General Equilibrium in a closed International Trade model

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by

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1. Introduction

In the fifties Arrow and Debreu [2,3], Gale [7], McKenzie [10,11], and others developed from basic axioms a theory on the existence of a market clearing price system in an economy where the agents act as price takers. Following the basic existence results on general equilibrium in an exchange economy many developments have been made. For a survey we refer to e.g. Debreu [4]. Work on general equilibrium theory is ongoing, for example a recent 'state of the art' survey of the case of infinitely many commodities is Aliprantis, Brown and Burkinshaw [1].

In general equilibrium theory an exchange economy is specified by characteristics on preferences and endowments of the traders. Given some price system which is a vector having as its components the prices for all commodities in the economy, the traders specify their planned demands and supplies of the commodities at this price system. The market excess demand for a commodity is the difference between the total demand and the total supply of each commodity. An equilibrium of the economy is a price system such that the excess demand is equal to zero for all commodities simultaneously. Then all markets clear and all planned demands and supplies can actually be made. The central result of the general equilibrium theory is that under rather weak conditions on the behaviour of the traders there exists an equilibrium price vector.

A central theme in the theory of international trade is the impact of exchange rate policy and monetary policy on the trade balance. Many authors have studied such problems in a general equilibrium macro-economic model either of an open one-country economy or of a closed two-country economy. In an open model it is assumed that excess demands or supplies can be imported, respectively exported at given (world market) prices. In a closed model commodity flows to or from the outside world are not possible. For a basic reference we refer to Dixit and Norman [5]. In most research in this area the authors are concerned with comparative statics and dynamics in the context of a Hicksian general equilibrium model. In such a model the existence of an equilibrium is taken for granted. In this paper we will consider the existence of an equilibrium in a closed more-country general equilibrium model. To show the existence of such an equilibrium we will reformulate the pure exchange equilibrium model as a more-country international trade model. The classical theory of international trade deals with only one kind of common money. In the so-called monetary approach introduced by Hamada [7], each country has its own money commodity. We will show that the difference between
the two models is only a matter of normalization of the prices. In the classical treatment all exchange rates are normalized to be equal to one. In the monetary approach, each country has a domestic money commodity with the domestic price equal to one. We will see in this paper that both models follow from the standard pure exchange economy. However, the monetary approach facilitates the analysis of exchange rate policy or monetary policy.

This paper has been organized as follows. In the next Section we give the basic Walrasian general equilibrium model of an exchange economy. In Section 3 we interpret this basic model as a model of international trade. In Section 4 we give a reformulation of the model in which we introduce the relation between the exchange rate and the trade balance. Section 5 is concerned with the existence of equilibrium under the monetary approach and the analogy with the classical approach. Finally, in Section 6 we discuss the existence of equilibrium under exchange rate policy.

2. Preliminaries

We consider an exchange economy with \( n+1 \) commodities, indexed \( k = 0,1,...,n \), and \( H \) agents, indexed \( h = 1,...,H \). Each agent represents a utility maximizing consumer or household. Agent \( h \), \( h = 1,...,H \), is characterized by a consumption set \( X^h \subset \mathbb{R}^{n+1} \), a utility function \( u^h \) on \( X^h \) representing her preferences, and a vector of initial endowments \( w^h \in X^h \). The set \( X^h \) is the set of all bundles of commodities possible for the \( h \)-th consumer. The function \( u^h \) assigns a real number, the utility of \( h \), to each consumption bundle in \( X^h \). The vector \( w^h \) is the bundle of commodities available to the consumer before any trade has taken place. We may interpret the initial endowments of agent \( h \) as the stock of commodities she holds after production has taken place. The problem we want to consider in this paper is the exchange of these commodities between agents without considering the process of production. Although equilibrium in an economy with production is more complicated than in an economy without production, we can restrict ourselves to the latter if we want to consider the problem of trade between agents. Since in this paper we are concerned with the latter problem, we restrict ourselves to a pure exchange economy.

For ease of notation, Let \( \Omega \) denote the nonnegative orthant \( \{q \in \mathbb{R}^{n+1} | q_j \geq 0, j = 0,...,n \} \) of the \((n+1)\)-dimensional Euclidean space. For all \( h \), we make the following assumptions, where \( w = \sum_h w^h \) is the vector of total initial endowments.

\[ A_1. \] The consumption set \( X^h \) is a closed, convex subset of \( \Omega \), containing the set 
\[ \{x \in \Omega | x_k \leq w_k, k = 0,...,n \}. \]
A2. The utility function $u^h$ is a strictly quasi-concave 1) continuous function from $X^h$ to $\mathbb{R}$, satisfying monotonicity, i.e., for all $x, y \in X^h$, $x_k \geq y_k$ for all $k$ and $x_k > y_k$ for at least one $k$ implies $u^h(x) > u^h(y)$.  

A3. For each commodity $k$, $w^h_k > 0$.

Given $p \in \Omega \{0\}$ being a vector of nonnegative prices with at least one positive component, the budget of agent $h$ at these prices is given by $y^h(p) = p^T w^h$, where $p^T a$ denotes the inner product $\sum_j p_j a_j$. So, $y^h(p)$ is the budget household $h$ would obtain from selling her initial endowments against prices $p$. Then household $h$ chooses a consumption bundle in $X^h$ that maximizes her utility given her budget, i.e., the household maximizes $u^h(x)$ subject to $p^T x \leq y^h(p) = p^T w^h$. So, the household chooses an utility maximizing consumption bundle in her budget set being the set of all consumption bundles having a value less or equal than the value of her initial endowments. Let $d^h(p) \in X^h$ be such a consumption bundle. Under the assumptions A1, A2, and A3 such a consumption bundle exists and is preferred to all other $x \in X^h$ satisfying the budget constraint, which implies that $d^h(p)$ is unique. Moreover, the assumptions imply that the demand function $d^h: \Omega \{0\} \to X^h$ is continuous and satisfies $p^T d^h(p) = p^T w^h$. The net demand of household $h$ is given by $d^h(p) - w^h$, being the difference between the optimal consumption given the prices $p$ under the budget restriction and the vector of initial endowments. Observe that the value of the net demand equals zero.

Definition 2.1 A general or Walrasian equilibrium is a vector of prices $p^*$, and a set of consumption vectors $x^h$, $h = 1,...,H$, such that

(a) $x^h = d^h(p^*)$ for $h = 1,...,H$,
(b) $\sum_h x^h = \sum_h w^h$.

Condition (a) says that in equilibrium each consumer maximizes her utility under her budget constraint. Condition (b) implies that in equilibrium for each commodity the total demand is equal to the total initial endowment.

Let $z(p)$ denote the total excess demand at price $p$, i.e.,

$$z(p) = \sum_h (d^h(p) - w^h).$$

It follows that $p^*$ is an equilibrium price vector if $z(p^*) = 0$. From the continuity of the demand functions $d^h$ it follows that $z$ is a continuous function from $\Omega \{0\}$ to $\mathbb{R}^{n+1}$. Moreover, from the utility maximization under A1, A2 and A3, it follows that for all

1) It is sufficient to assume quasi-concavity of the utility functions. However, assuming strict quasi-concavity implies that the demand functions are continuous functions instead of upper semi-continuous multifunctions.
\[ p \in \Omega \backslash \{0\}, \]

i) \[ p^T z(p) = 0 \quad (\text{Walras' law, i.e., the total value of the excess demands is equal to zero}), \]

ii) \[ z(\lambda p) = z(p) \quad \text{for all } \lambda > 0 \quad (\text{homogeneity of degree zero}), \]

iii) \[ z_k(p) > 0 \quad \text{if } p_k = 0, \quad k = 0, ..., n \quad (\text{desirability}). \]

Property ii) says that the excess demand does not change if all prices are multiplied by a positive factor. So, let \( p = (p_1, ..., p_{n+1})^T \) be a vector of equilibrium prices. Then also \( p^* = \lambda^{-1} p \) is a vector of equilibrium prices. Taking \( \lambda = \Sigma_k p_k \) we can normalize the sum of the prices to one. So, to find equilibrium prices we can restrict ourselves to the price space \( S^n = \{ p \in \Omega \backslash \{0\} \mid \Sigma_k p_k = 1 \} \) being the \( n \)-dimensional unit simplex. By using a fixed point theorem it can be proved that \( z \) has a zero point \( p^* \) on \( S^n \) (see e.g. Varian [14]). Hence there exists a vector of equilibrium prices. Property (iii) ensures that in equilibrium all prices are positive.

3. The pure exchange international trade model

We now interpret the pure exchange equilibrium model as a model of international trade. In this case a household \( h \) represents some country, so that \( d^h(p) \) yields the aggregate demand of country \( h \) at price vector \( p \).

When interpreting the general equilibrium model as an international trade model, we should distinguish between tradeable goods and non-tradeable goods. Tradeables or international goods can be exchanged between all countries, while non-tradeables or domestic commodities of a country can only be exchanged between residents of that country. The residents of a country have endowments only of the tradeables and of the non-tradeables of their own country. Moreover, the utility of a consumer in some country only depends on the tradeables and their own country's non-tradeables and is independent of the non-tradeables of foreign countries.

Let \( l+n(h) \) be the number of non-tradeables of country \( h \), \( h = 1, ..., H \) and let \( l+n(0) \) be the number of tradeables. So, the total number of commodities equals \( N = \Sigma_{h=0}^H (l+n(h)) \). For ease of notation in the remaining of the paper, we index the non-tradeables of country \( h \) by \( k = 1, ..., l+n(h) \) and we index the tradeables by \( k = 0, ..., n(0) \).

With \( a \in \mathbb{R}^N \) a vector of prices or quantities, we denote \( a \) by \( (a_0, a_1, ..., a_H) \) with \( a_0 \in \mathbb{R}^{n(0)+1} \) the vector of prices or quantities corresponding to the tradeable goods, and \( a_h \in \mathbb{R}^{n(h)+1} \) the vector of prices or quantities corresponding to the non-tradeables of country \( h \), \( h = 1, ..., H \). Country \( h \) does not possess non-tradeables of the
foreign countries, so that its vector of initial endowments \( w^h = (w^h_0, w^h_1, ..., w^h_H) \) satisfies \( w^h_j = 0 \) voor \( j \neq 0, h \). As observed above, the utility of the residents of country \( h \) only depends on \( x^0 \) and \( x^h \) if \( x = (x^0, x^1, ..., x^H) \) is their consumption vector. From this it follows that the excess demand function \( z^h = (z^h_0, z^h_1, ..., z^h_H) \) of country \( h \), given by

\[
z^h(p) = d^h(p) - w^h
\]

only depends on the prices of the tradeables and its own domestic goods and satisfies

(a) \( z^h_0(p) = z^h_0(p^0_0, p^h_0), \)
(b) \( z^h_j(p) = 0 \) voor \( j \neq 0, h, \)
(c) \( p^T z^h(p) = p^0_0 z^h_0(p^0_0, p^h_0) + p^h_0 z^h_0(p^0_0, p^h_0) = 0, \)

with \( z^h_0(p^0_0, p^h_0) \) the vector of excess demands of country \( h \) for tradeables and \( z^h_j(p^0_0, p^h_0) \) the vector of excess demands of country \( h \) for the domestic goods of country \( j \). Property (b) says that there is no excess demand for the non-tradeables of foreign countries and (c) states that the total value of the excess demand is equal to zero (Walras' law for country \( h \)). Observe that \( p^0_0 z^h_0(p^0_0, p^h_0) \) is the total value of the excess demands for tradeables in country \( h \).

The world excess demand function is given by \( z(p) = \Sigma_h z^h_0(p) \). It satisfies Walras' law \( p^T z(p) = 0 \) and can be written as \( z(p) = (z^0_0(p), z^1(p), ..., z^H(p)) \) with

(d) \( z^0_0(p) = \Sigma_h z^h_0(p^0_0, p^h_0), \)
(e) \( z^h(p) = z^h_0(p^0_0, p^h_0), h = 1, ..., H. \)

A price vector \( p^* = (p^{*}_0, p^{*}_1, ..., p^{*}_H) \) is an equilibrium price vector if \( z(p^*) \) is equal to zero. Although the assumption A3 and the assumption of monotonicity do not hold, it can be proved that there exists an equilibrium price vector with all prices positive. From (e) it follows that in equilibrium \( z^h(p^*^0_0, p^*^h_0) = 0 \) and hence together with (c) that

\[
p^*^T z^h(p^*) = p^*^0_0 z^h_0(p^*^0_0, p^*^h_0) + p^*^h_0 z^h_0(p^*^0_0, p^*^h_0) = p^*^0_0 z^h_0(p^*^0_0, p^*^h_0) = 0.
\]

So, in equilibrium the \( h \)-th country's total value of the excess demands for tradeables is equal to zero. Observe that the total value of the excess demands for tradeables is equal to the deficit on the trade balance of country \( h \). So, in equilibrium all countries have a zero trade balance. In the next section we will allow for equilibria with non-zero trade balances.
4. The trade balance and the exchange rate

In this section we will consider explicitly the relation between the trade balance and the exchange rate. The exchange rate of a country relates the prices of the domestic goods with the prices of the tradeables and will be defined more precisely later on in this section. Following Van der Laan [9] we will use the structure of the excess demand function to reformulate the international trade model. In this reformulation of the model the exchange rate will appear explicitly as one of the variables with the trade balance as the corresponding equation.

To introduce exchange rates in the model we take the Cartesian product of \( H+1 \) unit simplices, \( S = S_0 \times S_1 \times S_2 \times \ldots \times S_H \), with \( S_0 \) the \( n(0) \)-dimensional unit simplex and, for \( h = 1, \ldots, H \), \( S_h \) the \((1+n(h))\)-dimensional unit simplex. Related to an element \( q = (q_0, q_1, \ldots, q_H) \in S \), where \( q_0 = (q_{00}, \ldots, q_{0j})^T \) with \( j = n(0) \) and where for \( h \neq 0 \), \( q_h = (q_{h0}, \ldots, q_{hj})^T \) with \( j = 1+n(h) \), we define for all \( h \) a price vector \( \pi^h(q_0,q_h) = [(\pi^h_0(q_0,q_h)), (\pi^h_h(q_0,q_h))] \in \mathbb{R}^{n(0)+1} \times \mathbb{R}^{n(h)+1} \) for the tradeables and country \( h \)'s domestic commodities by

\[
\begin{align*}
(4.1) \quad & \pi^h_{0k}(q_0,q_h) = q_{00}q_{0k}, \quad k = 0, \ldots, n(0) \\
(4.2) \quad & \pi^h_{hk}(q_0,q_h) = q_{hk}, \quad k = 1, \ldots, 1+n(h).
\end{align*}
\]

We show that for each positive price vector \( p \in S^{N-1} \), there is only one element of \( S \) yielding the same relative prices for each country \( h \), \( h = 1, \ldots, H \). To do so, let \( p = (p_0, p_1, \ldots, p_H) \) be an element of \( S^{N-1} \), i.e., \( p_0 \in \mathbb{R}^{n(0)+1} \), \( p_h \in \mathbb{R}^{n(h)+1} \), \( h = 1, \ldots, H \) and the sum of all components \( \Sigma_h \Sigma_i p_{hi} \) equal to one. Now define \( q_0k \) by

\[
(4.3) \quad q_{0k} = p_{0k}/\Sigma_i p_{0i}, \quad k = 0, \ldots, n(0)
\]

and for \( h = 1, \ldots, H \), define \( q_{hk} \) by

\[
\begin{align*}
(4.4) \quad & q_{h0} = (\Sigma_i p_{0i})/(\Sigma_i p_{0i} + \Sigma_i p_{hi}) \\
(4.5) \quad & q_{hk} = p_{hk}/(\Sigma_i p_{0i} + \Sigma_i p_{hi}), \quad k = 1, \ldots, 1+n(h).
\end{align*}
\]

Clearly, \( \Sigma_k q_{0k} = 1 \), and for \( h = 1, \ldots, H \), also \( \Sigma_k q_{hk} = 1 \). Moreover, for \( h = 1, \ldots, H \), \( [\pi^h_0(q_0,q_h)), (\pi^h_h(q_0,q_h))] = [\lambda p_0, \lambda p_h] \), with \( \lambda = (\Sigma_i p_{0i} + \Sigma_i p_{hi})^{-1} \). So, for \( q \) defined by (4.3), (4.4) and (4.5) country \( h \) has at prices \( (\pi^h_0, \pi^h_h) \) defined by (4.1) and (4.2) the same relative prices for the tradeables and domestic goods as at prices \( (p_0, p_h) \).
Since the excess demand function is homogeneous of degree zero, this implies that at \((\pi^h_0, \pi^h_h)\) the excess demand of the \(h\)-th country will be the same as at \((p_0, p_h)\).

The component \(q_{0k}, k = 0, \ldots, n(0)\), can be interpreted as the price of the \(k\)-th tradeable denoted in some international currency, e.g. ECU's, whereas \(q_{hk}, k = 1, \ldots, 1+n(h)\), can be interpreted as the price of the \(k\)-th domestic commodity of country \(h\) in the own currency. By multiplying the components of \(q_0\) by \(q_{ho}\) we get the prices of the tradeables in the currency of country \(h\). So, the component \(q_{ho}\) measures the price in the home currency of a unit of the international currency so that we can interpret \(q_{ho}\) as the exchange rate between the international currency and the currency of country \(h\). An increase of \(q_{ho}\) increases the prices of the tradeables in terms of the domestic currency of country \(h\). Thus, an increase (decrease) of \(q_{ho}\) yields a depreciation (appreciation) of the home currency.

The next step is to modify the excess demand function. Therefore, we define
\[
\xi(q) = (\xi_0(q), \xi_1(q), \ldots, \xi_H(q))\text{ with } \xi_0(q) \in \mathbb{R}^{n(0)+1} \text{ and } \xi_h(q) \in \mathbb{R}^{n(h)+2} \text{ for } h = 1, \ldots, H \text{ by}
\]
\[
\xi_0(q) = \Sigma_h z^h_0(\pi^h(q_0, q_h)),
\]
and
\[
\xi_{ho}(q) = q_0 \Sigma_k z^h_{0k}(\pi^h(q_0, q_h)) = \Sigma_k q_{0k} z^h_{0k}(\pi^h(q_0, q_h)),
\]
\[
\xi_{hk}(q) = z^h_{hk}(\pi^h(q_0, q_h)), \quad k = 1, \ldots, 1+n(h).
\]

So, \(\xi_0(q)\) is the excess demand vector for the tradeables and \(\xi_{hk}(q), h,k \neq 0\) is the excess demand for the \(k\)-th domestic good of country \(h\) at prices defined by (4.1) and (4.2). Moreover \(\xi_{ho}(q)\) is the value of the excess demand of country \(h\) of the tradeables, i.e., the deficit on the trade balance, measured in the international currency. It follows straightforward that \(\xi(q)\) satisfies the following properties:

\begin{align}
\text{(4.6) for all } h \neq 0, q_h \Sigma_h \xi_h(q) = \\
\Sigma_k q_{0k} q_{0k} z^h_{0k}(\pi^h(q_0, q_h)) + \Sigma_k q_{hk} z^h_{hk}(\pi^h(q_0, q_h)) = 0,
\end{align}

because of Walras' Law and
\begin{align}
\text{(4.7) } q_0 \Sigma \xi_0(q) = \Sigma_k q_{0k} q_{0k} z^h_{0k}(\pi^h(q_0, q_h)) = \Sigma_h \xi_{ho}(q).
\end{align}

Property (4.6) shows again that \(\Sigma_k q_{0k} q_{0k} z^h_{0k}(\pi^h(q_0, q_h)) = 0\) if \(z^h_{hk}(\pi^h(q_0, q_h)) = 0\) for all \(k = 1, \ldots, n(h)+1\), i.e., the trade balance of country \(h\) is in equilibrium if all
domestic markets are in equilibrium. Property (4.7) shows that the value of the total excess demand for the tradeables equals the sum of the trade deficits of the countries.

Clearly, if $p^\ast = (p_0^\ast, p_1^\ast, ..., p_H^\ast)$ is a positive vector of equilibrium prices with $z(p^\ast) = (z_0(p^\ast), z_1(p^\ast), ..., z_H(p^\ast)) = 0$ then $\xi(q^\ast) = (\xi_0(q^\ast), \xi_1(q^\ast), ..., \xi_H(q^\ast)) = 0$ with $q^\ast$ as defined by (4.3), (4.4) and (4.5). So, a zero point of $z$ with all prices positive yields a corresponding zero point of $\xi$. Since there exists such a zero point of $z$, there also exists a zero point of $\xi$. At such a zero point $q^\ast$ the variables $q_{h0}^\ast$, $h = 1,...,H$, yield the equilibrium exchange rates of the countries. If for some $q$, $\xi(q) \neq 0$, then equilibrium can be found by adjusting the variables $q_{hj}$ on the product space $S$. In particular, country $h$ can reduce a deficit on its trade balance by a rise of the exchange rate $q_{h0}$. In Van der Laan [9] and Van den Elzen and van der Laan [6] it has been shown that there exists a convergent adjustment process.

5. The monetary approach

With the formulation of the international trade model on the product space of simplices we have introduced the variables $q_{h0}$, $h = 1,...,H$, as the exchange rates, while the corresponding components $\xi_{h0}(q)$ of the modified excess demand function yield the deficits on the trade balances. We now suppose that for each country $h$, one of its domestic commodities serves a money commodity, say the commodity indexed by $1+n(h)$. Although the consumers do not obtain direct utility from money, the amount of money held by a consumer appears in the utility function as savings for future consumption. So, money is held by the consumers for the sake of commodities they will buy at later dates. We now redefine the exchange rate as $e_h = q_{h0}/q_{hk}$ for $k = 1+n(h)$, i.e., money serves as the domestic currency and $e_h$ measures the value of the international currency in units of country $h$'s money. Since demand is homogeneous of degree zero, it follows from (4.1) and (4.2) that $\pi^h(q_0, \lambda q_h) = \lambda \pi^h(q_0, q_h)$. Hence, if $q^\ast = (q_0^\ast, q_1^\ast, ..., q_H^\ast)$ yields a vector of equilibrium prices, then also the vector $q' = (q_0', q_1', ..., q_H')$ with $q_0' = q_0^\ast$ and for $h = 1,...,H$, $q_h' = (q_{hk}^* q_h^\ast)^{-1} q_h^\ast$, where $k = 1+n(h)$, yields equilibrium prices. Let

$$P = \{(p_0, p_1, ..., p_H, e) \mid p_h \in R_{\geq 0}^{(h)+1}, h = 0, ..., H, e \in R^+_H, \Sigma_k p_{0k} = 1,$$

$$\text{and for } k = 1+n(h), p_{hk} = 1, h = 1, ..., H \}$$

be the set of prices and exchange rates with the prices of the money commodities normalized to one. Observe that for a vector $p = (p_0, p_1, ..., p_H, e) \in P$ instead of $q \in S$, (4.1) and (4.2) become
\begin{align}
(5.1) & \; \pi^{h}_{0k}(p_{0},p_{h},e_{h}) = e_{h}p_{0k}, \quad k = 0, \ldots, n(0), \\
& \pi^{h}_{hk}(p_{0},p_{h},e_{h}) = p_{hk}, \quad k = 1, \ldots, n(h), \\
(5.2) & \; \pi^{h}_{hk}(p_{0},p_{h},e_{h}) = p_{hk} = 1, \quad k = 1 + n(h).
\end{align}

Finally, for further simplicity, in the following we denote the initial endowment of money of country $h$ by $m_{h}$, i.e., $m_{h} = w_{h}^{h}$ with $k = 1 + n(h)$, and the excess demand for money at prices $p \in P$ of country $h$ by $m_{h}(p_{0},p_{h},e_{h})$, i.e., $m_{h}(p_{0},p_{h},e_{h}) = z^{h}_{hk}(\pi^{h}(p_{0},p_{h},e_{h}))$ with $k = 1 + n(h)$. The equations (4.6) becomes

\begin{align}
(5.3) & \; \text{for all } h \neq 0, \sum_{k \neq 1 + n(h)} e_{h}p_{0k}z^{h}_{0k}(\pi^{h}(p_{0},p_{h},e_{h})) + \\
& \quad \sum_{k \neq 1 + n(h)} p_{hk}z^{h}_{hk}(\pi^{h}(p_{0},p_{h},e_{h})) + m_{h}(p_{0},p_{h},e_{h}) = 0,
\end{align}

From the existence of positive equilibrium prices $q^{*} = (q_{0}^{*}, q_{1}^{*}, \ldots, q_{H}^{*})$ we obtain the next theorem.

Theorem 5.1. There exists a positive vector $p^{*} = (p_{0}^{*}, p_{1}^{*}, \ldots, p_{H}^{*}, e^{*}) \in P$ of prices and exchange rates such that

a\) $\sum_{h} z^{h}_{0k}(\pi^{h}(p_{0}^{*}, p_{h}^{*}, e_{h}^{*})) = 0,$

and, for $h = 1, \ldots, H$,

b1) $z^{h}_{hk}(\pi^{h}(p_{0}^{*}, p_{h}^{*}, e_{h}^{*})) = 0, \quad k = 1, \ldots, n(h),$

b2) $m_{h}(p_{0}^{*}, p_{h}^{*}, e_{h}^{*}) = 0,$

Theorem 5.1 says that for the monetary model in which each country has its own money commodity, there exist prices and exchange rates in which all markets and all money balances are equilibrium. From b), (5.3) and the fact that $e_{h}^{*} > 0$ it follows that

$\sum_{k} p_{0k}z^{h}_{0k}(\pi^{h}(p_{0}^{*}, p_{h}^{*}, e_{h}^{*})) = 0, \quad h = 1, \ldots, H,$

i.e., for each $h$ the trade balance is in equilibrium.

The exchange rate gives the value in home currency of one unit of the international currency. Until now we have not interpreted the meaning of the international currency, except that we have mentioned the ECU as an example of an international currency. Another possibility is to take one of the home currencies, e.g. the home currency of a large country as the international currency. Then, of course, the exchange rate of that country is equal to one. To do so, observe that the demands for commodities are homogeneous of degree zero in prices of the domestic non-money commodities, the exchange rate, and the initial money holdings. Now, let the currency of country 1 serve as the international currency. If $p^{*} = (p_{0}^{*}, p_{1}^{*}, \ldots, p_{H}^{*}, e^{*}) \in P$ is a
vector of equilibrium values, then $p^* = (p_0^*, p_1^*, p_2^*, ..., p_H^*, e^*) \in P$ with $e^*_h = e_h^*$ for $h \neq 1$, $e^*_1 = 1$ and $p^*_{1k} = p^*_{1k}/e^*_1$, $k = 1, ..., n(h)$ is a vector of equilibrium prices in case the initial holdings of money of country 1 become $m_1/e^*_1$. So, we can normalize the exchange rate of country 1 to one by multiplying the amount of money with the inverse of the exchange rate. Mathematically this means that the exchange rate can be set equal to one by choosing the unit of money appropriately. Economically this means that the exchange rate can be set equal to one by changing the amount of money. If the Central Bank doubles the amount of money by issuing new money, new equilibrium prices will arise in which both the exchange rate and the prices of the domestic commodities have been doubled and hence the home currency is depreciated with the same factor. Under the new prices the amounts of trade in equilibrium have not been changed because of the homogeneity. This result, which says that under floating exchange rates each country can choose its own rate of inflation without affecting the flows of the real trade is well known from international trade theory, see e.g. Dixit and Norman [5, page 217] or Van der Ploeg [13, page 95]. If all countries choose their money supplies in order to normalize their exchange rates equal to one, then one unit of a currency can be exchanged against one unit of each other currency and we obtain the classical international trade model with one common currency. In fact this common currency now becomes a tradeable. However, if money becomes a tradeable it is sufficient that the sum of all money balances is in equilibrium and hence condition b2) of Theorem 5.1 which says that all money balances must be in equilibrium is not needed any longer. This allows for deficits or surplusses on the money balances and hence on their counterparts the balances of trade. Going back to the monetary model this implies that floating exchange rates are not needed any longer to get equilibrium if we allow for non-zero trade balances. This makes the model applicable for analyzing exchange rate policy.

6. Exchange rate policy

In this section we consider the existence of an equilibrium in the monetary model under policy measures. Let $(p_0^*, p_1^*, ..., p_H^*, e^*) \in P$ be a vector of equilibrium values of prices and exchange rates. We now want to consider the case of exchange rate policy of one country, say country 1. So, country 1 sets an exchange rate $e_1 \neq e^*_1$. According to (5.1) the prices of the tradeables relative to the prices of the domestic goods change. This results in disequilibrium both on the markets of the domestic commodities and the tradeables. To restore equilibrium on the domestic markets the prices of the domestic goods have to be adjusted. Let $p_1$ be the vector of new equilibrium prices for the domestic goods. Then it follows from (5.3) that
since $z_{1k}(\pi^1(p^0,0) p^1_1, e_1)) = 0$ for $k \neq 1+n(h)$. Now suppose that country 1 is able to meet their demands and supplies of the tradeables on the world market. Then the value in domestic currency $\Sigma_k e_1 p^* 0_k z_{1k}(\pi^1(p^0,0) p^1_1, e_1))$ of the excess demands on tradeables, i.e., the surplus on the trade balance if $\Sigma_k e_1 p^* 0_k z_{1k}(\pi^1(p^0,0) p^1_1, e_1)) < 0$ or the deficit if $\Sigma_k e_1 p^* 0_k z_{1k}(\pi^1(p^0,0) p^1_1, e_1)) > 0$, is compensated by the increase or decrease $m_1(p^0,0, p^1_1, e)$ of the money holdings. To interpret this result we introduce a government or Central Bank who adjusts the money holdings by issuing or destroying money, see also Dixit and Norman [5, chapter 7]. The amount of the shift in money holdings is equal to the value of the excess demand for the tradeables. So, in case of a surplus, the creation of additional money is financed by the other countries. The Central Bank receives the surplus on the trade balance in the form of reserves in international currency and obtains in this way a claim on the other countries. The opposite holds in the case of a deficit. As a result of the exchange rate policy, country 1 faces a situation of equilibrium on all domestic markets, but disequilibrium on the markets for tradeables and a non-zero trade balance. The latter is compensated by a non-zero money balance for country 1.

We now consider the other countries. We suppose that all other countries have floating exchange rates. By adjusting prices and the other countries' exchange rates it is possible to reach equilibrium again on all commodity markets. However, at the resulting equilibrium the non-zero trade balance of country 1 induces non-zero trade balances for the other countries too. Let $p = (p_0, p_1, ..., p_H, e_1)$ be the vector of new 'equilibrium' prices. Since there is equilibrium on all commodity markets, we have that at the new prices the total value of the excess demands of the tradeables is equal to zero, i.e., the sum of all surplusses and deficits on the trade balances measured in the international currency is equal to zero. Moreover, it follows from (5.3) that for all $h$,

\begin{equation}
\Sigma_k e_h p^* 0_k z_{hk}(\pi^h(p_0, p_H, e_h)) + m_h(p_0, p_H, e_h) = 0,
\end{equation}

saying that the deficit or surplus on the trade balance of country $h$ measured in the own currency is compensated by a non-zero money balance. For the classical model we have noticed already this possibility.

It should be observed that a two-country model differs from a more-country model. In a two-country model the second country has to adjust the exchange rate in order to reach equilibrium for the tradeables again. If the second country has a fixed exchange rate equilibrium can not be reached. In this case the demands of country 2 are not affected by the exchange rate policy in country 1, which results in an excess demand
or supply for the tradeables. So, country 2 must have a floating exchange rate in order to compensate the shock triggered off by the policy by country 1. However, in a more-country model, it is not necessary that all other countries have a floating exchange rate. If some of the countries have a fixed exchange rate, the exchange rate policy by country 1 does not have any impact on these countries as long as they are small compared with the rest of the world and are able to meet their supplies and demands on the world market. So, in a more-country model it is possible that some countries keep their exchange rate fixed under a change in the exchange rate of one country. To reach equilibrium on the tradeables it is sufficient that some of the other countries have a floating exchange rate. It should be observed however, that due to general equilibrium effects, also the prices of the tradeables in the new equilibrium will differ from the old prices. Since the demand depends on the prices of the tradeables, this may change the net demand for tradeables of the countries with a fixed exchange rate and also the total value of the excess demand for tradeables by these countries. So, even a fixed exchange rate may result in a surplus or deficit on the trade balance of such a country. Another policy of such a country is to adjust its exchange rate in order to maintain a zero trade balance. If there is only one tradeable both policies are the same because then the price of the tradeable commodity is always set equal to one.

We now assume that there is only one tradeable, so that a country with a fixed exchange rate is not affected by the exchange rate policy of other countries, as long as it is small compared with the rest of the world and can meet its demand or supply of the tradeable at the fixed exchange rate. Of course, the assumption of only one tradeable is rather silly. However, we are not concerned primarily in the trade between countries in commodities, but in the effects of exchange rates policies on the trade balances, money holdings, and debts. The tradeable commodity can be interpreted as an aggregate or composite bundle of commodities.

Above we have considered the impact of a change in the exchange rate by one country in a situation of general equilibrium with zero trade balances. More generally, we now consider the case that some countries have an a priori fixed exchange rate, some other countries have floating exchange rates which respond to the excess demand or supply of the tradeable commodity, while the remaining countries have a policy of zero trade balance. Because of the assumption of only one tradeable commodity, in equilibrium the latter countries have a zero excess demand for this commodity and do not interfere with the other countries. So, we can delete the latter countries from the model. This results in a model with countries having a fixed exchange rate and countries having a floating exchange rate. Let \( H^f \subset \{1, \ldots, H\} \) be the set indices of countries with a fixed exchange rate and set \( e_h = e'_h \) for \( h \in H^f \). Then, with the set \( P \) as defined in Section 5, the set of prices becomes
\[ P^f = \{(p_0, p_1, \ldots, p_H, e) \in P | e_h = e_h^* \text{ for } h \in H^f\}. \]

We assume that the countries with fixed exchange rate are small compared with the rest of the world and the exchange rates do not differ too much from the equilibrium exchange rates \(^2\), so that they can meet their supplies and demands at these exchange rates. Then, extending the analysis given above, we obtain the following result.

**Result 6.1.** There exists a vector \( p^* = (p^*_0, p^*_1, \ldots, p^*_H, e^*) \in P^f \) such that,

- a) \[ \sum_h z^h_0((\pi^h p^*_0, p^*_h, e^*_h)) = 0, \]
- b) for all \( h, z^h_k(\pi^h(p^*_0, p^*_h, e^*_h)) = 0 \) \( k = 1, \ldots, n(h) \),
- c) for all \( h, e^*_h p^*_0 \tau z^h_0((\pi^h(p^*_0, p^*_h, e^*_h)) + m_h(p^*_0, p^*_1, e^*) = 0, \)
- d) \[ \sum_h p^*_0 \tau z^h_0((\pi^h(p^*_0, p^*_h, e^*_h)) = 0. \]

So, there exists a price vector \( p^* \in P^f \) with all non-money markets in equilibrium. Moreover, d) says that the total value of the surpluses and deficits measured in the international currency is equal to zero. However, the (domestic) money markets are not needed to be in equilibrium and they are the counterparts of the surpluses and deficits on the trade balances of the countries. Countries with a deficit on the trade balance have to reduce the holdings of domestic money in order to finance the deficit, whereas countries with a surplus have to expand their holdings of additional money. Moreover, the Central Banks of countries with a surplus get a claim on the Central Banks of countries with a deficit in terms of the international currency.

It should be observed that in general the "equilibrium" price vector \( p^* = (p^*_0, p^*_1, \ldots, p^*_H, e^*) \) is not unique. Clearly, if some \( h \notin H^f \) sets a fixed exchange \( e_h = e^*_h \neq e^*_h \), then there exists a new equilibrium price vector for this fixed exchanged rate. In general, a continuum of equilibrium price vectors will exist. As long as the number of countries not in \( H^f \) is greater or equal than two these countries have some possibility to change their exchange rates. For instance, the countries not in \( H^f \) can agree about linkages between their exchange rates. Suppose, they agree about a fixed ratio between their exchange rates, i.e., for \( h \notin H^f, e_h = ae^*_h \) for given \( e^*_h \). Under this policy the equilibrium price vector will be unique (or as usual in general equilibrium theory eventually an isolated odd number of price vectors).

In this paper we have shown the existence of equilibrium prices and exchange rates for a more-country international trade model. Moreover, we have seen

\(^2\) If the exchange rate is too low (almost zero) the demand goes to infinity and can not be met any longer.
above that under fixed exchange rates for some countries there exists an equilibrium price vector which clears the commodity markets but with non-zero trade balances. Result 6.1 shows that the countries not in $H^f$ have non-zero trade balances due to the fixed rate policies of the countries in $H^f$. In the static framework described in this paper zero trade balances can only be attained if the countries in $H^f$ give up their fixed exchange rate policies. So, the countries not in $H^f$ can not assure themselves of zero trade balances, unless they implement other measures, e.g. export restrictions in case that the countries in $H^f$ set relatively low exchange rates. This goes beyond the scope of this paper. However, we conclude with the observation that the countries not in $H^f$ does not need such restrictions if these countries prefer future consumption above current consumption. Because of the low exchange rates, in equilibrium the countries in $H^f$ will face deficits on their trade balances, i.e., they consume too many tradeables. In the short run the other countries suffer from this. However, as we have seen these countries get claims on the countries in $H^f$ in the form of reserves in international currency. So, the current overconsumption of the countries in $H^f$ goes at the expense of future consumption, while the countries not in $H^f$ give up a part of the current consumption in exchange for an additional claim on future consumption.

3) In a dynamic framework the money adjustments compensating the trade deficits and surplusses will restore zero trade balances in the long run. (see Dixit and Norman [6] and Neary [12]).
References


