Spatial Interaction Models
As a Unified Framework for Dynamic Transportation Models

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Abstract

In the past decades a wide variety of transportation planning issues has come to the fore, addressing both the supply and the demand side. In this context a (seemingly) wide variety of different transportation models - both static and dynamic, both micro- and macro-oriented - has been developed. The main question to be answered is whether this wide variety of transportation models (e.g., dynamic models of discrete choice, Lotka-Volterra models, dynamic programming models, etc.) can be viewed from an integrating perspective. The paper shows that the class of spatial interaction models provides a coherent framework incorporating most of the static and dynamic, micro and macro models that have been developed.
1. Introduction

It has often been stated that transport is a derived demand. In other words, transport demand is not an autonomous factor but is a result of major driving forces in society: demographic, economic, social and technological.

It is evident that these driving forces have many consequences for spatial behaviour of people. On the one hand they determine the evolutionary pattern of human activities (including their transport needs); on the other hand, they are also facing the related feed-back effects, which are to a large extent the result of strong bottlenecks such as lack of safety, low quality of life, resource exploitation, energy use, environmental degradation, etc. (see Nijkamp et al., 1989). In particular it is noteworthy that in the sixties these driving forces have moved transportation policy toward network and capacity expansion, while from the mid seventies onward the attention has been devoted to a more efficient use and management of existing networks.

In parallel to the broad expansion (in the sixties) of transport systems, the scientific development of research on transportation showed, in almost the same period, the need for and the emergence of macro-oriented mathematical tools (e.g., spatial interaction models, linear programming, etc.), which were capable of understanding and analyzing the transportation patterns and configurations, at least in a static context of an aggregate network.

Only in recent years, the awareness of the possibilities of a strong analytical support to management issues has moved planners in the direction of more emphasis on the practical usefulness of transport planning studies or models. At the same time new mathematical tools emerged (such as dynamic or multitemporal models of a discrete choice type, Lotka-Volterra models, master equation models, catastrophe or chaos models, etc.) with the aim of representing the spatial dynamics of mobility and transportation.

In this paper we will focus our attention on a particular tool largely used in transportation models, viz. the family of Spatial Interaction Models (SIMs). More precisely we will show that SIMs constitute a unified framework for many classes of models used in
transportation studies, in both a static and dynamic analysis.

The structure of the paper is the following. Section 2 will briefly outline the structure of a SIM in a static context and sketch its link with other approaches emerging from different disciplines. In particular, the connection with behavioural models such as Multinomial Logit (MNL) models and stochastic dominance approaches will be pointed out.

Section 3 will be devoted to the analysis of SIMs in a dynamic context by showing the dynamic properties of a SIM which may - under particular conditions - also lead to chaotic behaviour.

Section 4 will overview the dynamic transportation models developed so far in the light of their scale of analysis (macro-meso/micro). Moreover it will also indicate how these well-known models incorporate essentially a logistic structure and hence have strong connections with SIMs.

Then Section 5 will illustrate the evolution of transportation models by illustrating how SIMs continue to play a fundamental role in this context, being the fixed point for many of these models.

Finally Section 6 will contain some general conclusions.

2. Relevance of Spatial Interaction Models in 'Static' Transportation Models

The relevance of SIMs in transportation studies was outlined some years ago by Sen (1985), given their capability of forming the basis for network equilibrium models or their potential to combine interaction, network equilibrium and other travel or locational choice problems.

In this paper we will proceed along these lines by showing how SIMs play nowadays a fundamental role as the common basis for many other models widely used in transportation studies and applications, in both a static and dynamic context.

To this purpose we will give briefly here a presentation of SIMs, which may also be useful for a better understanding of our subsequent considerations. SIMs are simply models of flows $T_{ij}$ (flows of people, goods, messages, etc.) from some origin $i$ to some
destination \( j \), formulated as follows:

\[
T_{ij} = A_i B_j O_i D_j \exp (-\beta c_{ij})
\]  

(2.1)

with:

\[
A_i = \left[ \sum_j B_j D_j \exp (-\beta c_{ij}) \right]^{-1}
\]  

(2.2)

and

\[
B_j = \left[ \sum_i A_i O_i \exp (-\beta c_{ij}) \right]^{-1}
\]  

(2.3)

where:

\[ T_{ij} \] = the (unknown) trips from origin zone \( i \) to destination \( j \)

\[ O_i \] = the (given) trips generated from origin zone \( i \)

\[ D_j \] = the (given) trips attracted to destination zone \( j \)

\[ c_{ij} \] = a measure of the (given) unit cost of transport between

zone \( i \) and zone \( j \)

\[ \exp(-\beta c_{ij}) \] = a (non-linear) function of transport costs, where \( \beta \) is a

parameter to be determined by means of a calibration

process for the whole model.

\( A_i \) and \( B_j \) are so-called balancing factors ensuring the additivity

condition that the activity leaving origin zone \( i \) is equal to \( O_i \), and

the activity attracted to destination \( j \) is equal to \( D_j \), respectively.

To calibrate model (2.1) at some base year, with data on a

given set of origin zones \( O_i \) and destination zones \( D_j \), a parameter

value for the transport cost function \( f(c_{ij}) \) has to be found that

'best' reproduces the known base year trip pattern \( T_{ij} \) (see, e.g.,

Foot, 1986). Since the balancing factors \( A_i \) and \( B_j \) form a set of

simultaneous non-linear equations (see equations (2.2) and (2.3)), an

iterative procedure is necessary to solve the model. For this purpose

Hyman's method (Hyman, 1969) is usually applied.
Particular cases of model (2.1) are the singly-constrained SIM and the unconstrained SIM. The first model (a so-called production-constrained SIM) can be represented by the following equation:

\[ T_{ij} = A_i O_i D_j \exp(-\beta c_{ij}) \]  

(2.4)

where the balancing factor \( A_i \) now has the following specification:

\[ A_i = \left[ \sum_j D_j \exp(-\beta c_{ij}) \right]^{-1} \]  

(2.5)

The value of \( A_i \) \(^1\) serves to ensure that model (2.4) reproduces the volume of flows originating from zone \( i \), so that:

\[ O_i = \sum_j T_{ij} \]  

(2.6)

It is interesting to recall that in a static context SIMs can be derived from:

a) Newton's gravity law in physics (see, e.g., Ravenstein, 1885);

b) entropy maximization in statistical mechanics (see, Wilson, 1967);

c) linear and non-linear programming approaches (see Evans, 1973 and Nijkamp, 1979);

d) information minimization approaches (see Batten, 1983);

e) Alonso's theory of movement (see Alonso, 1978).

It should be noted that in this framework SIMs are essentially considered as aggregate models describing the macrostate of a system and based on macro data.

A further, very interesting derivation of SIMs, is also the one emerging from random utility theory in economics (see, for example Anas, 1983). In particular the formal analogy between multinomial

\(^1\) It is interesting to note that the inverse of \( A_i \) in (2.5) is usually interpreted as a measure of the accessibility of zone \( i \) (see, among others, Weibull, 1976).
t (MNL) models belonging to the class of Discrete Choice Models (DCMs) and SIMs has often been stressed in recent years (see, for a review, Nijkamp and Reggiani, 1989).

In fact, if we examine the structure of a production-strained SIM in its probabilistic form, i.e.:

\[ P_{ij} = \frac{T_{ij}}{O_i} = A_i B_j D_j \exp(-\beta c_{ij}) \]  

(2.7)

It is clear that, if we introduce in equation (2.7) the expression of the balancing factor \( A_i \) like in (2.2), the following equivalence results:

\[ P_{ij} = W_j \exp(\beta u_{ij}) / \sum_j W_j \exp(\beta u_{ij}) \]  

(2.8)

where \( W_j = B_j D_j \) can be interpreted as a weighting factor reflecting the attractiveness of a point of destination \( j \), and where \( u_{ij} = -c_{ij} \) can be considered as the deterministic part of the individual random utility underlying an MNL model (see, for the derivation of an MNL model, the seminal work of McFadden, 1974 and Domencich and McFadden, 1975).

Expression (2.8) is clearly an MNL model. For the sake of convenience we will suppose that the reader is familiar with the basic literature on MNL models and on DCMs. Consequently, we will summarize here some basic considerations related to the equivalence between (2.7) and (2.8):

If a singly constrained SIM can be considered equivalent to an MNL model, it also means that SIMs embed the limits inherent in MNL models such as the so-called IIA property.

Secondly, since << the same model without any aggregation error may be derived in disaggregate form from both entropy and utility maximization >> (see Batten and Boyce, 1986, p. 378), it follows that entropy and SIMs are not inherently less behavioural than stochastic utility models of DCMs (and MNL models in particular). Therefore, in this perspective, SIMs can be considered as aggregate models of human behaviour.
It should be noted that the above mentioned SIMs mainly deal with the demand side in a transportation system. However, it has also been pointed out (see Florian and Gaudry, 1983) that a phenomenon designated as supply at one particular level for a transportation system may become demand at another level. Consequently, relationships among different levels can be considered as input-output interactions, so that a precise distinction between supply and demand side can be made only at one particular level of the system. It turns out that the same SIM can be used in both demand and supply side analysis (see, for example, the concept of accessibility derived from spatial interaction analysis and applied to infrastructure systems; see Rietveld, 1989).

However, most models developed in the sixties and seventies and ending up with a SIM structure are still static/deterministic models of equilibrium and do not consider the time paths followed by the transportation system components as well as the uncertainty of the system and its network (see the special issue of Transportation Research: Boyce, 1985).

In this context it is interesting to emphasize the formal connections between SIMs and MNL models, since this link can place more emphasis on the analysis of individual motives and on the impact of micro random behaviour upon the functioning of transport systems (in view of the need to a better understanding of the stochasticity of transport systems).

In this context a final connection is also given by the integration of SIMs with rational screening methods related to risky alternatives (e.g., the stochastic dominance approach). This unifying approach which also shows the possibility of linking models of choice behaviour under certainty and models of choice behaviour under uncertainty (see Reggiani and Stefani, 1986) has recently also been applied to modal choice problems in a transportation system (see Reggiani and Stefani, 1989) by considering the attributes of the alternatives according to different states of nature.

Having now briefly reviewed the connections between SIMs and behavioural models, the next step will be to draw our attention to the broad category of dynamic models (DMs) developed more recently in transportation systems, in order to take into account newly emerging
relevant aspects of system dynamics, such as slow and fast dynamics, uncertainty, bifurcations, catastrophic changes, chaotic behaviour, fractal structures, etc. In particular we will point out a common similarity in these DMs - despite their different theoretical sources - viz. their close association, under particular conditions, with dynamic SIMs.

3. Analysis of Dynamic Properties in Spatial Interaction Models

It has been recently shown that dynamic SIMs can emerge as an equilibrium solution from an optimal control entropy approach (see Nijkamp and Reggiani, 1989). The structure of this emerging SIM is formally equivalent to (2.1), although in this case all variables are depending on time.

Obviously also in a dynamic framework the equivalence between SIMs and MNL models still persists, so that we get the following logit form:

\[ P_j = \frac{\exp(u_{j,t})}{\sum_{k} \exp(u_{k,t})} \]  \hspace{1cm} (3.1)

which is equivalent to a dynamic SIM and which represents the probability of choosing alternative j at time t.

In this context we may recall a recent result, i.e., that the rate of changes of \( P_j \) (formulated in 3.1) with respect to time (i.e. \( \dot{P}_j \)) can be expressed by a structure of the Volterra-Lotka type (see Sonis, 1988 and Nijkamp and Reggiani, 1990b), as follows:

\[ \dot{P}_j = \dot{u}_j P_j (1-P_j) - P_j \sum_{k} \dot{u}_k P_k \]  \hspace{1cm} (3.2)

It is also interesting to note that a particular case of the above dynamic MNL, i.e. a binary case, expressed by deleting the last term in (3.2) (i.e. the interaction term) is a logistic growth model. It is also well-known that the difference version of the standard logistic growth model has been thoroughly discussed in the literature for its capability of generating bifurcations and chaos for certain values of the growth parameter (see, e.g., the seminal work by May, 1976).
Therefore, a 'binary' logit model, belonging to the family of May models, shows the same 'chaotic' properties.

Given the above mentioned link between logistic functions, dynamic SIMs and dynamic MNL models, we will give in the next section a more in depth overview of the most frequently used dynamic transportation models in order to show their connection with a logistic structure and hence with SIMs.

4. Typology of Dynamic Transportation Models
4.1 Introduction

As we noted previously the last decade has shown a boom in the interest in the development of both behavioural and dynamic models, as it is generally expected that such models are capable to describe and represent the behavioural mechanisms underlying the evolutionary changes in complex transportation and network systems (see, e.g., Ben-Akiva, 1985). Consequently, a wide variety of multi-temporal or dynamic transportation models has arisen in the past decade with the aim of providing a stronger and more useful analytical support to planning processes than conventional static tools (such as static spatial interaction models, linear programming models, etc.). In this context it is noteworthy that SIMs tend to become again a focal point of analysis, since they can deal with the complicated and interwoven pattern of human activities in space and time.

However, despite all progress made in this new research direction (mostly from a theoretical viewpoint), still some important research questions remain, which largely concern the applicability of these dynamic models in relation to the scale of analysis at which various operational developments of these models are taking place. In particular, this important field of reflection concerns the advantages and disadvantages related to the use of macro-meso-micro approaches.

On the one hand, it is evident that aggregate representation may become extremely cumbersome and inefficient when it is necessary to represent complex systems, especially where there is considerable heterogeneity amongst the actors in those systems (see Clarke and Wilson, 1986). On the other hand, it is clear that the problems of data availability and computational processing requirements are often
in contrast with the need to use a micro-oriented approach. Moreover, the response of a population in aggregated models does not always correspond to an aggregation of the individual responses obtained from a micro model, so that it seems evident that the phenomena being studied require a careful consideration as regards the nature of their level of analysis (see again Clarke and Wilson, 1986). This problem has also been treated in analytical attempts focusing the attention on the interdependencies between micro- and macro-responses, which also depend on the interaction between demand and supply.

4.2 Macro-dynamic approaches

Several dynamic models of spatial structure have recently been developed at a macro level. We will give here a few illustrations. An example is the model developed by Allen et al. (1978), in which the evolutionary growth of zonal activities is assumed to follow a logistic pattern. Allen et al.’s model is a comprehensive model representing urban activities such as employment and residential population. A major finding in this model is that random fluctuations (e.g., changes due to infrastructure constructions) may alter the related urban evolution.

Another dynamic model of the logistic type is the one developed by Harris and Wilson (1978) and Wilson (1981). In this case the standard static spatial interaction model for activity allocation has been embedded into a dynamic evolutionary framework, again of a logistic type. Bifurcations and catastrophic behaviour emerge from this model, depending on particular values of the parameters. Obviously owing to this logistic structure also oscillations and cycles may occur.

These two important models have induced a wide spread production of related models both from theoretical and empirical viewpoints, also in a stochastic framework (see also Nijkamp and Reggiani, 1988). However, it should also be noted that the above mentioned two models primarily focus on the supply side, without clear dynamic equations for the demand side.

Another stream of research at the macro-level is the series of models based on ecological dynamics of the Volterra-Lotka type (see
Dendrinos and Mullally, 1981); in this formulation of interacting biological species, each species is characterized by a birth-death process of the logistic type. Recent papers on this topic show the integration between ecological models and optimal control models (Nijkamp and Reggiani, 1990a), between ecological models and random fluctuations of a white noise type (Campisi, 1986) or between ecological models, spatial interaction models of a gravity type and turbulence (Dendrinos, 1988).

Obviously, since a Lotka-Volterra system is a system of interrelated equations, we get by necessity here interaction mechanisms of supply and demand. Furthermore, given the related logistic form, it is also here again possible to get -for critical parameter values- oscillations and complex behaviour.

The last group of macro approaches can be found in the area represented by models based on optimal control approaches or dynamic programming analysis. Also here different forms of equilibrium/disequilibrium may emerge (e.g., saddle points, borderline stability) which show the possibility of unstable motions.

As a synthesis we may conclude that a common trend in these groups of macro-approaches is the development of models that are able to exhibit (under certain conditions) complex or chaotic behaviour and hence outcomes which are hardly foreseeable by modellers and planners. This lack of predictability of future events is clearly also a major concern in transportation planning. Thus another important research problem is emerging, i.e., the relevance of critical parameter values, such as their speed of change in a geographic or planning context in order to understand whether the system at hand is moving towards a predictable or complex (unpredictable) evolution.

4.3 Micro-meso dynamic approaches

In this Section we will briefly pay attention to the considerable body of literature based on micro simulation models (see Clarke and Wilson, 1986). Given the above mentioned drawbacks related to a macro-approach, a mixture of aggregate dynamic models in conjunction with micro-simulation (the micro-meso approach) has recently been advocated and adopted for various spatial applications.
(see also Birkin and Clarke, 1983). In this way also an integration between demand and supply results is possible. In other words, micro-meso dynamic approaches utilize individual data in conjunction with aggregate equations.

Another interesting micro-meso approach is the well-known logit model, based on a micro-economic foundation. It has recently been shown (see Section 3) that the growth over time of people choosing such alternatives as travel choice mode, destination, etc. according to a logit procedure follows again a logistic pattern. Such development can also lead to a chaotic or irregular behaviour for particular values of a utility function. The most important consequence of this result is the link between DCMs and hence the equivalent SIMs and logistic formulations of these models. Since most of the models referred to end up with a logistic shape, it is clear that SIMs can be considered as a comprehensive framework incorporating many advanced models also at a dynamic level.

In this area also the master equation/mean value equation models (see Haag and Weidlich, 1984; Haag, 1989) have to be mentioned. These models have been used extensively in spatial flow analysis. This framework models the uncertainties in the decision process of the individuals via the master equation approach. The mean values are then obtained from the master equation by an aggregation of the individual probability distributions. Thus this approach provides the link between micro-economic aspects and the macro-economic equation of motions for aggregate mean values.

In this context also compartmental analysis should be mentioned (see De Palma and Lefevre, 1984) which consists of equations which are the approximate mean-value equations. It has recently been shown (see Reiner et al., 1986) that also these meanvalue equations may display chaotic behaviour with strange attractors, given particular conditions for the group interaction. On the other hand, this result is not surprising, since the stationary solutions of mean value equations are strongly related to logit models. Hence it is plausible that interrelated logistic functions underlie the emerging chaotic motions.

After this brief review based on a typology of dynamic models and their potential in transportation planning with respect to the
scale of analysis, we will now show their evolution in comparison with the evolutionary pattern of SIMs.

5. Evolution of Spatial Interaction Models

The models discussed in the previous section can be unified in the broad area of DMs and then compared, in their evolution, to the groups of static SIMs and DCMs, in the light of life cycle concepts. As a synthesis, we can represent the series of the transportation models studied and adopted so far according to the scheme of overlapping generations illustrated in Figure 1.

In Figure 1 we have essentially depicted the generation and diffusion of the three main families of models which have received a great deal of attention in the last century, i.e., Spatial Interaction Models (SIMs), Discrete Choice Models (DCMs) and Dynamic Models (DMs).

From Figure 1 we can see that -while SIMs present a smooth development at the beginning of the century with more emphasis after the sixties- DCMs and DMs exhibit a rapid growth (from both a theoretical and empirical point of view). It should be noted that in DMs we have included the whole group of dynamic models treated so far, so that we can observe that after the mid seventies a broad number of mathematical models emerged.

Altogether we can notice an overlapping generation of models: in particular we can unify all these models in a unique-general logistic shape evolving nowadays where the points A and B mean the theoretical conjunction of DCMs and DMs with SIMs, respectively. However, we may rationally speculate that we are likely to approach a saturation level of the development of the above mentioned models. Probably, from this upper level, new tools will emerge in the next century, in response to new activity patterns. This upper level can likely be linked to the analytical structure of the models, since there are inevitable constraints in their formulation, so that also from a mathematical point of view the potential of such models will certainly reach a limit.

Since it has been underlined in the previous section that most of these models can be reformulated in terms of a logit-logistic formulation, and hence can be interpreted in a SIM structure, the broad potential of SIMs can be considered as such a limit.
Figure 1 Overlapping Generations of Models

Legend: SIMs = Spatial Interaction Models
DCMs = Discrete Choice Models
DMs = Dynamic Models
Y-axis = Development of the Models
X-axis = Time Period
6. Conclusions

In this paper the relevance of spatial interaction models in static and dynamic transportation models has been investigated. In particular the booming interest in dynamic transport models, which are able to describe and explain the evolutionary mobility patterns, was outlined, with particular reference to the emergence of new models displaying irregular behaviour and hence (seemingly) unpredictable movements. It has been argued that the different typologies of dynamic approaches (macro, meso/micro) can be reconducted, under certain conditions, to the well-known family of logistic models, capable to capture unstable behaviour, and hence to SIMs which seem to emerge as a reference framework for all the other models.

It seems also necessary in these dynamic models to focus the attention on the study of critical parameter values which can be manipulated by planners in order to avoid unpredictable or uncontrolled movements in transportation systems.

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