Summary

Title of the thesis: Rational Systems in Control and System Theory

In this thesis an algebraic approach to realization theory for the class of rational systems is presented. The results are applied in system identification and generalized for the class of so-called Nash systems.

Rational systems are dynamical systems whose dynamics and output functions are determined by rational functions. They are widely used as models of phenomena in life sciences, economy, physics, and engineering. The framework and motivation to study rational systems are presented in Chapter **??**. The notions of algebraic reachability and of algebraic/rational observability are introduced. For algebraic reachability of rational systems we provide a characterization in terms of polynomial ideals satisfying certain conditions. Both concepts, of reachability and of observability, are related to different notions of controllability, accessibility and observability of linear and nonlinear systems.

Realization theory is one of the central topics of control and system theory. Restricted to rational systems, it deals with the characterization of all rational systems which have a specified input-output behavior. Apart from the existence issues, the realization problem concerns properties of rational realizations such as canonicity and minimality, relations between different rational realizations of the same map, algorithms and procedures for constructing rational realization of desired properties. Furthermore, realization theory serves as a theoretical foundation for model reduction, system identification and control/observer design.

We deal with the realization problem for rational systems in Chapter **??**. We derive necessary and sufficient conditions for a response map to be realizable by a rational system. The characterization of the existence of rationally observable, canonical, and minimal rational realizations for a given response map is provided as well. We relate minimality of rational realizations to their rational observability, algebraic reachability, and canonicity. The relations between birational equivalence of rational realizations and their canonicity and minimality properties are determined. Namely, we show that all canonical rational realizations of the same response map

are birationally equivalent, and that birational equivalence preserves minimality of rational realizations.

In Chapter **??** we investigate realization theory of Nash systems. In particular, we introduce the class of Nash systems and then formulate and partially solve the realization problem for them. In analogy with results of Chapter **??** we derive necessary and sufficient conditions for the existence of Nash realizations of a response map. Further, the concepts of semi-algebraic observability and semi-algebraic reachability of Nash realizations are defined and their relationship with minimality is explained.

The problems of system identification deal with modeling a phenomenon based on the observed measurements. This involves the selection of a model structure, experimental design, identifiability analysis, parameter estimation and evaluation methods. In this thesis we consider only the identifiability problem for the deterministic classes of polynomial and rational systems and for noise-free data. Namely, in Chapter **??** we provide the characterization of structural and global identifiability of parametrizations of parametrized polynomial and parametrized rational systems. The corresponding method for checking identifiability is employed to investigate identifiability properties of systems modeling certain biological phenomena. Identifiability of a parametrization is a necessary condition for the uniqueness of parameter values determining a model fitting measurements. Without the existence of a unique solution to the parameter estimation problem it could happen that the methods for estimating parameters will never find the true values of the parameters. Therefore, verification of identifiability of a parametrization precedes estimation of numerical values of parameters, and thus formulation of a fully specified model of a phenomenon.

The thesis is concluded by Chapter **??** which provides directions for further research.