High-Resolution Inelastic Electron Scattering and the Isoscalar Nature of the M1 Transitions to the Jπ = 1+ State at Ex = 5.846 MeV in 208Pb

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The relative weight of proton and neutron spin-flip contributions to the M1 excitation of the recently discovered Jπ = 1+ state at Ex = 5.846 MeV has been determined by comparison of the momentum-transfer dependence of the measured electron-scattering form factor (qem = 0.44±1.59 fm−1) to results from a simple two-state model and from random-phase-approximation calculations using a spin- and spin-isospin-dependent effective separable interaction. The M1 transition is shown to be predominantly of isoscalar nature.

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The discovery of the Jπ = 1+ state at Ex = 5.846 MeV in 208Pb in resonance fluorescence and (p,p') and (d,3He) experiments1−2 has given some momentum to the everlasting search3 for magnetic dipole ground-state transition strength in 208Pb. In this note we focus on the particular problem of the relative size of proton and neutron spin-flip contributions in the M1 transition from the ground state to the state at Ex = 5.846 MeV, i.e., whether this transition is of isoscalar or isovector character.

In order to state the problem, let us recall briefly that in 208Pb the independent-particle shell model3 predicts two (almost degenerate) configurations |πh11/2h9/2⟩ and |νi3/2i1/2⟩. Because of the residual interaction these result in two Jπ = 1+ states with wave functions

\[ |1^+⟩ = α |πh_{11/2}h_{9/2}⟩ - β |νi_{3/2}i_{1/2}⟩, \]

\[ |1^+⟩ = β |πh_{11/2}h_{9/2}⟩ + α |νi_{3/2}i_{1/2}⟩, \]

and \( α^2 + β^2 = 1 \). The first, Eq. (1a), should carry little excitation strength since proton and neutron spin-flip components interfere destructively. The transition leading into this state is therefore—in loose analogy to the classification in light nuclei—called isoscalar (ΔT = 0). Consequently, the transition into the second state, Eq. (1b), should be of isovector character (ΔT = 1) with proton and neutron components being in phase.4 We recall further that random-phase-approximation (RPA) predictions5−9 with pure central forces do not yield Jπ = 1+ states below an excitation energy Ex ≈ 7 MeV. There are, however, two predictions10,11 which produce a Jπ = 1+ state at Ex ≈ 5.5 MeV, i.e., fairly close to the location of the state detected in the recent experiments1,2. In both predictions the proton and neutron contributions interfere destructively pointing therefore to an isoscalar nature of the transition. The older prediction10 is based on the Tamm-Dancoff approximation (TDA) and yields B(M1) ≈ 0.12μ2T while the newer prediction11 hereafter called WJS, utilizes the RPA with tensor correlations from π and ρ exchange and results in B(M1) ≈ 0.4μ2T. A calculation in this model of WJS with a larger particle-hole (p-h) space than used in Ref. 11 yields Ex = 5.49 MeV but a larger B(M1) ≈ 0.77μ2T. The magnitudes of the form factor and hence of the transition strength reflect mainly the uncertainty of the tensor interaction. As Love et al.12 pointed out, π + ρ exchange gives a good description of the low-momentum behavior of a realistic tensor force deduced from phase shifts but systematically overestimates the high-momentum behavior. To ascertain the sensitivity the π + ρ interaction was reduced by 10% (keeping g0 = g6 = 0.6 fixed) resulting in a Jπ = 1+ state at Ex = 6.03 MeV, i.e., about 180 keV above the experimental value, with B(M1) ≈ 1.44μ2T.

Finally, in a recent more realistic two-state model than stated above, the importance of the mixing of...
isoscalar and isovector $M1$ excitation modes using an effective separable p-h interaction and the RPA has been pointed out.\textsuperscript{13} The calculation\textsuperscript{14} leads to a $J^π = 1^+$ state at $E_x = 5.82$ MeV and a transition strength of $B(M1) = 1.10 \mu_κ^2$. Note that about half of this strength is accounted for by the isoscalar-isovector mixing. One of us (J.W.) repeated the RPA calculation employing a large model space and a b force with a strength equivalent to the parameters of the separable interaction in Ref. 13. The resulting isoscalar strength of $1.08 \mu_κ^2$ and the excitation energy of $E_x = 5.87$ MeV are in very good agreement with those of the schematic model.\textsuperscript{13}

The low excitation energy of the experimentally found $J^π = 1^+$ state has been the first argument for the fact that we are probably dealing with an isoscalar mode. This is of course not a very strong argument; neither is the reasoning on the basis of the transition strength. The experimental strength\textsuperscript{1} \(B(M1) = 1.6 \pm 0.5 \mu_κ^2\) is about 30\% larger than Vergados's prediction.\textsuperscript{10} Considering the uncertainty of the latter due to the delicate coupling between the involved proton and neutron spin-flip contributions, and the fact that the admixture of 2\% lp-1h components to the wave function of this lowest $J^π = 1^+$ state has been essential in the description of the transition, we have to look for additional constraints. One of those is provided by the fact that the state is also seen in a (d,d') experiment.\textsuperscript{15} Another constraint could in principle be the observation of the $J^π = 1^+$ state at $E_x = 5.846$ MeV in the pickup experiment $^{209}$Bi($d,^3$He)$^{208}$Pb. The spectroscopic factor of the $|\pi_{\uparrow\uparrow}\rangle$ component found\textsuperscript{2,16} is larger than 0.75, leading to $\alpha > 0.87$ and hence essentially to the situation depicted in Eq. (1a) above. However, this large spectroscopic factor is clearly in contrast to another recent investigation\textsuperscript{17} of the reaction $^{209}$Bi($d,^3$He)$^{208}$Pb in which $\alpha < 0.5$ is deduced. Even if this problem of vastly different spectroscopic factors were discarded, the large value of $\alpha > 0.87$ would result in $B(M1) > 4.6 \mu_κ^2$, at variance with the experimental observation. Finally, our recent attempt\textsuperscript{18} to investigate the form factor of this transition in low-momentum-transfer inelastic electron scattering has been only partly successful. Although a description of the magnitude and the $q$ dependence of this transition has been consistent with an isoscalar interpretation, the possibility of a relatively weak isovector transition with a small lp-1h contribution due to the strong 2p-2h admixtures to the wave function could not be ruled out completely.

We therefore extended the previous ($e,e'$) experiment\textsuperscript{18} to higher momentum transfers at the new electron accelerator of NIKHEF-K at Amsterdam. A circular, 99\% enriched $^{208}$Pb (10 mg/cm$^2$) target of 45 mm diameter, rotating in the beam, has been exposed to electron beams of up to 30 $\mu$A intensity. Five spectra at $E_e = 76.8$, 90.8, 105.7, 119.2, and 137.7 MeV were taken at $\theta = 154^\circ$. The inelastically scattered electrons were detected with a quadrupole–double-dipole magnetic spectrometer operated in the energy-loss mode.\textsuperscript{19}

A low-energy spectrum\textsuperscript{18} and two spectra at the higher bombarding energies are shown in Fig. 1. The achieved high resolution in the NIKHEF-K measurement yielded an excellent signal-to-background ratio, making possible the evaluation of very small cross sections. (Note that the background due to the radiative tail has been subtracted only in the DALINAC spectrum.) The spectra were decomposed with the line shape of the elastic line and the position of known states as input parameters. The $J^π = 1^+$ state at $E_x = 5.846 \pm 0.005$ MeV can be analyzed without any difficulty in all spectra. Inelastic cross sections have been determined relative to simultaneously measured elastic ones (Table I).

We add here a remark on the analysis and results of the spectra taking, e.g., the outermost $n_{el} = 1.122$ fm$^{-1}$ (Fig. 1). Firstly, the spectrum might indicate that only three points lie above the background at the energy of the 5.846-MeV line. The original spectrum (in counts

![Graph](image-url)

**FIG. 1.** Three $^{208}$Pb($e,e'$) sample spectra. The low-energy spectrum (upper part) has been taken at Darmstadt, while the two spectra at higher energy are from Amsterdam. The $J^π = 1^+$ state at $E_x = 5.846$ MeV is indicated by an arrow.
TABLE I. Bombarding energy $E_0$, scattering angle $\theta$, effective momentum transfer $q_{\text{eff}}$, energy resolution $\Delta E_{1/2}$, and derived inelastic cross section with its experimental uncertainty. The lowest four energy spectra are from Darmstadt, the five spectra at higher energy from Amsterdam.

<table>
<thead>
<tr>
<th>$E_0$ [MeV]</th>
<th>$\theta$ [degrees]</th>
<th>$q_{\text{eff}}$ [fm$^{-1}$]</th>
<th>$\Delta E_{1/2}$ [keV]</th>
<th>$(d\sigma/d\Omega)_{\text{m}}$ [fm$^2$/sr]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.3</td>
<td>165</td>
<td>0.438</td>
<td>25.6</td>
<td>3.68$\times$10$^{-6}$</td>
<td>±18</td>
</tr>
<tr>
<td>36.4</td>
<td>165</td>
<td>0.582</td>
<td>28.7</td>
<td>4.43$\times$10$^{-7}$</td>
<td>±25</td>
</tr>
<tr>
<td>49.8</td>
<td>165</td>
<td>0.722</td>
<td>26.5</td>
<td>2.43$\times$10$^{-7}$</td>
<td>±55</td>
</tr>
<tr>
<td>61.2</td>
<td>165</td>
<td>0.840</td>
<td>43.2</td>
<td>2.05$\times$10$^{-7}$</td>
<td>±25</td>
</tr>
<tr>
<td>76.8</td>
<td>154</td>
<td>0.982</td>
<td>22.6</td>
<td>1.45$\times$10$^{-7}$</td>
<td>±40</td>
</tr>
<tr>
<td>90.8</td>
<td>154</td>
<td>1.122</td>
<td>24.4</td>
<td>2.44$\times$10$^{-8}$</td>
<td>±70</td>
</tr>
<tr>
<td>105.7</td>
<td>154</td>
<td>1.270</td>
<td>28.9</td>
<td>3.68$\times$10$^{-8}$</td>
<td>±60</td>
</tr>
<tr>
<td>119.2</td>
<td>154</td>
<td>1.404</td>
<td>30.7</td>
<td>1.81$\times$10$^{-8}$</td>
<td>±80</td>
</tr>
<tr>
<td>137.7</td>
<td>154</td>
<td>1.588</td>
<td>30.2</td>
<td>3.92$\times$10$^{-9}$</td>
<td>±190</td>
</tr>
</tbody>
</table>

*per channel and not per energy*, after being decomposed, shows, however, that the area of the line is made up by thirty counts yielding a statistical error of about 20%. Secondly, since the absolute energy scale is only determined by ± 5 keV, the position of the lines in the neighborhood were allowed to vary in the fit to yield a maximum and minimum value for the area of the 5.846 MeV-line. This resulted in an additional uncertainty and in the rather conservative errors quoted in Table I.

Three theoretical form-factor curves, calculated in distorted-wave Born approximation, are compared in Fig. 2 with the experimental data. (i) The RPA prediction using an equivalent interaction to the one employed in the isoscalar-isovector mixing model\textsuperscript{13} provides a very good description of the shape (except for one datum point at $q_{\text{eff}} = 1.270$ fm$^{-1}$ which lies outside of all model predictions) and magnitude of the measured form factor. The RPA calculation (WJS) for a pure isoscalar mode (not shown here) also describes the form factor rather well but it underestimates its magnitude by a factor of 1.4. A slight reduction of the $\pi + \rho$ exchange tensor force results in a somewhat poorer description and the experimental strength is overestimated by roughly 30%. It is, however, interesting to note here that for the first time the magnitude of the $M1$ transition strength is theoretically comparable to the experimental value. Recall that for isovector $M1$ transitions, a different behavior is found in medium heavy and heavy nuclei, i.e., those transitions are strongly quenched with respect to theoretical predictions.\textsuperscript{20} The transition strength deduced from the data with the help of the RPA prediction is $B(M1) = 1.01\mu_2$ with an uncertainty of about ±8% from the overall fit to the data.

(ii) The two-state model prediction for an isoscalar mode, Eq. (1a), with fitted coefficients $\alpha = 0.77$ and $\beta = -0.64$, also describes both the shape and the magnitude of the measured form factor very well (dashed curve). Note that these coefficients can also be reproduced within the isoscalar-isovector mixing model.\textsuperscript{13} (iii) In order to test if the $J^p = 1^+$ state could possibly be predominantly excited through an isovector mode [cf. Eq. (1b)], we assumed a mechanism\textsuperscript{18} whereby the $1^+$ state is strongly pushed down in energy by the in-

![FIG. 2. Comparison of the experimental form factor of the $J^p=1^+$ state with various theoretical predictions. The solid line shows a distorted-wave Born-approximation calculation using the RPA with a particular isoscalar-isovector interaction strength; the dashed and dash-dotted lines result from two-state model wave functions assuming a predominant isoscalar and isovector mode, respectively.](image-url)
teraction with many high-lying 2p-2h configurations via the tensor force. The result is $\beta = -0.18$ and $\alpha = 0.05$ and a form factor which fails completely to describe the experimental points at high momentum transfer (dash-dotted curve in Fig. 2). It is clear that the additional points at higher momentum transfer have been decisive in the question of the relative importance of proton and neutron amplitudes in the transition.

The slightly different behavior of the three different theoretical form factors at low momentum transfers yields for the photon point slightly different transition strengths. This is taken as a measure for the model dependence of the deduced transition strength in the present $(e,e')$ experiment. We obtain $B(M1) \Gamma = (1.01^{+0.42}_{-0.33}) \mu^2_B$ (statistical errors are not included) in reasonable agreement with the result from the resonance fluorescence experiment.

We conclude that the present results on the momentum-transfer dependence of the form factor of the $E_2 = 5.846$ MeV g.s. $M1$ transition determine this transition to be predominantly of isoscalar nature $[\alpha > 0.5, \beta < 0$ in Eq. (1a)]. The isovector interpretation is ruled out. It is interesting to note that recent $(p,p')$ experiments are only satisfactorily described with the isoscalar amplitudes determined from $(e,e')$ and hence support the findings of the present experiment.

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4Note, that the definition of the two-state model in Eqs. (1a) and (1b) is based on the RPA phase convention $[\alpha > 0$, $\beta > 0$] and is the same as in J. Wambach, A. D. Jackson, and J. Speth, Nucl. Phys. A348, 221 (1980). It differs from the usual definition where the isoscalar combination is given by $\alpha |\pi\rangle + \beta |\nu\rangle$ and the isovector one by $-\beta |\pi\rangle + \alpha |\nu\rangle$ with $\alpha, \beta > 0$, $|\pi\rangle$ and $|\nu\rangle$ being the proton and neutron spin-flip amplitudes, respectively.
11Wambach, Jackson, and Speth, Ref. 4.
14The slight difference in excitation energy and strength as compared to the corresponding values in Ref. 13 is due to a numerical error in the value for the isovector gyromagnetic factor $\mu^s$ in Ref. 13.