to this signal extraction problem will be based on the agents’ expectations about the relevant price movement, which are in turn based on their information sets. Given the incompleteness of the latter, the agents will make expectational errors. Such errors can be one of two kinds: either the agents mistakenly interpret a real change as a monetary one, or

\textsuperscript{39} As Maddock (1979, p. 158n4) observed, "[i]t is quite common in this [i.e. New Classical] literature to switch back and forth between income and unemployment by replacing log unemployment for log income minus log trend income." Therefore, the NRO and the NRU are considered to be reciprocal.

\textsuperscript{40} For instance, see Sargent and Wallace (1975, pp. 221 - 24).
they think that a monetary change is in fact a real one. In the former case, output and unemployment will be adjusted to the price movement as soon as the correct information becomes available. The economy then adjusts itself to its new NRE, albeit it with a lag, and there will be no business cycle. However, things will be different if economic agents respond to monetary changes as if they were real ones. Monetary changes will then have real effects, so that it seems as if the agents suffer from money illusion. The agents' information sets thus play a crucial role in EBCT.  

According to the NCE, rational economic agents do not make systematic expectational errors, because such errors are easily detectable. After all, if the rate of inflation has been underestimated for a sequence of periods, then agents will be inclined to adjust their expectations formation function, so that the systematic component of their expectational error is eliminated. The problem then arises how unsystematic, serially uncorrelated, shocks can account for the persistence which characterizes the economic data over the business cycle. The NCE solves this problem by adopting a solution which had already been given in the 1930s. Slutsky (1937) and Frisch (1933) had already shown that random shocks could lead to serially correlated movements of the variables in the system under consideration, due to some propagation mechanism. Thus, the crucial distinction, as Lucas and Sargent (1978, p. 313) noted, is that between 'sources of impulses' and 'propagation mechanisms'. The latter ensure that serially uncorrelated disturbances will have serially correlated effects. Instances of such mechanisms have been provided by Lucas (1975), Blinder and Fisher (1981), and Kydland and Prescott (1980), among others. These mechanisms are instances of what Barro (1981b, p. 48) called 'adjustment-costs explanations' of persistence. They explain persistence by introducing 'friction' into the economic system.

It should be noted that the persistence discussed above only applies to

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41 Lucas (1987, p. 94, italics in original) emphasized this property of the NCE when discussing the (New-) Keynesian assumption that nominal prices are rigid because of some form of collective bargaining. He objected that "[t]he central issue for a theory of nominal price rigidity ... is not the nature of the game agents are assumed to be engaged in, but rather the information agents are assumed to have about the state of the system at each date."

42 Lucas (1975) accounted for persistence effects by incorporating physical capital, which is affected by random changes in the growth rate of money and the ensuing Mundell-Tobin effect. Once installed, the physical capital stock can only be adjusted at the rate of depreciation. Blinder and Fisher (1981) used the gradual adjustment of inventory stocks of finished goods as a mechanism to ensure serial correlation. Kydland and Prescott (1980, p. 175) explain persistence by arguing that "... there are long lags from the time when changes in its determinants call for an increase in the capital stock until the time when the new capital starts yielding services."
deviations from the mean level of endogenous variables. This follows from the fact that New Classicals formulate their views as mathematical models, which means that the Cournot problem must be 'solved'. One may discern two strategies which the NCE uses to address the Cournot problem, namely the 'representative-agent approach' and the 'islands approach'.

The representative-agent approach

Early New Classical models were so-called 'representative-agent models'. The concept of the representative agent was first explicitly introduced by Marshall.\footnote{In Marshall's (1947, pp. 317 - 18, italics in original) view, "... a Representative firm is that particular average sort of firm, at which we need to look in order to see how far the economies, internal and external, of production on a large scale have extended generally in the industry and country in question."} His 'representative firm' is some sort of non-mathematical, fictitious average entity (1947, p. 318). In contrast, the NCE defines the representative agent (whether individual, household, or firm) as the mathematical mean of the group (subsystem) as a whole.\footnote{For instance, Lucas and Rapping (1969a, p. 25) deflate the aggregate labour supply function by an index of the number of households, Lucas (1972, p. 68) assumes that each period \( N \) identical individuals are born who each supply \( n \) units of labour which yield \( n \) units of output.} This means that the concept is used as a hypostatisation of the (sub)system as a whole.\footnote{This was already stressed by Runde and Torr (1985, p. 217), who argued that the rational expectations approach (i.e. the NCE) does not analyze individual, but rather the market's behaviour. In a general-equilibrium analysis this means that the representative individual is merely another way of depicting the system as a whole.} As Hoover (1988, p. 242) observed, New Classicals treat aggregates and index numbers as if they obey the principles of microeconomics. They restrict their analyses to the mathematical mean, accounting for deviations from this mean by adding a probability distribution. The expected value of these deviations are zero, which means that there are no \textit{a priori} reasons why expectations and actions of different individuals will differ from each other. The representative agent by definition forms rational expectations which equal the mathematical mean of the aggregate expectations, albeit in a probabilistic sense. This property may be explained by arguing that the expectation deviations will cancel out, presumably due to the 'Law of Large Numbers'.\footnote{Haltiwanger and Waldman (1989) argue that this will only hold under very special circumstances which concern the nature and intensity of the interrelations between the actions of the individuals. They distinguish two types of interrelations, namely \textit{congestion} and \textit{synergism}. The former applies when the incentive for agent \( i \) to engage in an activity will be lower if the number of
elements of the population under consideration belong to the same 'class', i.e., if they do not differ from each other in any relevant aspect. The assumption that the agents are identical will be called the homogeneity postulate. This postulate is adopted in the New Classical 'representative-agent models', which means that these models cannot analyze distributional effects. That is, the 'representative-agent approach' of the NCE concentrates on the magnitude of aggregates, disregarding (changes in) their composition. It implies that redistributions between economic agents are considered to be irrelevant in the explanation of industrial fluctuations. This restriction of the domain of New Classical Economics could be circumvented by introducing several 'representative agents'. This strategy is adopted in the 'islands approach', which can be regarded as a more sophisticated version of the 'representative-agent approach'.

The islands approach
The 'islands approach' uses Phelps's (1970) suggestion that markets can be seen as islands. In particular, this 'islands parable' was adopted by Lucas (1972, 1973, 1975), Lucas and Prescott (1974), and Barro (1976, 1980). These models consider an economy in which a single output is produced, which must be divided between private consumption, government consumption, and next period's capital. The population consists of identical households, so that their utility functions do not differ. The conjunction of these two assumptions implies that economic agents do not differ from each other with respect to their utility functions and their productive activities. However, this does not mean that there are no differences at all between these agents. As Lucas (1975, p. 180) stated, the 'islands' or markets are "imperfectly linked both physically and informationally", so that information is assumed to be homogeneous across agents in a given market, but heterogeneous across markets.\footnote{According to Pesaran (1989, p. 57), this feature follows from the New Classical assumption that "[a]ll firms [or individuals] observe current equilibrium prices in their local markets." Such prices clear the market under consideration, thus conveying all information on that market. As on a given market individuals derive their information from observing the same equilibrium price, they...} The 'island approach' thus...
assumes that economic agents only differ from each other as regards their information sets. Hence, it disregards 'intra-market' distributional effects. The study of such effects between markets, however, appears to remain possible.

As was stated above, the EBCT explains industrial fluctuations in terms of expectational errors. These errors are caused by discrepancies between changes in the relative prices and in the general price level. That is, the economic agents must determine how much of a given change in their respective local prices must be attributed to a change in the general price level, and how much of the change in local prices reflects changes in real factors. The relevant decision variable thus is the difference between local price changes and global price changes: \( d_t(z) = p_t(z) - p_v \), where the subscripts denote the time period under consideration, and \( z \) is an index of location. In 'islands models' agents have complete local and incomplete global information. Their information sets thus include only \( p_t(z) \), and not \( p_v \). They must then form expectations about the latter price change. These expectations are rational, so that the agents are assumed to know the 'true' probability distribution of \( d_t(z) \). According to Pesaran (1989, p. 57), this means that the problem of heterogeneity of information across markets is circumvented. After all, the relevant decision variable is \( d_t(z) \), and all agents are assumed to know its 'true' distribution. As there can only be one 'true' distribution, this means that the respective information sets do not differ in any relevant (i.e. systematic) manner. Information about the relevant decision variable turns out to be homogeneous across markets. The problem of the heterogeneity of information across markets in the 'islands' models is thus circumvented by assuming that all economic agents know the 'true' probability distributions of \( d_t(z) \).

The fact that the agents know the 'true' probability distributions of changes in the money supply, and in the distribution of individuals over both markets is merely another way of stating that expectations are formed rationally in the sense of Muth (1961). Muth's REH holds that "... expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the 'objective' probability distributions of outcomes)" (p. 316).

Barro (1976, p. 85) explicitly assumed that "... all markets have the same ex ante distribution of price ..."

McCallum (1979) and Machlup (1983) have criticized the New Classical assumption that economic agents know the 'true' probability distributions, because it implies that they also know the 'correct' structure ('model') of the economy. It is not even clear why all agents would use the
In conclusion, the ‘islands approach’ and the ‘representative-agent approach’ adopt identical assumptions as regards the differences between economic agents. In both approaches economic agents are assumed not to differ with respect to their utility functions, their abilities to gather information (and hence their relevant information sets), their expectations formation functions (and, given the information sets, their expectations) and their productive activities. This implies that the New Classical ‘solution’ to the Cournot problem leads to the elimination of Cantillon effects from the domain of business cycle theory. This elimination appears to be the price paid for abstraction and mathematical formalization.

5. Conclusions

According to Blaug (1985, p. 155), "... the Cantillon effect ... denies 'the homogeneity postulate' by asserting that changes in prices produced by cash injections vary with the nature of the injection and, moreover, that changes in absolute prices are almost always associated with alterations in relative prices." Our analysis allows for a more explicit formulation of this homogeneity postulate. In particular, the NCE assumes that economic agents are homogeneous with respect to:

1. the utility functions,
2. the abilities to gather information (and hence the information sets),
3. the expectations formation processes (and, given (2), the expectations), and
4. their productive activities (i.e. the order of goods they produce).

These assumptions reflect that the EBCT explains cyclical fluctuations in terms of aggregate variables, thereby implying that distributional effects do not play an important role. In contrast, the ABCT also incorporates changes in the composition of the aggregate variables. It stresses the complexity of social reality, but in doing so it completely surrenders to the impossibility of solving the Cournot same (New Classical) 'model', and hence why expectations would be Muth-rational. Frydman (1983, p. 115) concluded that "... the assumption that agents form rational expectations appears to conflict with the fact that the economy is decentralized."

51 Of course, whether this price should be paid depends on the empirical importance of the distribution effects to be eliminated. This empirical question differs from the purely theoretical one which is addressed in this paper.
problem in principle. The ABCT allows for the inclusion of Cantillon effects and the ensuing (comparative-static) non-neutrality of money, but it does so at the cost of analytical - or at least mathematical - rigour. In contrast, the NCE undertakes a practical attempt to solve, or rather to circumvent, the so-called Cournot problem by introducing simplifying assumptions. This restricts the domain of economic explanation because it eliminates Cantillon effects. As a result, the NCE must assume that anticipated changes in the money supply rule are neutral in a comparative-static sense. This is the price to be paid for the methodological prescription that economic theories should be formulated as mathematical models.

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52 This emphasis is consistent with the Austrian view that economic agents differ from each other in relevant aspects. These differences may even concern the ‘models’ which agents use.


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