In this paper we will be mainly concerned with the longitudinal quark spin distribution \( \lambda(x, k_T) \) and the two DF’s \( g_2^q(x, k_T^2) \) and \( g_1^q(x, k_T^2) \) describing it. Following Ref. [3], we first consider the polarized SIDIS in the simple quark-parton model. We will use the standard notation for DIS variables: \( l \) and \( l' \) are the momenta of the initial and the final state lepton; \( q = l - l' \) is the exchanged virtual photon momentum; \( P \) (\( M \)) is the target nucleon momentum (mass), \( S \) its spin; \( P_h \) is the final hadron momentum; \( Q^2 = -q^2 \); \( s = Q^2/xy \); \( x = Q^2/2P \cdot q \); \( y = P \cdot q/P \cdot l \); \( z = P \cdot P_h/\rho \cdot q \). The reference frame is defined with the \( z \) axis along the virtual photon momentum direction (antiparallel) and \( x \) axis in the lepton scattering plane, with positive direction chosen along lepton transverse momentum. Azimuthal angles of the produced hadron, \( \phi_h \), and of the nucleon spin, \( \phi_s \), are counted around \( z \) axis (for more details see Refs. [3] or [8]). In this paper as independent azimuthal angles we will choose

Longitudinal quark polarization in transversely polarized nucleons

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Accounting for transverse momenta of the quarks, a longitudinal quark spin asymmetry exists in a transversely polarized nucleon target. The relevant leading quark distribution \( g_1^q(x, k_T^2) \) can be measured in semi-inclusive deep-inelastic scattering. The average \( k_T^2 \) weighted distribution function \( g_1^q(x, k_T^2) \) can be obtained directly from the inclusive measurement of \( g_2 \). [0556-2821(96)03813-1]

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Intrinsic transverse momentum (\( k_T \)) plays an important role in the quark distribution functions (DF’s) used to describe a polarized nucleon [1,2]. For the leading (twist-two) part of the deep inelastic scattering cross section one already needs six DF’s to describe the quark state in a polarized nucleon. They depend on \( x \) and \( k_T^2 \), which parametrize the quark momentum in a nucleon with momentum \( P \), \( k = xP + k_T \). We will adopt the notation of Ref. [2] for these “new” six independent DF’s: \( f_1^q \), \( g_{1L}^q \), \( g_{1T}^q \), \( h_1^q \), \( h_{1L}^q \), and \( h_{1T}^q \) (\( q \) denotes the quark flavor). For a polarized nucleon the spin vector is written as \( S_N = \lambda P/M + S_T \), satisfying \( \lambda^2 - S_T^2 = 1 \). The probability \( \mathcal{P}_N(x, k_T^2) \), the longitudinal spin distribution \( \lambda(x, k_T) \), and the transverse spin distributions \( s_{1L}^q(x, k_T) \) of the quark in a polarized nucleon are given by

\[
\mathcal{P}_N(x, k_T^2) = f_1^q(x, k_T^2),
\]

\[
\mathcal{P}_N^q(x, k_T^2) \lambda^q(x, k_T) = g_{1L}^q(x, k_T^2) \lambda - g_{1T}^q(x, k_T^2) S_T - \frac{k_T S_T}{M},
\]

\[
\mathcal{P}_N^q(x, k_T^2) s_{1L}^q(x, k_T) = h_{1L}^q(x, k_T^2) S_T + \left[ h_{1T}^q(x, k_T^2) - h_{1L}^q(x, k_T^2) \right] \frac{k_T S_T}{M}.
\]

These DF’s have a clear physical interpretation: For example, \( g_{1L}^q \) describes the quark longitudinal polarization in a transversely polarized nucleon. Such a polarization can be non-vanishing only if the quark transverse momentum is nonzero. This DF cannot be measured in deep-inelastic scattering (DIS) at leading order in \( 1/Q \). It can be measured in polarized semi-inclusive deep-inelastic scattering (SIDIS) as first shown in [3], where it appears as an azimuthal asymmetry. Measurements of the other “new” DF’s were proposed in the doubly-polarized Drell-Yan process [1,2], and in the polarized SIDIS [3,4] using the so-called Collins effect [5].

The quark fragmentation is described by two fragmentation functions (FF’s): spin-independent and transverse-spin-dependent ones.

The “ordinary” \( f_1^q(x) \), \( g_{1L}^q(x) \), and \( h_1^q(x) \), and the “new” leading-twist DF’s are related by \( k_T \) integration:

\[
f_1^q(x) = \int d^2k_T f_1^q(x, k_T^2),
\]

\[
g_{1L}^q(x) = \int d^2k_T g_{1L}^q(x, k_T^2),
\]

\[
h_1^q(x) = \int d^2k_T \left[ h_{1T}^q(x, k_T^2) - \frac{k_T^2}{2M^2} h_{1L}^q(x, k_T^2) \right].
\]
The phase space element in the transverse direction is $d^2 P_{\perp} = |P_{\perp}|d|P_{\perp}|d\phi_5$. Integrating $A_{LT}$ over $\phi_5$ we are left with the contribution proportional to $\mathcal{H}_{S_{1L}}^{0}$:

$$
\int \frac{d\phi_5^S}{2\pi} A_{LT} = D(y) \frac{\mathcal{H}_{S_{1L}}^{0}}{\mathcal{H}_{f_1}^{0}}.
$$

One can also define the asymmetry

$$
\langle A_{LL}(x,y,z) \rangle = \frac{\int d^2 P_{\perp} (d\sigma^- - d\sigma^+) \int d^2 P_{\perp} (d\sigma^+ + d\sigma^-)}{\int d^2 P_{\perp} (d\sigma^+ + d\sigma^-)} = D(y) \sum_q e_q^2 g_{1T}^q(x) D_h^b(z) \sum_q e_q^2 f_1^q(x) D_h^p(z).
$$

This asymmetry was measured by the Spin Muon Collaboration (SMC) [9] and provides the flavor analysis of the quark longitudinal-spin DF’s in a longitudinally polarized nucleon [10,11]. The future measurements are planned by the HERMES [12] and the HMC [13] Collaborations.

The target-transverse-spin asymmetry is given by

$$
A_{LT}(x,y,z,P_{\perp},\phi_5^S) = D(y) \cos \phi_5^S \frac{\mathcal{H}_{S_{1L}}^{0}}{\mathcal{H}_{f_1}^{0}},
$$

and can in principle be disentangled measuring the asymmetry at different values of $\phi_5^S$ and performing a Fourier analysis. For example, let us integrate Eq. (16) weighted by $\cos \phi_5^S$ over $\phi_5^S$. We obtain

$$
\int 2^0 d\phi_5^S \frac{1}{2\pi} \cos \phi_5^S A_{LT}(x,y,z,P_{\perp},\phi_5^S) = D(y) \frac{\mathcal{H}_{S_{1L}}^{0}}{\mathcal{H}_{f_1}^{0}}.
$$

It is useful to define the transverse-spin asymmetry weighted with $S_{LT} \cdot P_{\perp} / M = (|P_{\perp}| / M) \cos \phi_5^S$:

$$
\langle \frac{|P_{\perp}|}{M} \cos \phi_5^S A_{LT} \rangle (x,y,z)
$$

$$
\langle \frac{|P_{\perp}|}{M} \cos \phi_5^S \rangle (x,y,z)
$$

$$
\int d^2 P_{\perp} \frac{|P_{\perp}|}{M} \cos \phi_5^S (d\sigma^1 - d\sigma^-) = \frac{z D(y) \sum_q e_q^2 g_{1T}^q(x) D_h^b(z)}{\sum_q e_q^2 f_1^q(x) D_h^p(z)}.
$$

where

$$
g_{1T}^{q(1)}(x) = \int d^2 k_T \frac{k_T}{2M} g_{1T}^q(x,k_T^2).
$$

Note that relations (15) and (18) are valid for any $k_T$ dependence of DF’s and FF’s. In principle, it is possible to separate contributions from different quark flavors by measuring the asymmetry (18) for different produced hadrons in the same way as proposed in [10,11].

Next we turn to a quantitative estimates of the asymmetries, starting with the longitudinal asymmetry. For this we
consider the production of \( \pi^+ \) meson on the proton. The dominant contribution will come from scattering on the \( u \) quark. In order to estimate \( A_L \) we use the parametrization of Brodsky, Burkardt, and Schmidt (BBS) for \( g_1 \) and \( f_1 \). The result is shown in Fig. 1.

For an estimate of the transverse asymmetry we need the DF's \( g_1 T q(x,k_T^2) \). In contrast with the \( k_T \)-integrated DF's \( f_1 q(x) \) and \( g_1 q(x) \) there are no measurements of the function \( g_1 T q(x,k_T^2) \). As shown in Refs. \([6,8]\) the \( (k_T^2/2M^2) \)-weighted \( k_T \)-integrated function \( g_1^{(1)}T(x) \), which appears in Eq. (18), is directly related to the DF \( g_2^q(x) \):

\[
g_2^q(x) = \frac{d}{dx} g_1^{(1)}T(x). \tag{21}
\]

This relation just follows from constraints imposed by Lorentz invariance on the antiquark-target forward scattering amplitude and the use of QCD equations of motion for quark fields. We will use this relation for our quantitative estimates. We note that the effects of higher-order QCD corrections for the transverse-momentum-dependent functions, however, require further investigation \([15]\). For the function \( g_2 \) the QCD corrections have been extensively studied \([16]\).

In our first estimate for the transverse asymmetry \([\text{Eq. (18)}]\) we use recent data on \( g_2 \) and the relation in Eq. (21). Such data are available from the SMC Collaboration \([19]\) and the E143 Collaboration at SLAC \([20]\). Particularly the latter data allow a rough estimate of the function

\[
g_1^{(1)}T(x) \approx \frac{1}{D(y)} \langle A_{LL} \rangle(x,y,z) \approx \frac{g_1^q(x)}{f_1^q(x)} \tag{20}
\]

The result obtained by averaging the two sets of data at different angles and adding statistical and systematic errors quadratically is shown in Fig. 2.

Our second estimate for this distribution function comes from the representation for \( g_2^q(x) \) in terms of other \( k_T \)-integrated functions. For \( g_2^q = g_2^1 + g_2^2 \) one has

\[
g_1^{(1)}T(x) = \frac{1}{2} \sum_q e_q^2 g_1^{(1)}T(x) = \int d^2 k_T \frac{k_T^2}{2M^2} g_1T(x,k_T^2)
\]

\[
= - \int_x^1 dy g_2(y). \tag{22}
\]

The result obtained by averaging the two sets of data at different angles and adding statistical and systematic errors quadratically is shown in Fig. 2.
\[ g_T^q(x) = \int_x^1 dy \frac{g_q^\parallel(y)}{y} + \frac{m_q}{M} \left[ h_1^q(x) - \int_x^1 dy \frac{h_1^q(y)}{y^2} \right] + \frac{g_0^\parallel(y)}{y}, \]  

where \( m_q \) is the quark mass, \( \overline{g}_T^q \) is the so-called interaction-dependent part of the DF \( g_T^q(x) \). The term \( (m_q/Mx)h_1^q(x) \) on the right-hand side (RHS) of Eq. (23) was found many years ago by Feynman [17] and represents the contribution of the transverse spin distribution to \( g_T(x) \). The most well-known contribution in Eq. (23) is the first term found by Wandzura and Wilczek [18]. Using Eq. (21) an estimate of \( g_{1T}^{1(1)}(x) \) is obtained from Eq. (23), keeping only the first term (Wandzura-Wilczek) as this does not contradict the data. In that case \( g_T(x) = g_T^{WW}(x) \) where

\[ g_T^{WW}(x) = g_1(x) + g_2^{WW}(x) = \int_x^1 dy \frac{g_1(y)}{y}, \]  

leading to

\[ g_{1T}^{1(1)WW}(x) = - \int_x^1 dy g_2^{WW}(y) = x g_T^{WW}(x). \]  

We use the parametrization of DF’s from Ref. [14]. The result is shown as the curve in Fig. 2. Using other parametrizations for \( g_1 \) does not substantially change this result.

Assuming the \( u \)-quark dominance for the \( \pi^+ \) production on the proton, the estimate for the transverse spin asymmetry,

\[ \frac{1}{zD(y)} \left( \frac{P_{h/L}}{M} \cos \phi_T \right)_{(x,y,z)} \approx g_{1T}^{u(1)}(x), \]  

can be obtained (see Fig. 3). In this paper we have considered the azimuthal asymmetry in one-particle inclusive polarized lepton production. The longitudinal spin asymmetry averaged over the transverse momenta of the produced hadrons gives independent ways to study the polarized quark distributions as has been pointed out before [10,11]. As we have shown the transverse-spin asymmetry provides information on the quark-longitudinal spin distribution in a transversely polarized target, the DF \( g_{1T}^{1}(x,k_T^2) \). This information appears in a \( \cos(\phi_T - \phi_h) \) asymmetry for the produced particles. A Fourier analysis of this asymmetry weighted with the modulus of the transverse momentum of produced particles gives the \( k_T^2 \)-integrated \( \frac{k_T^2}{2M^2} \)-weighted function \( g_{1T}^{u(1)}(x) \) which is at the tree level directly related to \( g_1^Q(x) \). This provides an alternate way of obtaining the latter DF, although a careful analysis of the QCD corrections is needed.

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