Double Crossing of the Ground Rotational Band and Super Band

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(Received 1 November 1977)

We suggest that the crossing of the ground rotational band by a second rotational-aligned band may in fact be a double crossing. From extrapolations of the rotational energy formula, we show that one could expect such a second crossing around $I = 30$ in $^{156}$Er. This provides an alternative explanation in terms of a double band crossing for the second discontinuity recently observed in the moment of inertia about $I = 28$ in $^{156}$Er.

The general understanding of the phenomena of backbending, where there occurs a sudden large increase in the moment of inertia, $\theta$, and reversal of the monotonic increase in energy level spacings, is that the ground band is crossed by an excited super band with a larger $\theta$ than that of the ground band. The various interpretations of the nature of the second bands have been discussed in several reviews.\(^1\) In the light-mass rare-earth nuclei, the rotational alignment effect\(^2\) involving the $\nu(i_{13/2})^2$ configuration most probably produces the backbending observed around $I = 14^+$ in the yrast cascades (see Stephens, Faessler, and co-workers\(^1\)).
For $I < I_{\text{cross}}$ the levels are considered to belong to the GRB (ground rotational band) and above $I_{\text{cross}}$ to the SB (super band). An intriguing question has been what happens for $I \approx 20$. Very recently Lee et al.\textsuperscript{4} have observed yrast states to spin 28\textsuperscript{4} and probably 32\textsuperscript{4} in 158Er. They observe a second discontinuity between spin 26 and 28. As possible explanations of this second discontinuity, they considered the sudden collapse of the pairing correlation.\textsuperscript{5} While this cannot be ruled out, they point out that calculations\textsuperscript{6,7} suggest that pairing effects do not cause sudden changes in $\delta$. Since the first backbending in this case is probably from the alignment of a pair of $i_{13/2}$ neutrons, they suggest that the more likely possibility is that at higher spin additional pairs of high-$j$ nucleons are aligned to cause the second discontinuity in 158Er. They suggest that the second pair would be additional $i_{13/2}$ neutrons or $h_{11/2}$ protons.

In this Letter we wish to point out another alternative explanation of the second discontinuity. As we shall see, if the SB is really based on an aligned two-particle (high $j$)\textsuperscript{8} configuration with $I_0 = 2j - 1 \gg 0$, then the SB should cross the GRB not once but twice.

The energy of the GRB levels can be described by the expansion

$$E_{\text{GRB}}(I) = \sum_{n=1}^{\infty} \alpha_n \omega_n^{2n}, \quad n = 1, 2, 3, \ldots$$

(1)

Generally a four-term expansion is used\textsuperscript{8}

$$E_{\text{GRB}}(I) = \alpha \omega_1^2 + \beta \omega_1^4 + \gamma \omega_1^6 + \delta \omega_1^8$$

(2)

with the parameters derived from the experimental energies for $I \leq 10$. In Fig. 1, it is seen that in a nucleus where no backbending or band crossing is observed,\textsuperscript{8,9} up to $I = 18$ in 176Hf and $I = 24$ in 238U, the formula nicely describes the experimental data. While it is not known to how high spin Eq. (2) may be useful for a rotational band, the general trends predicted by it may be qualitatively reasonably good up to even higher spins than 18 to 24 for the GRB's based on the 176Hf and 238U data.

We fitted Eq. (2) to the experimental data for $I \leq 10$ and calculated $E_{\text{GRB}}(I)$ for the GRB to high spin for comparison with the experimental yrast cascades in three nuclei with strong, medium, and weak backbending, 158Er, 158Dy, and 174Hf.\textsuperscript{10,11} The results are given in Table I, with the results for 158Er also shown in Fig. 2. In all three cases one finds that there is a double crossing of the GRB and the SB. Also one finds that the stronger

![FIG. 1. The experimental level energies as a function of spin for 176Hf and 238U, connected by a theoretical curve derived by fitting Eq. (2) to the experimental energies for $I \leq 10$ for 176Hf and $I \leq 12$ for 238U. The differences between the curves and the data points are $\approx 1$ keV for $I \leq 16$ and gradually increased to only 17 keV at $I = 24$ for 238U and are 2 and 10 keV at 16$^+$ and 18$^+$ in 176Hf. These differences are too small to be observed in the figure.](image-url)
TABLE I. The experimental yrast levels in $^{158}$Er, $^{158}$Dy, and $^{174}$Hf are given along with the calculated energies (Ref. 8) of the ground-state rotational band as obtained from Eq. (2) fitted to $I^* \leq 10^3$. All energies in keV.

<table>
<thead>
<tr>
<th>Spin</th>
<th>$^{158}$Er Levels</th>
<th>$^{158}$Dy Levels</th>
<th>$^{174}$Hf Levels</th>
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<td></td>
<td>Expt.</td>
<td>GRB$_{calc}$</td>
<td>Expt.</td>
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<tr>
<td>2</td>
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<td>529</td>
<td>317.5</td>
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<tr>
<td>6</td>
<td>970</td>
<td>972</td>
<td>637.6</td>
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<td>8</td>
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<td>1497</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>32</td>
<td>9899</td>
<td>9810</td>
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well. More recently we have observed the GRB above the crossing by two rotational aligned bands in $^{68}$Ge and the $8^+$ and $(10^+)$ GRB states are nicely predicted by fits of Eq. (2) to the states up to spin $6^+$. Since this paper was submitted, other new data have become available which clearly support that the expansion is good above the crossing. In neighboring $^{84}$Er, Johnson et al. observe states above and below the crossing of the ground band and super band at $I=16$. Fitting the $I \leq 10$ states to Eq. (2), we find that the fitted energies agree exactly for $I=10$ and are as follows above that with the fitted energies in parentheses: $12^+$, 2083.8 (2083.9); $14^+$, 2702.6 (2708.1); $16^+$, 3411.2 (3383.7); $18^+$, 4121.2 (4102.0); $20^+$, 4868.4 (4856.7); $22^+$, 5651.5 (5642.7) keV.

There is a jump of 27.5 keV in the experimental energy over the fitted one at the crossing point where mixing would push up this level. After that the experimental levels are similarly 10–20 keV higher than the fit as may be expected from the pushing up of the $16^+$ level. Thus, the expansion works for the next three states above the first crossing point to offer strong support to our use of it above the first crossing in $^{158}$Er.

The reason for the prediction of a second crossing of the GRB and SB is that at high $I$ the $\gamma$-ray energies, $E_\gamma$, for the GRB based on Eq. (2) do not increase as fast as $E_\gamma$ for the same $I$. After $I_{GRB}$, $E_\gamma (I) > E_\gamma (20^+)$ (because $\theta_{SB} > \theta_{GRB}$), but at some $I_{GRB}$ again (this can lead to $\theta_{GRB} > \theta_{SB}$, which is possible because $\theta_{eff}$ is not a real moment of inertia, and there are many reasons to decrease $E_\gamma$ in addition to a real increase in the moment of inertia.) In $^{158}$Er

![Graph](image-url)

FIG. 2. The experimental yrast level energies for $^{158}$Er. The GRB is crossed about $I=12$ by SB. A fit of the experimental levels with $I=10$ to Eq. (2) yielded the theoretical curve for the GRB shown as a dashed curve. The experimental yrast curve is crossed a second time by the calculated GRB curve about $I=30$. 746
up to \( I = 32 \), \( \theta_{\text{eff}} \) is always less than \( \theta_{\text{rot}} \). However, if the SB is a rotational aligned one, then the above behavior is expected because of the large difference in the collective (rotational) angular momentum of the states of the same \( I \) in each band; e.g., for the GRB, \( R = I \) but for an aligned SB, \( R = I - I_0 \) with \( I_0 = 8 - 12 \) so that the correction \( \Delta E \) to \( E_{\text{GRB}}(I) \), \( \Delta E = \gamma \omega R^6 + \delta \omega R^8 \) (which experimentally is found to be negative), is much larger for the GRB levels than that for the SB levels at a given \( I \) even though this equation should not strictly hold for the SB which is not a pure \( K = 0 \) band. Thus for a rotational aligned SB, \( E_{\gamma}^{\text{GR}} \) can increase more strongly with \( I \) at high spin than \( E_{\gamma}^{\text{GRB}} \) and lead to a double crossing of the two bands. For \(^{159}\)Er, one can turn the argument around to say the following: If the observed second discontinuity in the yrast cascade for \(^{159}\)Er is a result of a second crossing of the GRB and SB, then the SB is a rotational aligned band. Of course, perturbation of the GRB by the SB should occur to some degree and may be responsible for the difference between the expected second backbending about \( I = 28 \) and our prediction at \( I = 32 \). In any case the above arguments indicate that for weak backbending, when a rotational aligned band crosses the GRB, one should expect that the states above the crossing will be mixed for many spins, and even for strong backbending, the states in the two bands should come back together so that mixtures of these two bands may occur again at higher spin. Our proposed second crossing of the GRB and SB levels can be tested by experimentally seeking to follow the GRB to levels with spins above the first crossing to see how the energies of these levels compare with those predicted by Eq. (2) for \(^{159}\)Er.

This work was supported in part by a grant from the U. S. Energy Research and Development Administration.

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14I. N. Mikhailov, K. Neergard, V. V. Paechkevich, and S. Frauendorf, to be published.
