This thesis is devoted to the analysis of mathematical models for flame balls. Flame balls are tiny (4mm), stable, stationary, spherically symmetric flames that occur in combustible gas mixtures (such as lean hydrogen-air mixtures), having low Lewis numbers. They are visible only in a micro-gravity environment. The Lewis number is a measure of the rate of diffusion of fuel into the flame ball relative to the rate of diffusion of heat away from the flame ball, and is an important parameter throughout this thesis. The reaction zone, where the fuel is burning, occurs at the boundary of the ball and is assumed to be thin. Through this flame sheet, fuel and oxygen diffuse inward while heat and products diffuse outward.

The main variables we are interested in to describe the behavior, are the temperature, the fuel mass fraction and the flame ball radius. The classical governing equations for combustion are rederived in Chapter 1 as a reaction-diffusion system (RDS). Since we assume the flame to occur in a narrow region, it is possible to reduce this RDS to a free boundary problem (FBP), the free boundary being the a priori unknown radius, which may vary in time. Going from one formulation to the other is a hard issue and requires involved asymptotic analysis. In this thesis, we will consider the FBP formulation.

A first mathematical model to describe flame balls was derived in 1944 by Zeldovich, and he showed that such flames were unstable, and hence certainly not observable. Nevertheless, forty years later, flame balls were experimentaly found by Ronney. Therefore, a physical mechanism for stabilising the flame balls had to be sought. It has been argued that radiative effects could strongly influence the stability properties of flame balls. One can give an heuristic argument explaining why radiation can stabilise flame balls. We remark first that the total heat release is proportional to the flame ball surface area and that the total radiative heat loss is proportional to the flame ball volume. Hence, if the flame radius is large, the surface area to volume ratio is small, thus the ratio of total heat release to total radiative heat loss is large, and the flame ball becomes weaker and shrinks. Conversely, if the radius is small, the flame ball grows stronger and expands. Hence, flame balls with volumetric heat losses could be stable to radial disturbances.

New models were then derived introducing heat loss terms in the temperature equation, where the power radiated by a body is proportional to the fourth power of the temperature (Stefan-Boltzmann law). The analysis
of such models leads to stability results.

In this thesis, we go one step further in describing the radiation effects. Indeed, radiation does not involve only emission of photons but also absorption. A more physical model of this phenomenon is described by the Eddington equation (radiative transfer equation) which we couple to the equations for the temperature and the fuel mass fraction. Radiation is even more important when the medium is filled with dust. The opacity of the medium is then another natural parameter to consider. The model and more physical explanations are discussed in Chapter 1.

Thus we are interested in studying the FBP involving the radiative transfer equation and, more specifically, we would like to prove stability results. Throughout this thesis, we suppose that flame balls are radially symmetric.

We start by proving the existence of stationary solutions. This is the objective of Chapter 2, in which we first prove that for each fixed radius, there exist a unique associated temperature profile if the reaction rate is not specified. The fuel mass fraction equation is decoupled from the system, and therefore we can compute explicitly its solution. The final step is to determine the radius of the flame given a specific temperature dependent reaction rate. In this way we can prove that there exist stationary solutions, and moreover, for a generic choice of the parameters, the number of solutions is odd. The existence of multiple solution is shown in various bifurcation diagrams.

In order to study stability properties, we are interested in perturbing slightly a fixed stationary solution. The perturbations that we consider in this thesis are radial (nonradial perturbations are part of work still in progress). For this purpose, we consider the FBP and linearise around a fixed stationary solution. It leads to a linear problem for which spectral properties have to be derived. To understand the “dynamics” of the eigenvalues considering different parameter values, we construct a special function known as the Evans function. It is an analytical function and its zeros are the eigenvalues. The analysis consists of two steps. We first consider the linearised Eddington equation and show that, for some specific range of the parameters, a branch of solutions of the bifurcation diagram is stable. This case is more accessible because we can derive explicit formulas. We then extend these results to the nonlinear Eddington equation (black body radiation model). In this case there are no explicit formulas and the computations are more involved but lead to similar stability results. Compared to the linear case, a smaller branch of the bifurcation diagram is stable under radial perturbations. We moreover show that stability depends strongly on the Lewis number. More precisely, if the ratio of radiative and thermal
fluxes is greater or equal to 1, then stability cannot occur. This analysis is performed in Chapter 5.

Chapter 3 is the most theoretical chapter of this thesis. We prove rigorously instability results, using semi-group techniques. Part of a branch of the bifurcation diagrams obtained in Chapter 2 corresponds to unstable solutions. The analysis performed in this chapter relates first the spectral stability to linear stability. Then, deriving proper estimates, we can deduce instability results from the previous linear analysis. Because the linearisation of the FBP leads to a fully nonlinear problem, it introduces theoretical difficulties. Indeed, instability results can still be proved in this setting, but stability issues are, on the theoretical level, an open problem.

Finally, Chapter 4 takes another approach. Considering the linearised Eddington law, under suitable assumptions, it is possible, from the initial FBP to derive an integro-differential equation describing the growth of the radius of the flame ball. The analysis of this model equation allows us to understand the behavior of flame balls for long times. After a mathematical analysis of this equation, we perform numerics and show that, when two steady flame balls exist, the smaller one is unstable while the larger one is stable.